Atmospheric Thermodynamics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$pV = NkT = \nu R^* T, \tag{1}$$

N is the number of molecules.

$$p = \rho R_m T. \tag{2}$$

First Law:

$$dq = c_p dT - v dp, \tag{3}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{5}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface assuming L_v is constant:

$$e_s = e_{s0}\exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \tag{6}$$

Rate of change of e_s as a function of T:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{7}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{8}$$

Equivalent Potential temperature:

$$\theta_e = \theta\exp\left(\frac{L_v r_v}{c_p T}\right) \tag{9}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{10}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{11}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s \frac{4217}{(T - 32.19)^2} \tag{12}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{13}$$

Measures of water vapour, mass mixing ratio:

$$r_v = \frac{R_d e}{R_v(p - e)} = \epsilon e/(p - e). \tag{14}$$

Virtual temperature:

$$T_v \equiv T\left(\frac{1 + \frac{r_v}{\epsilon}}{1 + r_v}\right) \tag{15}$$

Speed of sound c in an inviscid fluid:

$$c = \sqrt{\frac{C_p}{C_v} RT} \tag{16}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets neglecting the aerosol and curvature effects:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S - 1) \tag{17}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \approx e_s(\infty)\left(1 + \frac{a}{rT} - \frac{b}{r^3}\right) \tag{18}$$

Radiation

The Planck Function:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{\frac{ch}{k\lambda T}} - 1)} \tag{19}$$

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{20}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{21}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{22}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{23}$$

Chapter 1

Tables

Table 1.1: Table of thermodynamical constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Specific heat capacity of water	c_p	4185	J kg^{-1} K^{-1}
Specific heat capacity of sea water	c_p	$\approx$ 3985	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Saturation vapour pressure at $T_0 = 0^o$C	e_{s0}	611.2	Pa
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Surface tension of liquid water	$\sigma_{l,v}$	7.5×10^{-2}	Nm^{-1}

Table 1.2: Table of radiation constants

Planetary albedo of Earth	α_p	0.3	
Planetary albedo of Mercury	α_p	0.07	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between Earth and the Sun	r_d	149.6×10^6	km
distance between Mercury and the Sun	r_d	58×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Questions

1. Convection

   Using the tephigram profile

   i (1pt) What is the dew point temperature of the surface air? (1 pt)

   ii (2pts) A surface parcel of air on the tephigram is lifted until it reaches the level of free convection (LFC), after which point it undergoes free convection until it reaches the level of neutral bouyancy (LNB).

   Draw the trajectories that marks the properties of temperature and humidity of the updraft air parcel on the tephigram

   iii (3pt)

   Now using this trajectory, determine the pressure of the

   a. Lifting condensation level (LCL)

   b. Level of free convection (LFC)

   c. Level of neutral buoyancy (LNB)

   iv (4pts)

   At 600 hPa, equal masses of cloudy air and environment air mix. The parcel is then brought to saturation by the evaporation of precipitation. What is the final temperature of the air parcel estimated from the tehpigraph (mark the point on the tephigram)? Can the parcel form a saturated convective downdraft?

   v (2pts) What are the underlying orthogonal axes of a tephigram?

   vi (3pt)

   In the United states, instead of a tephigram, it is more common to use a "skewT logp" diagram. As the name suggests, this diagram is based a grid of Temperature lines (rotated, hence "skew") and lines of the logarithm of pressure (the log p part). Thus in these diagrams, temperature and pressure are straight lines, and dry adiabats are curved!

   Using any of the equations given at the start of the exam paper, show that a "skewT logp" diagram is a true thermodynamical diagram, that is, that the area enclosed by a closed path in the chart is equivalent to proportional to the work done by/on a parcel following that trajectory.

2. thermo

   i (5pt) I'm fed up with the ICTP mensa food and so I decided to run up to the guesthouse and cook myself some pasta for lunch.

I'm in a hurry as I only have an hour before the next diploma lecture starts, so I make sure I put a lid on the saucepan of water to bring it to the boil more quickly. Explain TWO key ways in which placing a lid over the saucepan helps it to come to the boil much more quickly.

ii (3pt) One of the most annoying things about wearing a mask during the pandemic has been the fact that my glasses are always steaming up as I breathe. Trying to keep you answer concise, please explain the physics behind this phenomenon.

iii (4pt) What are the weather conditions (i.e. the parameters of the atmosphere) that will make the steaming up worse and WHY! (you will only get the marks if you explain the reasoning behind your answer)

3. Clouds

   A cloud exists in the lower troposphere (e.g. ignore ice processes) with a liquid water content of L (units kg m^{-3}), a physical depth z (units m), droplet number concentration of N (units m^{-3}), mean droplet radius $\overline{r}$ (units m), standard deviation of the droplet radius distribution $\sigma(r)$ (units m), and a vertically integrated liquid water path of W (units kg m^{-2}). Describe **how** the following specific changes would change the precipitation rate [increase, no change, decrease] generated from the cloud and why. [Note, if a variable is not mentioned in the question, it may or may not change]. If you do not specify the physical mechanism, no marks will be awarded for getting the precipitation change correct!

