Atmospheric Physics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$pV = NkT = \nu R^* T, \tag{1}$$

N is the number of molecules.

$$p = \rho R_m T. \tag{2}$$

First Law:

$$dq = c_p dT - v dp, \tag{3}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{5}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{6}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{7}$$

Equivalent Potential temperature:

$$\theta_e = \theta \exp\left(\frac{L_v r_v}{c_p T}\right) \tag{8}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{9}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{10}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T - 32.19)^2} \tag{11}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{12}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S - 1) \tag{13}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \tag{14}$$

($a = 3.3 \times 10^{-7}$ m K and $b = 1.47 \times 10^{-23}$ m^3)

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{15}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{16}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{17}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{18}$$

1 Appendices

1.1 Tables of constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v} = \frac{m_v}{m_d}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Planetary albedo of Earth	α_p	0.3	
Planetary albedo of Mercury	α_p	0.07	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between Earth and the Sun	r_d	149.6×10^6	km
distance between Mercury and the Sun	r_d	58×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Table 1: Constants

Questions

1. Radiation

   i (1pt) What is the definition of the *single scattering albedo*?

   ii (2pt) Explain why clouds are effectively black bodies in the infrared despite the fact that the single scattering albedo is around 0.5

   iii (1pt) Give one reason why is the sky blue and not red?

   iv (2pt) Name two reasons why the sky is blue and not violet?

   v (3pt) At night in winter in Trieste, which of the following situations will result in the warmest night minimum temperature and explain why in a FEW SENTENCES (not 5 pages!) : (a) clear sky conditions (b) optically thick high cloud (c) optically thin high cloud (d) optically thick low cloud ?

   vi (2pt) Which clouds have the greatest impact on the net global radiation budget of Earth: low clouds or high clouds? - explain your answer in a few sentences.

2. Radiation

   i (2pt) Derive the solid angle of a hemisphere. Show your workings.

   ii (3pt) What is the solid angle of the moon if it has an angular diameter of 0.52^o?

   (For this, you may assume that the angle is small compared to the hemisphere diameter of 180^o to derive a good approximation, or you can use the area of a spherical cap, which is $A = 2\pi hr$, see Fig. 1 - either approach is acceptable for full points)

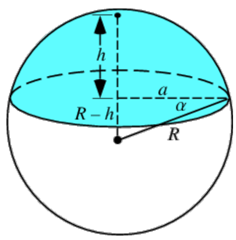

Figure 1: Area of a spherical cap is $A = 2\pi hr$.

   iii (1pt) what is isotropic radiation ?

   iv (3pt) If we have a situation where radiation is anisotropic and the radiance is a function of the zenith angle, L(θ, ϕ)=$L_0 cos(\theta)$, what is the irradiance E?

v (3pt) Mercury orbits the sun at a mean distance of 58 million km. What is the irradiance (in W m^{-2}) at Mercury's surface? (to a good approximation, Mercury has no atmosphere, assume the sun temperature is 5800K and that the sun is a black body).

vi (2pt) If Mercury's albedo is 0.07, what is the mean surface temperature? (to a good approximation, Mercury has no atmosphere).

3. Microphysics

   i (2pt) A chance collision of 100 water vapour molecules forms a liquid droplet. What is the radius of the droplet?

   ii (2pt) In order to form a stable cloud droplet from the chance collisions of water vapour molecules, very high relative humidities are required of several hundred percent. Explain why such high relative humidities are **not** observed in the atmosphere at the surface.

   iii (2pt) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water (of infinite radius, so we write $e_s(\infty)$). But in fact the saturation vapour over a water droplet of radius r and with a mass M of dissolved aerosol is modified by two factors - in a few sentences explain what the two correction factors refer to (use a sketch if helpful).

   iv (3pt) A solute liquid droplet with a radius of 0.05 micron (μm) is present in an environment with a relative humidity of 95% and a temperature of $T = 273K$. Calculate the saturation ratio with respect to the droplet. Will the droplet grow or shrink?

4. Microphysics

   i (2pt) collision and coalescence can lead to the growth of raindrops. We model this assuming a drop of radius R falls at its terminal velocity V_t through a cloud of smaller droplets. Give the relationship for the volume of air swept out by the larger droplet in time t.

   ii (2pts) We assume the cloud droplets of radius r are much smaller than the raindrop, $r << R$, and that their terminal velocities are also negligibly small (they are at rest), What is the rate of change of mass dM/Dt of the raindrop for a cloud of absolute liquid water density L (kg m^{-3})?

   iii (3pts) The rate of change of mass is related to the raindrop radius by $\frac{dM}{dt} = 4\pi R^2 \rho_L \frac{dr}{dt}$. Assuming a terminal fallspeed of $V_t = X_1 r^2$

where $X_1 = 1.2 \times 10^8 \text{s}^{-1}\ \text{m}^{-1}$, calculate the time taken for a droplet to grow in a cloud with mean absolute liquid water density of $L = 1.0 \times 10^{-3}$ kg m^{-3}.

(a) from 10μm to 20 μm by this process

(b) from 20μm to 30 μm by this process

What does this tell you about when this process dominates droplet growth?

iv (3pt) The collection efficiency is not unity. If we assume that for our cloud drop size of r, we can represent the collection efficiency as $E(R, r) = \sqrt{\frac{R}{R_0}}$, where R_0 is 100μm, how long would it now take to grow the drop from 20μm to 30 μm?

5. Convection

i (3pts) A layer of air has a potential temperature of 303K and a mixing ratio of 5 g/kg at its LOWER boundary at p=700 hPa, and a potential temperature of 305K, a mixing ratio of 1 g/kg at its UPPER boundary. If the whole layer is lifted will it eventually undergo convection or not ?

ii (2pts) At what time of day (local time) do you expect to see the most stable boundary layer (assume no large-scale weather patterns are passing through, i.e. situation of stable high pressure)? Explain why in one or two sentences? (a) 9am (b) 3pm (c) 9pm (d) 3am

iii (2pts) At what time of day (local time) do you expect to see the deepest boundary layer (assume no large-scale weather patterns are passing through, i.e. situation of stable high pressure)? Explain why in one or two sentences? (a) 9am (b) 3pm (c) 9pm (d) 3am