

# Atmospheric Physics Exam

## Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!! Formulae**

Most formulae required are given here:

**Thermodynamics** Ideal gas law:

$$pV = NkT = \nu R^*T, \quad (1)$$

$N$  is the number of molecules.

$$p = \rho R_m T. \quad (2)$$

First Law:

$$dq = c_p dT - v dp, \quad (3)$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T} \left( \frac{dT}{dz} + \frac{g}{c_p} \right). \quad (4)$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \quad (5)$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface assuming  $L_v$  is constant:

$$e_s = e_{s0} \exp \left[ \frac{L_v}{R_v} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right]. \quad (6)$$

Rate of change of  $e_s$  as a function of  $T$ :

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \quad (7)$$

Potential temperature:

$$\theta = T \left( \frac{p_0}{p} \right)^{\frac{R_d}{c_p}}. \quad (8)$$

Equivalent Potential temperature:

$$\theta_e = \theta \exp \left( \frac{L_v r_v}{c_p T} \right) \quad (9)$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g \left( \frac{\theta - \theta_{env}}{\theta_{env}} \right) \quad (10)$$

where  $\theta$  is potential temperature and  $env$  refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio  $r_s$  ( $\text{kg kg}^{-1}$ ) as a function of pressure  $p$  (in Pa) and temperature  $T$  (measured in Kelvin):

$$r_s(T) = \frac{380}{p} \exp \left( 17.5 \frac{(T - 273.16)}{(T - 32.19)} \right) \quad (11)$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s \frac{4217}{(T - 32.19)^2} \quad (12)$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s} \quad (13)$$

Measures of water vapour, mass mixing ratio:

$$r_v = \frac{R_d e}{R_v (p - e)} = \epsilon e / (p - e). \quad (14)$$

Virtual temperature:

$$T_v \equiv T \left( \frac{1 + \frac{r_v}{\epsilon}}{1 + r_v} \right) \quad (15)$$

### Microphysics

Approximate diffusion equation for radius  $r > 1 \mu\text{m}$  droplets neglecting the aerosol and curvature effects:

$$\frac{dr}{dt} \simeq \frac{D e_s(\infty)}{\rho_L r R_v T} (S - 1) \quad (16)$$

Saturation vapour pressure over a solute droplet of radius  $r$ :

$$e_s^r(sol) = e_s(\infty) \left( 1 - \frac{b}{r^3} \right) \exp \left( \frac{a}{rT} \right) \approx e_s(\infty) \left( 1 + \frac{a}{rT} - \frac{b}{r^3} \right) \quad (17)$$

### Radiation

The Planck Function:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5 (e^{\frac{hc}{\lambda T}} - 1)} \quad (18)$$

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \quad (19)$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho \sec\theta dz. \quad (20)$$

Transmittance  $\tau$  is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \quad (21)$$

solid angle

$$\Omega = \frac{A}{r^2} \quad (22)$$

# Chapter 1

## Tables

Table 1.1: Table of thermodynamical constants

Avogadro's constant	$N_A$	$6.02 \times 10^{23}$	$\text{mol}^{-1}$
Specific heat capacity at constant pressure for dry air	$c_p$	1005	$\text{J kg}^{-1} \text{K}^{-1}$
Specific heat capacity at constant volume for dry air	$c_v$	718	$\text{J kg}^{-1} \text{K}^{-1}$
Specific heat capacity of water	$c_p$	4185	$\text{J kg}^{-1} \text{K}^{-1}$
Specific heat capacity of sea water	$c_p$	$\approx 3985$	$\text{J kg}^{-1} \text{K}^{-1}$
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v}$	0.622	
Latent heat of vaporization	$L_v$	$2.5 \times 10^6$	$\text{J kg}^{-1}$
Latent heat of sublimation	$L_s$	$2.83 \times 10^6$	$\text{J kg}^{-1}$
Latent heat of sublimation	$L_s$	$2.83 \times 10^6$	$\text{J kg}^{-1}$
Gas constant for dry air	$R_d$	287.06	$\text{J kg}^{-1} \text{K}^{-1}$
Gas constant for vapour	$R_v$	461.5	$\text{J kg}^{-1} \text{K}^{-1}$
Density of liquid water	$\rho_l$	1000	$\text{kg m}^{-3}$
Molar mass of water	$m_v$	18.02	$\text{g mol}^{-1}$
Universal Gas Constant	$R$	8.314	$\text{J K}^{-1} \text{mol}^{-1}$
Saturation vapour pressure at $T_0 = 0^\circ\text{C}$	$e_{s0}$	611.2	Pa
Vapour diffusion coefficient	$D$	$\approx 2.2 \times 10^{-5}$	$\text{m}^2 \text{s}^{-1}$
Surface tension of liquid water	$\sigma_{l,v}$	$7.5 \times 10^{-2}$	$\text{Nm}^{-1}$

