Atmospheric Physics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$pV = NkT = \nu R^* T, \tag{1}$$

N is the number of molecules.

$$p = \rho R_m T. \tag{2}$$

First Law:

$$dq = c_p dT - v dp, \tag{3}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{5}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface assuming L_v is constant:

$$e_s = e_{s0}\exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \tag{6}$$

Rate of change of e_s as a function of T:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{7}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{8}$$

Equivalent Potential temperature:

$$\theta_e = \theta\exp\left(\frac{L_v r_v}{c_p T}\right) \tag{9}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{10}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{11}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T - 32.19)^2} \tag{12}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{13}$$

Measures of water vapour, mass mixing ratio:

$$r_v = \frac{R_d e}{R_v(p - e)} = \epsilon e/(p - e). \tag{14}$$

Virtual temperature:

$$T_v \equiv T\left(\frac{1 + \frac{r_v}{\epsilon}}{1 + r_v}\right) \tag{15}$$

Speed of sound c in an inviscid fluid:

$$c = \sqrt{\frac{C_p}{C_v}RT} \tag{16}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets neglecting the aerosol and curvature effects:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S - 1) \tag{17}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \approx e_s(\infty)\left(1 + \frac{a}{rT} - \frac{b}{r^3}\right) \tag{18}$$

Radiation

The Planck Function:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{\frac{ch}{k\lambda T}} - 1)} \tag{19}$$

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{20}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{21}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{22}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{23}$$

Chapter 1

Tables

Table 1.1: Table of thermodynamical constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Specific heat capacity of water	c_p	4185	J kg^{-1} K^{-1}
Specific heat capacity of sea water	c_p	$\approx$ 3985	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Saturation vapour pressure at $T_0 = 0^o$C	e_{s0}	611.2	Pa
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Surface tension of liquid water	$\sigma_{l,v}$	7.5×10^{-2}	Nm^{-1}

Table 1.2: Table of radiation constants

Planetary albedo of Earth	α_p	0.3	
Planetary albedo of Mercury	α_p	0.07	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between Earth and the Sun	r_d	149.6×10^6	km
distance between Mercury and the Sun	r_d	58×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Questions

1. Thermodynamics

   i (3pt) If the atmosphere warms by 5^o, does the pressure at 5km increase or decrease? What is the rough magnitude of the change, assuming an average temperature of 280K and a surface pressure of 1013 hPa?

   ii (5pts) You measure the relative humidity at 80%. It starts to rain, what does this do to the relative humidity (increase, no change, decrease) and why? What happens to the temperature (increase, no change, decrease) and why? After an infinite amount of time, what is the final temperature called?

   iii (5pts) A parcel of air at p=900hPa, has a temperature of 10^oC, a mass mixing ratio of 5 g kg^{-1}. It is vertically lifted to reach it's lifting condensation level (i.e. saturated). (a) What is the temperature at which the parcel becomes saturated? (b) What is this temperature called? (c) What is the pressure at which condensation is reached?

   iv (4pts) An parcel of dry air is at a temperature of 15^oC and a pressure of 1013 hPa. Heat is added to the parcel to cause it to expand. It expands at constant pressure to 1.5 times its original volume. (a) What is the new temperature of the parcel? (b) What is the amount of heat per unit mass that was added to the parcel? (c) How much work per unit mass was done by the parcel during this expansion? (d) What was the change in specific internal energy of the air parcel?

2. Convection

i (4pts)

The following figure shows measurements in convective clouds (circles) taken in an environment (dash line) in a phase space of Q (total water mixing ratio=waterv vapor+cloud water) and θ_q (an analogue to the equivalent potential temperature θ_e).