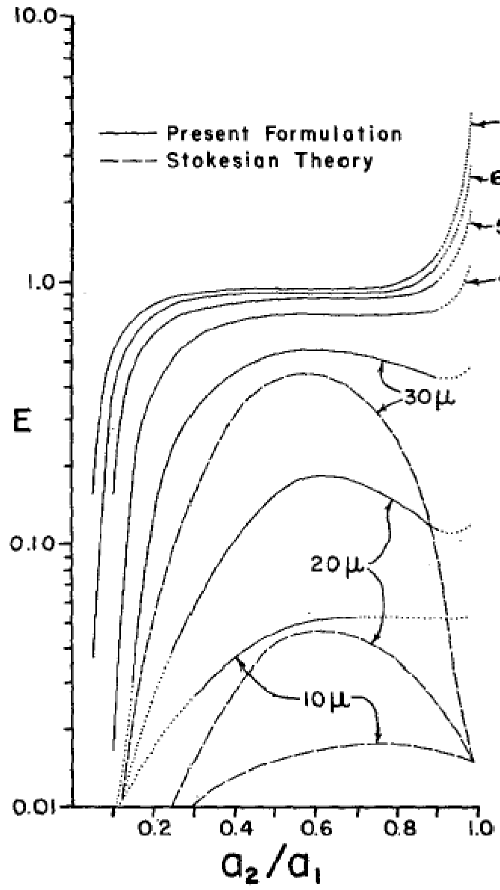
   i (2pts) Increasing the cloud physical depth z, keeping L, $\overline{r}$ and N constant (in this case W must increase, for example).

   ii (2pt) Increasing N, keeping L and z constant.

   iii (2pts) Increasing the cloud physical depth z, keeping W and $\overline{r}$ fixed.

   iv (2pts) Increasing $\sigma(r)$, keeping N, $\overline{r}$, L, z and W fixed.

4. The graph shows calculations of collection efficiency (using two different methods, the "present method" derived in the paper, and the classical Stokesian method) when a droplet of radius a_1 falls through a cloud consisting of drops of radius a_2.



i (2pt) Explain why (use a sketch if helpful) when the largest droplet a_1 has a radius of 10 μm, the efficiency calculated is considerably below 1

ii (2pt) Explain how it is possible to have collection efficiencies that **exceed** unity (upper right of plot!)

5. Microphysics

i (2pt) A chance collision of 250 water vapour molecules forms a liquid droplet. What is the radius of the droplet?

ii (2pt) In order to form a stable cloud droplet from the chance collisions of water vapour molecules, very high relative humidities are required of several hundred percent. Explain why such high relative humidities are **not** observed in the atmosphere at the surface.

iii (2pt) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water (of infinite radius, so we write $e_s(\infty)$). But in fact the saturation vapour over a water droplet of radius r and with a mass M of dissolved impurity is modified by two factors $\left(1 - \frac{b}{r^3}\right)$ and $\exp\left(\frac{a}{rT}\right)$, where a and b are constants, with b depending on the mass and type of aerosol. In a few sentences, explain what the two correction factors refer to (use a sketch if helpful).

iv (3pt) A solute liquid droplet with a radius of 0.05 micron (μm) is present in an environment with a relative humidity of 95% and a temperature of $T = 273K$. Calculate the saturation ratio with respect to the droplet. Will the droplet grow or shrink?

v (4pt) If the constants for the correction factors for a given solute haze particle are $a = 3.3 \times 10^{-7}$ m K and $b = 1.47 \times 10^{-23}$ m^3 and the haze particle resides in air at a temperature of 273 K. What is the radius AND relative humidity at which the haze particle becomes activated?

vi Another droplet is instead formed by heterogeneous nucleation on a wettable but **insoluble** [this word "insoluble" is important!] CCN in an environment that is 0.4% supersaturated (i.e. S=1.004) at a temperature T=273 K. What is the minimum aerosol radius for this to occur?