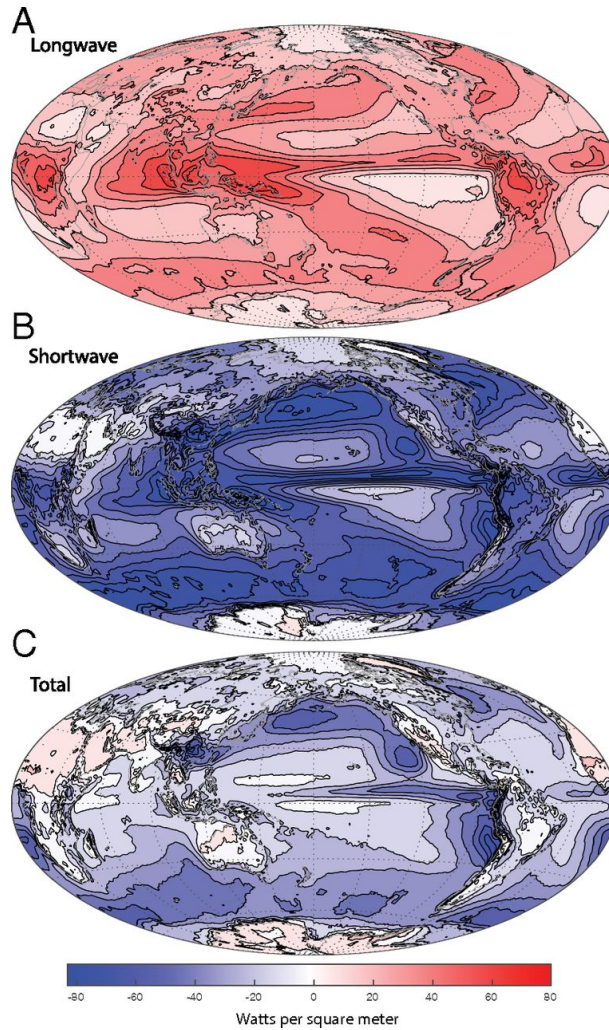
Table 1.2: Table of radiation constants

Planetary albedo of Earth	$\alpha_p$	0.3	
Planetary albedo of Mercury	$\alpha_p$	0.07	
Speed of light	$c$	$3 \times 10^8$	$\text{m s}^{-1}$
Planck Constant	$h$	$6.625 \times 10^{-34}$	$\text{J s}$
Boltzmann constant	$k$	$1.3806 \times 10^{-23}$	$\text{J K}^{-1}$
Stefan Boltzmann constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{Wm}^{-2} \text{K}^{-4}$
radius of the earth	$r_e$	6340	km
radius of the sun	$r_s$	$0.7 \times 10^6$	km
distance between Earth and the Sun	$r_d$	$149.6 \times 10^6$	km
distance between Mercury and the Sun	$r_d$	$58 \times 10^6$	km
Solar Constant	$S_0$	1370	$\text{W m}^{-2}$

## Questions

### 1. Clouds

Look at the figure below.



i (2pt)

Look at panel A, the Pacific ocean: What kind of cloud is the band of strong positive OLR forcing associated with? Why is the forcing positive?

ii (2pt)

In those same locations, the SW forcing (panel B) is of the opposite sign, why is that ?

iii (2pt)

Look at panel C, which gives the sum (OLR+SW) - where does the cloud forcing have the largest magnitude and why? Is this a

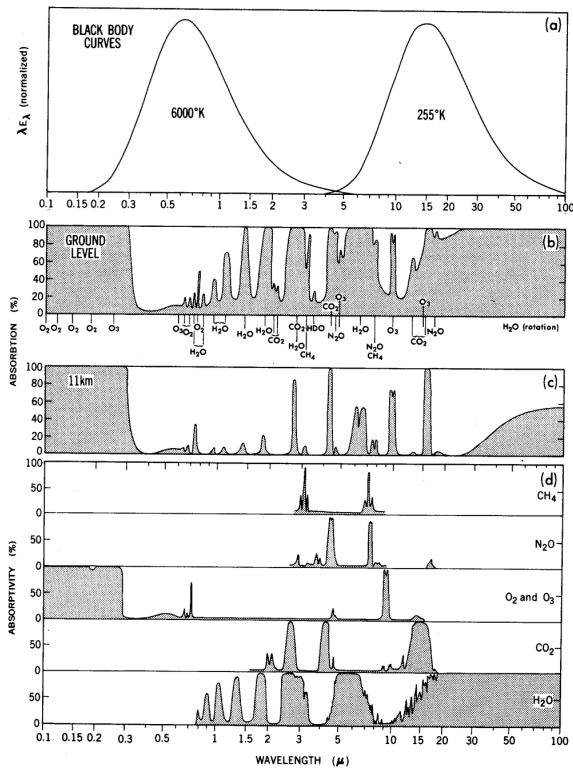


Figure 1.1: Upper panel shows the transmitted radiation in comparison to theoretical blackbody curves. Lower panels show the percentage absorptance. The X-axis shows wavelength, labeled on the top of the upper panel with units of microns.

cooling or a warming? (you can mark notes on the plot and hand it in if it helps).

