Atmospheric Thermodynamics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$pV = NkT = \nu R^* T, \tag{1}$$

N is the number of molecules.

$$p = \rho R_m T. \tag{2}$$

First Law:

$$dq = c_p dT - v dp, \tag{3}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{5}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface assuming L_v is constant:

$$e_s = e_{s0}\exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \tag{6}$$

Rate of change of e_s as a function of T:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{7}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{8}$$

Equivalent Potential temperature:

$$\theta_e = \theta\exp\left(\frac{L_v r_v}{c_p T}\right) \tag{9}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{10}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{11}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T - 32.19)^2} \tag{12}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{13}$$

Measures of water vapour, mass mixing ratio:

$$r_v = \frac{R_d e}{R_v(p - e)} = \epsilon e/(p - e). \tag{14}$$

Virtual temperature:

$$T_v \equiv T\left(\frac{1 + \frac{r_v}{\epsilon}}{1 + r_v}\right) \tag{15}$$

Speed of sound c in an inviscid fluid:

$$c = \sqrt{\frac{C_p}{C_v}RT} \tag{16}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets neglecting the aerosol and curvature effects:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S - 1) \tag{17}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \approx e_s(\infty)\left(1 + \frac{a}{rT} - \frac{b}{r^3}\right) \tag{18}$$

Radiation

The Planck Function:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{\frac{ch}{k\lambda T}} - 1)} \tag{19}$$

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{20}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{21}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{22}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{23}$$

Chapter 1

Tables

Table 1.1: Table of thermodynamical constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Specific heat capacity of water	c_p	4185	J kg^{-1} K^{-1}
Specific heat capacity of sea water	c_p	≈ 3985	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Saturation vapour pressure at $T_0 = 0^o$C	e_{s0}	611.2	Pa
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Surface tension of liquid water	$\sigma_{l,v}$	7.5×10^{-2}	Nm^{-1}

Table 1.2: Table of radiation constants

Planetary albedo of Earth	α_p	0.3	
Planetary albedo of Mercury	α_p	0.07	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between Earth and the Sun	r_d	149.6×10^6	km
distance between Mercury and the Sun	r_d	58×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Questions

1. TEPHIGRAM: Global Warming in the Tropics. In this question use your tephigram, if you need to show any working please mark it on the bottom or back of the tephugram sheet, and do not forget to write your name on the top.

    i (2pts) In the Tropics the air at 1000hPa is measured to have a temperature of 25^oC and a relative humidity (RH) of 80%. Plot these data on the tephigram and state the value of mixing ratio of the air?

    ii (2pts) A parcel of air from 1000hPa is then lifted and expands and cools in an adiabatic process. State in words what an adiabatic process is

    iii (1pt) Use the tephigram to find out the lifting condensation level of the parcel and give the result

    iv (1pt) After the lifting condensation level the parcel is lifted in saturated ascent until it reaches 300hPa. What is the temperature at 300hPa

    v (4pt) Global warming results in the mean temperature at 1000hPa to warm by $+3^o$C. Assuming RH does not change (i.e. still 80%), plot the new profile of a lifted surface (1000 hPa) parcel in this warmed world. What is the new temperature at 300hPa?

    vi (1pt) Is the difference at 300hPa greater or less than the 3C warming at 1000hPa? This change in warming with height is known as the lapse rate feedback and will be discussed in the term 2 course.

2. thermo

    i (3pt) One of the most annoying things about wearing a mask during the pandemic has been the fact that my glasses are always steaming up as I breathe. Trying to keep you answer concise, please explain the physics behind this phenomenon.

    ii (4pt) What are the weather conditions (i.e. the parameters of the atmosphere) that will make the steaming up worse and WHY! (you will only get the marks if you explain the reasoning behind your answer)

3. Radiation

    i (6pts)

Derive a simple model of radiative transfer through the atmosphere with the following assumptions:

1) the earth's surface is a black body 2) the atmosphere is divided into TWO layers, both assumed to be BLACK BODIES in the infra-red but TRANSPARENT to solar radiation

What is the surface temperature if the system is in equilibrium? Is this higher or lower than today's mean surface temperature and explain why?

ii (4 pts)

Now assume that of the net TOA solar flux $0.25S_0(1-\alpha_p)$, 10% is absorbed in the first layer, and 10% is absorbed in the second layer and 80% is absorbed at the surface. How does this change the surface equilibrium temperature?

4. i (2pt) The equation for the Gibbs free energy for a cluster formation is:

$$\Delta G = 4\pi r^2 \sigma_{l,v} - \frac{4R_vT}{3v_l}\pi r^3 ln(S) \tag{1.1}$$

Briefly explain what the two terms on the right hand side represent in terms of the physics (i.e. don't just name them).

ii (2pt) Starting from this equation, derive an expression (show your working) for the critical droplet radius above which pure liquid water droplets will grow by diffusion, for a given saturation value $S = \frac{e}{e_s}$

iii (2pt) A cloud is formed by heterogeneous nucleation on wettable, **insoluble** [this word "insoluble" is important!] aerosols in an environment that is 0.4% supersaturated (i.e. S=1.004) at a temperature T=273 K. What is the minimum aerosol radius for this to occur?

5. Thermodynamics

i (5pt) Specify which quantities, among the following ones, are conserved during (a) **isobaric cooling until fog forms**, (b) **pseudoadiabatic moist saturated ascent**: mixing ratio r_v, vapour pressure e, (dry bulb) temperature T, potential temperature θ, dew point temperature T_d, wet bulb temperature T_w, wet bulb potential temperature θ_w, adiabatic equivalent temperature T_e, equivalent potential temperature θ_e

ii (4pts) An parcel of dry air is at a temperature of 15^oC and a pressure of 1013 hPa. Heat is added to the parcel to cause it to expand. It expands at constant pressure to 1.5 times its original volume. (a) What is the new temperature of the parcel? (b)

What is the amount of heat per unit mass that was added to the parcel? (c) How much work per unit mass was done by the parcel during this expansion? (d) What was the change in specific internal energy of the air parcel?

6. Radiation and Surface Energy Budgets

   i (4pt) The blackbody emission from the surface can be linearized about some reference temperature T_0.

   $$\sigma T_s^4 = \sigma T_0^4 + 4\sigma T_0^3 (T_s - T_0) + ... \tag{1.2}$$

   And the sensible heat flux (SH) at (and which cools) the surface can be written as

   $$SH = C_p \rho C_d U (T_s - T_a) \tag{1.3}$$

   If the surface temperature rises by 1^{o}C, by how much will the longwave and sensible cooling increase? Assume that T_0 = 288K, T_a =286K (and is fixed), ρ = 1.2 kg m^{-3}, C_d = 10^{-3} and U=5 m s^{-1}.

   ii (1pt) Given your answers, which is the greater control on surface temperature over daily timescales?

   iii (5pt) Apart from the linearization procedure (which is accurate for small perturbations) what do you think is the largest source of error in this estimate of the cooling by the OLR the SH fluxes (i.e. which assumption(s) is/are inaccurate) and why? Would correcting for these inaccuraries reduce or increase the efficiency of the process for cooling?