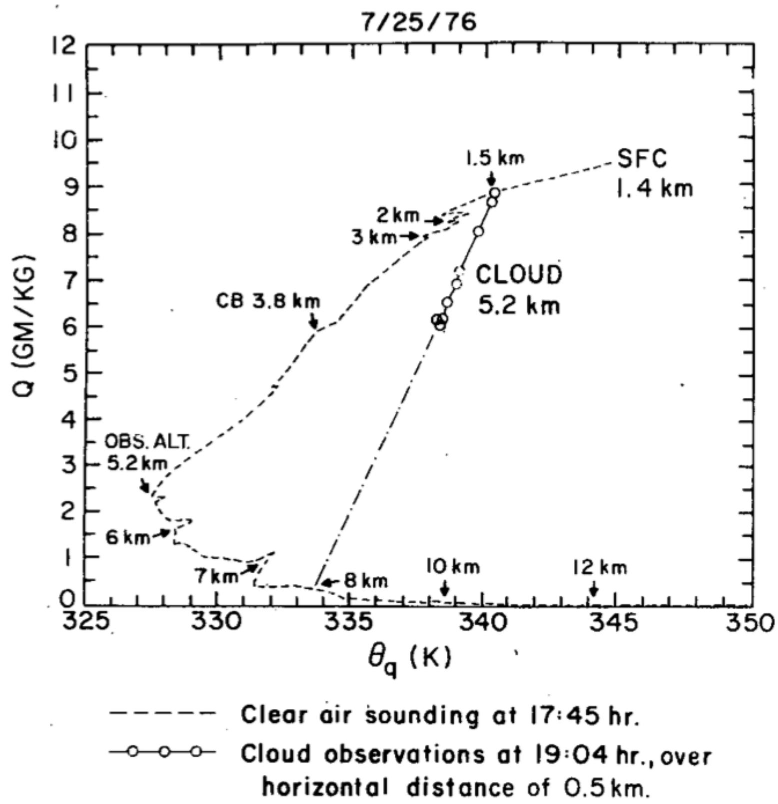

Figure 1.1: Comparison of the total mixing ratio and the wet equivalent potential temperature computing from data collected inside a growing cumulus, taken from

Describe what the figure is showing and what it tells us about physical processes in convective clouds?

3. Convection

   i (4pts)
   Starting from the vertical momentum equation for ideal invisid fluid:

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{1.1}$$

   (where w=vertical velocity, ρ density, p pressure, z vertical coordinate, g accelertion due to gravity), derive the pressure gradiant and bouyancy acceleration terms for a parcel of air, assuming that the local density and pressure variations in the parcel are small compared to their mean values in the ambient environment, thus allowing you to neglect all second order terms. Recall that you can write

$$p = \overline{p} + p' \tag{1.2}$$

$$\rho = \overline{\rho} + \rho' \tag{1.3}$$

   where the overbar refers to the mean ambient values and hyphen to the local perturbation values in the parcel.

4. Convection

   i (2pts) Take a look at the attached tephigram. Marking your answer on the tephigram, what is the convective trigger temperature of this profile?

   ii (2pts) Where in the Pacific do you think this profile is from, the Western Pacific or Eastern Pacific and state why.

5. Clouds

   A cloud exists in the lower troposphere (e.g. ignore ice processes) with a liquid water content of L (units kg m^{-3}), a physical depth z (units m), droplet number concentration of N (units m^{-3}), mean droplet radius $\overline{r}$ (units m), standard deviation of the droplet radius distribution $\sigma(r)$ (units m), and a vertically integrated liquid water path of W (units kg m^{-2}). Describe **how** the following specific changes would change the precipitation rate [increase, no change, decrease] generated from the cloud and why. [Note, if a variable is not mentioned in the question, it may or may not change]. If you do not specify the physical mechanism, no marks will be awarded for getting the precipitation change correct!

   i (2pts) Increasing the cloud physical depth z, keeping L, $\overline{r}$ and N constant (in this case W must increase, for example).

ii (2pt) Increasing N, keeping L and z constant.

iii (2pts) Increasing the cloud physical depth z, keeping W and $\overline{r}$ fixed.

iv (2pts) Increasing $\sigma(r)$, keeping N, $\overline{r}$, L, z and W fixed.

6. i (2pt) The equation for the Gibbs free energy for a cluster formation is:

$$\Delta G = 4\pi r^2 \sigma_{l,v} - \frac{4R_v T}{3v_l}\pi r^3 ln(S) \tag{1.4}$$

Briefly explain what the two terms on the right hand side represent in terms of the physics (i.e. don't just name them).

ii (2pt) Starting from this equation, derive an expression (show your working) for the critical droplet radius above which pure liquid water droplets will grow by diffusion, for a given saturation value $S = \frac{e}{e_s}$

iii (2pt) A cloud is formed by heterogeneous nucleation on wettable, *insoluble* aerosols in an environment that is 0.2% supersaturated (i.e. S=1.002). What is the minimum aerosol radius for this to occur?

7. Radiation

i (1pt) In radiation, what is a black body?

ii (2pt) What does it mean for an atmospheric slab to act as a grey body? What does Kirchoff's law tell us about grey bodies?

iii (2pt) the atmosphere is said to have an "atmospheric window" between 8 and 13 microns, what does this refer to?

iv (6pts) We assume the Earth's atmosphere consists of TWO slabs in the vertical, which are both considered to be black bodies in the infra-red and transparent to solar radiation. Derive the surface temperature under these assumptions in equilibrium. Is the value higher or lower than today's average temperature and why? What is incorrect about this model?

8. Radiation

i (2pt) The Moon has a surface area of $3.8\times10^7 km^2$ and has a mean orbital radius about the Earth of 3.8×10^5 km. What is the solid angle of the moon?

ii (2pt) What fraction of the total visible sky does the full moon's solid angle represent? Assume a flat earth (i.e. the visible sky is a hemisphere).

iii (3pt) Mercury orbits the sun at a mean distance of 58 million km. What is the irradiance (in W m^{-2}) at Mercury's surface? (to a good approximation, Mercury has no atmosphere, assume the sun temperature is 5800K and that the sun is a black body).

iv (2pt) If Mercury's albedo is 0.07, what is the mean surface temperature? (to a good approximation, Mercury has no atmosphere).