## 2. Radiation

- i (4pt) Take a look at Figure 1.1. Name the key contributions to the **short-wave** extinction coefficient of the atmosphere.
- ii (2pt) Take a look at Figure 1.1. If you wanted to design a satellite to measure the sea surface temperature using infra-red (long) wavelengths, which wavelengths would you use and why?
- iii (2pt) Name two line broadening mechanisms.
- iv (3pt) Combustion of fossil fuels emits  $CO_2$ , does this increase or decrease the atmospheric infra-red emissivity?  
Is the change in emissivity linearly or logarithmically related to increasing  $CO_2$  and why?

### 3. Radiation

i (6pts)

Derive a simple model of radiative transfer through the atmosphere with the following assumptions:

1) the earth's surface is a black body 2) the atmosphere is divided into TWO layers, both assumed to be BLACK BODIES in the infra-red but TRANSPARENT to solar radiation

What is the surface temperature if the system is in equilibrium? Is this higher or lower than today's mean surface temperature and explain why?

### 4. Microphysics

i (2pt) A chance collision of 250 water vapour molecules forms a liquid droplet. What is the radius of the droplet?

ii (2pt) In order to form a stable cloud droplet from the chance collisions of water vapour molecules, very high relative humidities are required of several hundred percent. Explain why such high relative humidities are **not** observed in the atmosphere at the surface.

iii (2pt) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water (of infinite radius, so we write  $e_s(\infty)$ ). But in fact the saturation vapour over a water droplet of radius  $r$  and with a mass  $M$  of dissolved impurity is modified by two factors  $\left(1 - \frac{b}{r^3}\right)$  and  $\exp\left(\frac{a}{rT}\right)$ , where  $a$  and  $b$  are constants, with  $b$  depending on the mass and type of aerosol. In a few sentences, explain what the two correction factors refer to (use a sketch if helpful).

iv (4pt) The constants for the correction factors for a given haze particle are  $a = 3.3 \times 10^{-7} \text{ m K}$  and  $b = 1.47 \times 10^{-23} \text{ m}^3$ . The haze particle resides in air at a temperature of 273 K. What is the radius AND relative humidity at which the haze particle becomes activated?

### 5. Convection

i (2pt) Describe two contributions to the formation of convective downdrafts

ii (2pt) Why are convective downdrafts important for convective triggering?

iii (2pt) (If you like with the aid of a sketch), describe how updraft entrainment affect the LNB and CAPE.



## 6. Convection

- i (6pt) Clearly define the terms (you can mark on tephigram if you prefer)
    - a. Lifting condensation level (LCL)
    - b. Level of free convection (LFC)
    - c. Convective inhibition energy (CIN)
    - d. Level of neutral buoyancy (LNB)
    - e. Convective Trigger temperature
    - f. The convective available potential energy (CAPE)
  - ii (2pts) Take a look at the tephigram. The sun heats the surface until the trigger temperature is reached, what is that temperature? (PLEASE MARK YOUR CALCULATION ON THE TEPHIGRAM AND HAND IN).
  - iii (2pt) If the trigger temperature is reached and convection takes place, what is the cloud top temperature? What is that temperature otherwise known as?
  - iv (2pt) Would you use the trigger temperature to forecast whether squall lines would occur or not? You must justify your answer.
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7.
    - i (2pt) At cold temperature below  $-40^{\circ}\text{C}$ , ice crystals in the atmosphere can form by homogeneous freezing nucleation from the liquid phase. Ice can also form by heterogeneous nucleation on aerosols known as ice nuclei. From the two processes (hetero and homogeneous from liquid), which results in the highest ice crystal number concentration ( $\text{m}^{-3}$ ) and why?
    - ii (2pt) Assuming the ambient conditions allow both nucleation processes to occur, state at least TWO FACTORS that will determine which process will dominate.
    - iii (2pt) Can homogeneous nucleation occur at vapor pressures that are subsaturated with respect to liquid water? Please explain your answer to get awarded the mark.
    - iv (1pt) State ONE characteristic that an aerosol requires in order to act as an ice nuclei.
    - v (1pt) Apart from ice itself, name a substance that can act as an efficient ice nuclei.