Atmospheric Thermodynamics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!** Note that the number of points available are stated at the start of each question.

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$p = \rho R T. \tag{1}$$

First Law:

$$du = dq - pdv \Rightarrow dq = c_v dT + pdv = c_p dT - vdp, \tag{2}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{3}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{4}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{5}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{6}$$

Dry Static Energy:

$$s = c_p T + gz \tag{7}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta_v - \theta_{v-env}}{\theta_{v-env}}\right) \tag{8}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T-273.16)}{(T-32.19)}\right) \tag{9}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T-32.19)^2} \tag{10}$$

Mixing Ratio r_v

$$r_v = \frac{\rho_v}{\rho_d} \tag{11}$$

specific humidity

$$q_v = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v} \tag{12}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{13}$$

Virtual Temperature

$$T_v = T(1 + \frac{1-\epsilon}{\epsilon}r_v) \tag{14}$$

Virtual potential Temperature

$$\theta_v \equiv T_v \left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{15}$$

Isobaric Equivalent temperature T_{ie}

$$T_{ie} = T + \frac{L_v}{c_p}r_v \tag{16}$$

Adiabatic Equivalent temperature T_e

$$T_e = T\exp\left(\frac{L_v r_v}{c_p T}\right). \tag{17}$$

surface heat fluxes

$$H = \rho C_p C_d \Delta\theta\Delta V \tag{18}$$

$$LE = \rho L C_d \Delta r_v \Delta V \tag{19}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S-1) \tag{20}$$

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{21}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{22}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{23}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{24}$$

Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v} = \frac{m_v}{m_d}$	0.622	
Density of liquid water	ρ_l	1000	kg m^{-3}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between earth and sun	r_d	149.6×10^6	km
Solar Constant	S_0	1370	W m^{-2}
Planetary albedo	α_p	0.3	
Planck Constant	h	6.625×10^{-34}	J s
Constant c_1 in Planck's Law	c_1	3.74×10^{-16}	W m^{-2}
Constant c_2 in Planck's Law	c_2	1.45×10^{-2}	m K

Table 1: Constants

Questions

1. (3pt) If at 0^oC the density of dry air alone is 1.275 kg m^{-3} and the density of water vapour alone is 4.770 $\times 10^{-3} kg m^{-3}$, what is the total pressure exterted by a mixture of the dry air and the water vapour at 0^oC?

2. (2 pts) Explain in a short paragraph why hot weather causes more human discomfort when the air is humid than when it is dry.

3. (2 pts) Explain in a short paragraph why in cold climates in the winter, the air indoors tends to have very low relative humidities.

4. (2 pts) Explain in a short paragraph why a bicycle pump tends to get hot when you pump up your tyre.

5. (2pts) From first principles, derive the definition of the **specific heat of moist air at constant volume**, c_{vm}, in terms of the specific heat of dry air at constant volume c_v and the specific heat of water vapour at constant volume c_{vv}.

6. (5pts) A dry air parcel has a temperature of 298 K. (a) if it is lifted from 1000 hPa to 900hPa, what is its final temperature? (b) Assume the parcel is moving through an isothermal atmosphere of T=290K which is in hydrostatic balance, and that pressure perturbations can be neglected, what is the **change** in the parcel dry static energy s?

7. (3pt) An air parcel of moist (but non-cloudy) air has a temperature of 298K. It is neutrally buoyant sitting in an environment that has a temperature of 298.4 ^oC and a mixing ratio of 10 g kg^{-1}. What is the mixing ratio in the air parcel?

8. (3pts) One kilogram of dry air is warmed from 20C to 70C under a constant pressure of 1000 hPa. Calculate (a) the heat absorbed by the gas (b) the work done on the gas (c) the change in internal energy. (Show your working).

9. (2pt) Air has an initial temperature T_1 and a mixing ratio r. All the water is condensed into liquid droplets *isobarically.* This process increases the temperature to a final temperature T_2. What is the name given to this temperature? Why is this temperature not really relevant for cloud processes?

10. (1pt) What is the criterion for dry air to be considered stable to vertical perturbations?