Atmospheric Physics Exam

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!**

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$p = \rho R_m T. \tag{1}$$

First Law:

$$dq = c_p dT - v dp, \tag{2}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{3}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{4}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{5}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{6}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{7}$$

where θ is potential temperature and env refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{8}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s \frac{4217}{(T - 32.19)^2} \tag{9}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{10}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S - 1) \tag{11}$$

Saturation vapour pressure over a solute droplet of radius r:

$$e_s^r(sol) = e_s(\infty) \left(1 - \frac{b}{r^3}\right) \exp\left(\frac{a}{rT}\right) \tag{12}$$

($a = 3.3 \times 10^{-7}$ m K and $b = 1.47 \times 10^{-23}$ m^3)

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{13}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{14}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{15}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{16}$$

1 Appendices

1.1 Tables of constants

Avogadro's constant	N_A	6.02×10^{23}	mol $^{-1}$
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v} = \frac{m_v}{m_d}$	0.622	
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Molar mass of water	m_v	18.02	g mol^{-1}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Planetary albedo of Earth	α_p	0.3	
Speed of light	c	3×10^8	m s^{-1}
Planck Constant	h	6.625×10^{-34}	J s
Boltzmann constant	k	1.3806×10^{-23}	J K^{-1}
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between earth and sun	r_d	149.6×10^6	km
Solar Constant	S_0	1370	W m^{-2}

Table 1: Constants

Questions

1. Radiation

   i (2pt) The simple model of the greenhouse effect considers the atmosphere as a single slab that emits as a grey body in the infra-red with temperature T_a and fractional emittance ϵ_{IR}=0.6. We now instead divide the atmosphere into THREE layers with equal emissivity $\epsilon_{IR1} = \epsilon_{IR2} = \epsilon_{IR3}$. What is the value of ϵ_{IR1}?

   ii (4pts) Assuming that the atmosphere is transparent to solar radiation, and that the average incoming solar radiation at the TOA is $\frac{S_0}{4}(1-\alpha_p)$, where S_0 is the solar constanst and α_p is the albedo, and that equilibrium exists with the temperature in the three layers noted as T_1, T_2, T_3, derive the energy balance equation for the interface at top of the atmosphere and at the earth's surface. (DO NOT ATTEMPT TO SOLVE THESE EQUATIONS).

   iii (2pts) If the full set of balance equations were solved (DO NOT ATTEMPT THIS) to derive T_1, T_2, T_3, would the resulting temperature profile be convectively unstable, neutral or stable? Why is the derivation of T_1, T_2, T_3 using the assumption of radiative equilibrium a poor model of the atmosphere?

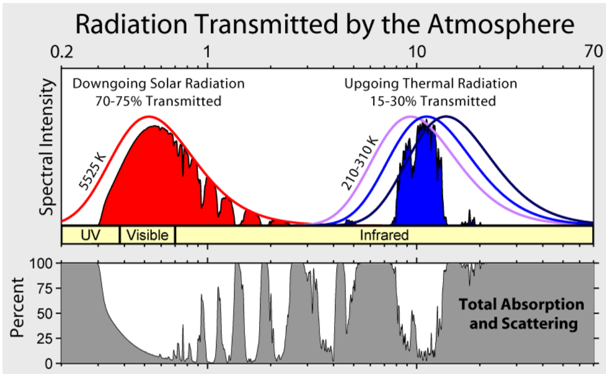

Figure 1: Upper panel shows the transmitted radiation in comparison to theoretical blackbody curves. Lower panels show the percentage absorptance. The X-axis shows wavelength, labled on the top of the upper panel with units of microns.

2. Radiation

   i (2pt) Take a look at Figure 1. If you wanted to design a satellite to measure the sea surface temperature using infra-red (long) wavelengths, which wavelengths would you use and why?

   ii (3pt) If man emits CO_2 to the atmosphere by burning fossil fuels does this increase or decrease the atmospheric infra-red emissivity? Is the change in emissivity linearly or logarithmically related to increasing CO_2 and why?

   iii (3pt) As man emits CO_2 to the atmosphere by burning fossil fuels, what happens to the mixing ratio of water vapour in the troposphere and why? State why this change is important when attempting to predict future temperatures as a result of CO_2 emission.

   iv (2pt) As man emits CO_2 to the atmosphere by burning fossil fuels, what happens to the mean temperature in the stratosphere and why?

   v (2pt) Does sea ice at high latitudes act as a positive or negative feedback in the climate system? Explain why this is in one or two sentences.

3. Microphysics

   i (2pt) A chance collision of 300 water vapour molecules forms a liquid droplet. What is the radius of the droplet?

   ii (2pt) In order to form a stable cloud droplet from the chance collisions of water vapour molecules, very high relative humidities are required of several hundred percent. Explain why such high relative humidities are **not** observed in the atmosphere at the surface.

iii (2pt) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water (of infinite radius, so we write $e_s(\infty)$). But in fact the saturation vapour over a water droplet of radius r and with a mass M of dissolved aerosol is modified by two factors - in a few sentences explain what the two correction factors refer to (use a sketch if helpful).

iv (3pt) A solute liquid droplet with a radius of 0.05 micron (μm) is present in an environment with a relative humidity of 95% and a temperature of $T = 273K$. Calculate the saturation ratio with respect to the droplet. Will the droplet grow or shrink?

v (2pt) Explain what it means for a haze particle to become activited

vi (2pt) Write down the an equation in terms of radius r that should be solved to calculate the radius at which the droplet becomes **activated**. (DO NOT ATTEMPT TO SOLVE IT!)

4. Microphysics

i (2pt) At cold temperature below -40C, ice crystals in the atmosphere can form by homogeneous nucleation from the liquid phase. Explain why ice supersaturations on the order of 40 to 60% are required for this process to occur?

ii (2pt) Ice can also form by heterogeneous nucleation on aerosols known as ice nuclei. From the two processes (hetero and homogeneous from liquid), which results in the highest ice crystal number concentration (m^{-3}) and why?

iii (1pt) State ONE characteristic that an aerosol requires in order to act as an ice nuclei.

iv (1pt) Apart from ice itself, name a substance that can act as an efficient ice nuclei.

v (2pt) Assuming the temperature is cold enough (less than -40C) for homogeneous nucleation from the liquid phase to take place, state two factors that will determine which process from the two ice nucleation mechanisms will dominate.

5. Convection

i (2pts) Take a look at the tephigram. The sun heats the surface until the trigger temperature is reached, what is that temperature? (PLEASE MARK YOUR CALCULATION ON THE TEPHIGRAM AND HAND IN).

ii (2pt) If the trigger temperature is reached and convection takes place, what is the cloud top temperature? What is that temperature otherwise known as?

iii (2pt) Would you use the trigger temperature to forecast whether squall lines would occur or not? You must justify your answer.

6. Convection

i (2pt) Take a look at the clouds in figure 2. What can you say about the thermodynamic state of the upper atmosphere from the presence of these clouds?

Figure 2: Examples of clouds formed by convectively unstable layers. (a) Altocumulus (b) Altocumulus undulus, resulting from vertical wind shear (c) cirrocumulus (source: www.theairlinepilots.com)

ii (2pt) In mid-latitudes, the presence of these clouds is a sign that rainfall may be on its way in the next day or so. Explain why this is the case.