Atmospheric Physics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$pV = NkT = \nu R^{*} T, \tag{1}$$

N is the number of molecules.

$$p = \rho R_m T. \tag{2}$$

First Law:

$$dq = c_p dT - v dp, \tag{3}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{5}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface assuming L_v is constant:

$$e_s = e_{s0} \exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \tag{6}$$

Rate of change of e_s as a function of T:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{7}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{8}$$

Equivalent Potential temperature:

$$\theta_e = \theta \exp\left(\frac{L_v r_v}{c_p T}\right) \tag{9}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{10}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{11}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T - 32.19)^2} \tag{12}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{13}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets neglecting the aerosol and curvature effects:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S - 1) \tag{14}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \tag{15}$$

($a = 3.3 \times 10^{-7}$ m K and $b = 1.47 \times 10^{-23}$ m^3)

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{16}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{17}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{18}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{19}$$

Tables

Table 1: Table of thermodynamical constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Latent heat of sublimation	L_s	2.83×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Saturation vapour pressure at $T_0 = 0^o$C	e_{s0}	611.2	Pa
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Surface tension of liquid water	$\sigma_{l,v}$	7.5×10^{-2}	Nm^{-1}

Table 2: Table of radiation constants

Planetary albedo of Earth	α_p	0.3	
Planetary albedo of Mercury	α_p	0.07	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between Earth and the Sun	r_d	149.6×10^6	km
distance between Mercury and the Sun	r_d	58×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Questions

1. i (2pt) The simple model of the greenhouse effect considers the atmosphere as a single slab that emits as a grey body in the infra-red with temperature T_a and fractional emittance ϵ_{IR}=0.6. We now instead divide the atmosphere into THREE layers with equal emissivity $\epsilon_{IR1} = \epsilon_{IR2} = \epsilon_{IR3}$. What is the value of ϵ_{IR1}?

   ii (4pts) Assuming that the atmosphere is transparent to solar radiation, and that the average incoming solar radation at the TOA is $\frac{S_0}{4}(1-\alpha_p)$, where S_0 is the solar constanst and α_p is the albedo, and that equilibrium exists with the temperature in the three layers noted as T_1, T_2, T_3, derive the energy balance equation for the interface at top of the atmosphere and at the earth's surface. (DO NOT ATTEMPT TO SOLVE THESE EQUATIONS).

   iii (2pts) If the full set of balance equations were solved (DO NOT ATTEMPT THIS) to derive T_1, T_2, T_3, would the resulting temperature profile be convectively unstable, neutral or stable? Why is the derivation of T_1, T_2, T_3 using the assumption of radiative equilibrium a poor model of the atmosphere?

2. (4pts) A cloud droplet takes 10 minutes to grow from a size of 2μm to 20 μm at a temperature of T=0^{o}C. Ignoring the aerosol and curvature effects, and assuming droplet growth does not impact the ambient humidity, what is the ambient supersaturation in percent?

3. i (2pts) Briefly describe the processes that lead to the formation of convective **cold pools**.

   ii (2pts) Briefly state reasons why convective cold pools are important to understand

4. For the following question, assume the normal solar irradiance above the atmosphere is equal to the solar constant.

   i (2pt) In a certain clear sky the zenith transmissivity for solar radiation is 0.75. What is the transmissivity of the atmosphere when the sun is 25^o above the horizon?

   ii (2pt) What is the irradiance at sea level of a surface normal to the direction of the sun?

   iii (2pt) What is the irradiance at sea level of a horizontal surface?

5. i (4pt) briefly describe the TWO main mechanisms by which ice crystals can form in the atmosphere

   ii (2 pt) which of these results in the highest ice crystal number concentration and why?

iii (2pt) Assuming the ambient conditions allow both nucleation processes to occur, state at least TWO FACTORS that will determine which process will dominate.

6. i (2pt) The equation for the Gibbs free energy for a cluster formation is:

$$\Delta G = 4\pi r^2 \sigma_{l,v} - \frac{4R_v T}{3v_l}\pi r^3 ln(S) \tag{20}$$

Briefly explain what the two terms on the right hand side represent in terms of the physics (i.e. don't just name them).

ii (2pt) Starting from this equation, derive an expression (show your working) for the critical droplet radius above which pure liquid water droplets will grow by diffusion, for a given saturation value $S = \frac{e}{e_s}$

iii (2pt) A cloud is formed by heterogeneous nucleation on wettable, *insoluble* aerosols in an environment where $S = 1.01$. What is the approximate aerosol radius

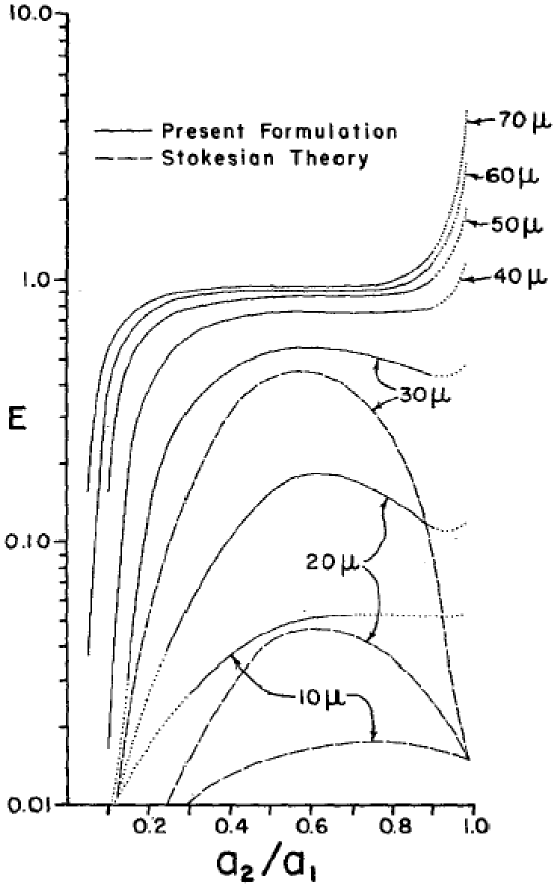
7. Take a look at the photo of stratocumulus cloud in the boundary layer. This photo was taken near Genova, a coastal town in west Italy during the Christmas break at around 8am local time. There was a situation of high pressure at the time. Each morning I would wake to find the sky overcast with this stratocumulus layer that had formed during the night (hence the name nocturnal stratocumulus). The cloud layer would become thinner and would dissipate around 11am local time each day and the rest of the day would be bright and sunny. The next morning I would wake to find the stratocumulus there again. Please answer the following questions BRIEFLY!

i (2pt) Explain why you think the cloud layer forms at night.

ii (2pts) Explain what drives the vertical instability.

iii (2pts) Explain why these clouds never formed precipitation (luckily, as I was out jogging when I took the photograph!)

8. The graph shows calculations of collection efficiency (using two different methods, the "present method" derived in the paper, and the classical Stokesian method) when a droplet of radius a_1 falls through a cloud consisting of drops of radius a_2.



i (2pt) Explain why (use a sketch if helpful) when the largest droplet a_1 has a radius of 10 μm, the efficiency calculated is considerably below 1

ii (2pt) Explain how it is possible to have collection efficiencies that **exceed** unity (upper right of plot!)

iii (1pt) For the case of $a_1 = 30\mu m$, and using the Stokesian theory derived curves, what value does E have when $a_2/a_1 = 1$?