Atmospheric Physics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$pV = NkT = \nu R^* T, \tag{1}$$

N is the number of molecules.

$$p = \rho R_m T. \tag{2}$$

First Law:

$$dq = c_p dT - v dp, \tag{3}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{5}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface assuming L_v is constant:

$$e_s = e_{s0}\exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \tag{6}$$

Rate of change of e_s as a function of T:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{7}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{8}$$

Equivalent Potential temperature:

$$\theta_e = \theta\exp\left(\frac{L_v r_v}{c_p T}\right) \tag{9}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{10}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{11}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T - 32.19)^2} \tag{12}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{13}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets neglecting the aerosol and curvature effects:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S - 1) \tag{14}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \approx e_s(\infty)\left(1 + \frac{a}{rT} - \frac{b}{r^3}\right) \tag{15}$$

Radiation

The Planck Function:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{\frac{ch}{k\lambda T}} - 1)} \tag{16}$$

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{17}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{18}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{19}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{20}$$

Chapter 1

Tables

Table 1.1: Table of thermodynamical constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Saturation vapour pressure at $T_0 = 0^o$C	e_{s0}	611.2	Pa
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Surface tension of liquid water	$\sigma_{l,v}$	7.5×10^{-2}	Nm^{-1}

Table 1.2: Table of radiation constants

Planetary albedo of Earth	α_p	0.3	
Planetary albedo of Mercury	α_p	0.07	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between Earth and the Sun	r_d	149.6×10^6	km
distance between Mercury and the Sun	r_d	58×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Questions

1. Convection

   i (4pts) With the aid of a sketch, please describe a typical diurnal cycle of the planetary boundary layer over land, taking care to explain: i) the mixed layer, (ii) the residual layer and (iii) the stable boundary layer.

   ii (2pt) Is the magnitude of the diurnal variation of the boundary layer depth larger or smaller over the ocean compared to the land? Explain briefly the reason for your answer.

2. Convection

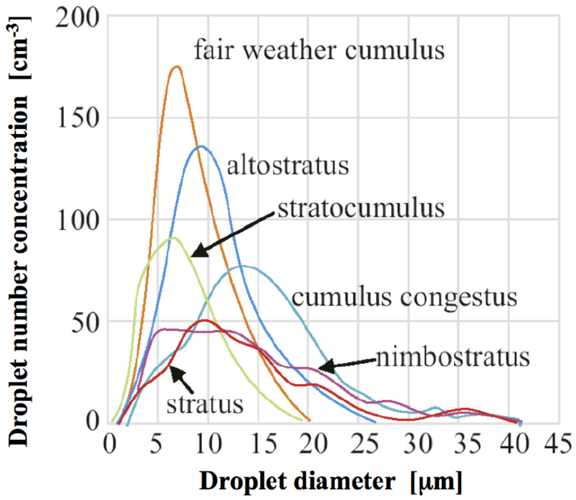
   For this question, mark your calculations on the attached tephigram and remember to hand it in with your answers. Recall that the isopleths of saturation mixing ratio are the FAINT DASHED LINES running diagonally labled on the LOWER border of the tephigram.

   i (4pts) The sun heats the surface during the day until the trigger temperature is reached and deep convection occurs (recall that this assumes that there is no significant surface latent heat flux). Assuming non-reversible, psedu-adiatic ascent of parcels within in the cloud, what is the cloud top pressure and temperature (i.e. the level of neutral buoyancy)?

   ii (4pt) Now we assume that a single mixing event occurs between the cloud and its environment at 700hPa, such that a mixed parcel is formed consisting of 50% cloudy updraft air and 50% environmental air. The mixed parcel continues its ascent to its level of neutral buoyancy, what is the pressure of the modified level of neutral buoyancy for this mixed parcel?

3. Clouds

   i (3pts) Look at the figure below. Explain which cloud types are most likely to rain and why.



ii (3pt)
(A) In a mixed-phase cloud with both cloud liquid drops and ice crystals present, do ice crystals grow or decay? (B) How is this growth/decay process accelerated by the presence of the liquid droplets?

iii (4pt)
(A) Ice crystals can potentially form by three processes, heterogeneous nucleation, homogeneous nucleation from the vapour phase, and homogeneous nucleation from the liquid phase. Which mechanisms operate in real clouds? (B) which nucleation process creates the highest ice crystal concentration and why?

iv (2pt) Aircraft can sometimes form cloud from the engine exhaust called contrails. If the contrails do not dissipate, what does it tell you about the state of the atmosphere the aircraft is flying through? Is the aircraft likely to be in the upper troposphere or lower stratosphere?

4. For the following question, assume the normal solar irradiance above the atmosphere is equal to the solar constant 1340 W m^{-2}.

i (2pt) In a clear sky situation the zenith transmissivity for solar radiation is 0.85. What is the transmissivity of the atmosphere when the sun is 10^o above the horizon (i.e. the solar declination angle is 80^o ?

ii (2pt) What is the irradiance at sea level of a surface normal to the direction of the sun?

iii (2pt) What is the irradiance at sea level of a horizontal surface?

5. i (3pt) Mercury orbits the sun at a mean distance of 58 million km. What is the global average irradiance (in W m^{-2}) at Mercury's surface? (to a good approximation, Mercury has no atmosphere, assume the sun temperature is 5800K and that the sun is a black body).

ii (2pt) If Mercury's albedo is 0.07 and it is assumed to be a black body, what is the mean surface temperature?