Atmospheric Physics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$pV = NkT = \nu R^* T, \tag{1}$$

N is the number of molecules.

$$p = \rho R_m T. \tag{2}$$

First Law:

$$dq = c_p dT - v dp, \tag{3}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{5}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface assuming L_v is constant:

$$e_s = e_{s0}\exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \tag{6}$$

Rate of change of e_s as a function of T:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{7}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{8}$$

Equivalent Potential temperature:

$$\theta_e = \theta\exp\left(\frac{L_v r_v}{c_p T}\right) \tag{9}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{10}$$

where θ is potential temperature and env refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T-273.16)}{(T-32.19)}\right) \tag{11}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T-32.19)^2} \tag{12}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{13}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets neglecting the aerosol and curvature effects:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S-1) \tag{14}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \approx e_s(\infty)\left(1 + \frac{a}{rT} - \frac{b}{r^3}\right) \tag{15}$$

Radiation

The Planck Function:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{\frac{ch}{k\lambda T}} - 1)} \tag{16}$$

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{17}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{18}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{19}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{20}$$

Chapter 1

Tables

Table 1.1: Table of thermodynamical constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Saturation vapour pressure at $T_0 = 0^o$C	e_{s0}	611.2	Pa
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Surface tension of liquid water	$\sigma_{l,v}$	7.5 $\times 10^{-2}$	Nm^{-1}

Table 1.2: Table of radiation constants

Planetary albedo of Earth	α_p	0.3	
Planetary albedo of Mercury	α_p	0.07	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between Earth and the Sun	r_d	149.6×10^6	km
distance between Mercury and the Sun	r_d	58×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Questions

1. Microphysics

   i (2pt) A chance collision of 150 water vapour molecules forms a liquid droplet. What is the radius of the droplet?

   ii (2pt) In order to form a stable cloud droplet from the chance collisions of water vapour molecules, very high relative humidities are required of several hundred percent. Explain why such high relative humidities are **not** observed in the atmosphere at the surface.

   iii (2pt) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water (of infinite radius, so we write $e_s(\infty)$). But in fact the saturation vapour over a water droplet of radius r and with a mass M of dissolved impurity is modified by two factors $\left(1-\frac{b}{r^3}\right)$ and $\exp\left(\frac{a}{rT}\right)$, where a and b are constants, with b depending on the mass and type of aerosol. In a few sentences, explain what the two correction factors refer to (use a sketch if helpful).

   iv (2pt) Explain what it means for a haze particle to become activited

   v (4pt) The constants for the correction factors for a given haze particle are $a = 3.3 \times 10^{-7}$ m K and $b = 1.47 \times 10^{-23}$ m^3. The haze particle resides in air at a temperature of 273 K. What is the radius AND relative humidity at which the haze particle becomes activated?

2. i (2pt) At cold temperature below -40C, ice crystals in the atmosphere can form by homogeneous freezing nucleation from the liquid phase. Ice can also form by heterogeneous nucleation on aerosols known as ice nuclei. From the two processes (hetero and homogeneous from liquid), which results in the highest ice crystal number concentration (m^{-3}) and why?

   ii (2pt) Assuming the ambient conditions allow both nucleation processes to occur, state at least TWO FACTORS that will determine which process will dominate.

   iii (2pt) Can homogeneous nucleation occur at vapor pressures that are subsaturated with respect to liquid water? Please explain your answer to get awarded the marks (yes/no will not suffice!).

   iv (4pt) Heteorogeneous ice nucleation can occur by deposition, condensation, contact and immersion freezing. Which of these pathways permits nucleation at vapor pressure values that are **sub**saturated with respect to liquid water and why.

3. Radiation

   i (2pt) At the (infrared) wavelength of 10 microns, which has the highest radiance: the sun or the Earth? Briefly explain why.

   ii (3pt) If we plot the **normalized** Planck function for the sun and Earth as a function of wavelength, there is almost no overlap between the two PDFs, briefly explain why this is (use a sketch if you like) and what the implications of this are.

   iii (2pt) A parallel beam radiation of wavelength λ is passing vertically through a layer of 1000m in thickness, containing a homogeneous gas with a density of 0.01 kg m^{-3} with an absorption coefficient k_λ^e=0.1 m^2 kg^{-1} and a zero scattering coefficient.

   What is the optical thickness of the layer?

   iv (1pt) What is the transmissivity τ of the layer?

   v (2pt) How thick would the layer have to be to absorb half of the incidence radiation beam?

4. Convection

   i (6pt) Clearly define the terms

   a. Lifting condensation level (LCL)

   b. Level of free convection (LFC)

   c. Convective inhibition energy (CIN)

   d. Level of neutral buoyancy (LNB)

   e. Convective Trigger temperature

   f. The convective available potential energy (CAPE)

   ii (2pt) Describe two contributions to the formation of convective downdrafts

   iii (2pt) Why are convective downdrafts important for convective triggering?

   iv (2pt) (If you like with the aid of a sketch), describe how updraft entrainment affect the LNB and CAPE.