Atmospheric Thermodynamics Exam 2011

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!** Note that the number of points available are stated at the start of each question.

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$p = \rho R T. \tag{1}$$

First Law:

$$dq = c_p dT - v dp, \tag{2}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{3}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{4}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{5}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{6}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta_v - \theta_{v-env}}{\theta_{v-env}}\right) \tag{7}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T-273.16)}{(T-32.19)}\right) \tag{8}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T-32.19)^2} \tag{9}$$

Mixing Ratio r_v

$$r_v = \frac{\rho_v}{\rho_d} \tag{10}$$

specific humidity

$$q_v = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v} \tag{11}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{12}$$

Virtual Temperature

$$T_v = T(1 + \frac{1-\epsilon}{\epsilon}r_v) \tag{13}$$

Virtual potential Temperature

$$\theta_v \equiv T_v\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{14}$$

Isobaric Equivalent temperature T_{ie}

$$T_{ie} = T + \frac{L_v}{c_p}r_v \tag{15}$$

Adiabatic Equivalent temperature T_e

$$T_e = T\mathrm{exp}\left(\frac{L_v r_v}{c_p T}\right). \tag{16}$$

surface heat fluxes

$$H = \rho C_p C_d \Delta\theta \Delta V \tag{17}$$

$$LE = \rho L C_d \Delta r_v \Delta V \tag{18}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S-1) \tag{19}$$

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{20}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{21}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{22}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{23}$$

Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Gas constant for Helium	R_{He}	2077	J kg^{-1} K^{-1}
Gas constant for Hydrogen	R_{H_2}	4124	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v} = \frac{m_v}{m_d}$	0.622	
Density of liquid water	ρ_l	1000	kg m^{-3}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between earth and sun	r_d	149.6×10^6	km
Solar Constant	S_0	1370	W m^{-2}
Planetary albedo	α_p	0.3	
Planck Constant	h	6.625×10^{-34}	J s
Constant c_1 in Planck's Law	c_1	3.74×10^{-16}	W m^{-2}
Constant c_2 in Planck's Law	c_2	1.45×10^{-2}	m K

Table 1: Constants

Questions

In all these questions, for simplicity, assume the definition of relative humidity defined in terms of mixing ratio: $RH = \frac{r_v}{r_s}$.

1. (2 pt) Dry air at 800 hPa is measured to have a temperature of 23^{o}C, while at 700 hPa the temperature is 20^{o}C, is this layer of the atmosphere dry stable, neutral or unstable? Show your calculation.

2. (3pt) Our by-now familiar air parcel of moist (but non-cloudy) air has a temperature of 26.9^{o}C. It is neutrally buoyant sitting in an environment that has a temperature of 27.0 ^{o}C and a mixing ratio of 11 g kg^{-1}. What is the mixing ratio in the air parcel?

3. (2pt) the temperature of the air parcel is increased by 0.5 ^{o}C, so that it is no longer neutrally buoyant. Assuming the resulting acceleration is maintained (i.e. constant) for 10 seconds, what is the final parcel velocity?

4. (2pt) name of all the following quantities that are conserved in *pseudoadiabatic moist saturated* ascent: mixing ratio r_v, pressure p, wet bulb potential temperature θ_w, (dry bulb) temperature T, potential temperature θ, dew point temperature T_d, vapour pressure e, wet bulb temperature T_w, adiabatic equivalent temperature T_e, equivalent potential temperature θ_e?

5. (1pt) Emma's mother buys her a helium filled balloon at the fairground in winter, where we just so happen to have standard temperature and pressure (STP) conditions. What is the density of the balloon, (neglecting the weight of the rubber balloon itself)?

6. (1pt) If Emma's father buys her a hydrogen balloon (careful not to go close to a flame!) of the same size, and both balloons are released, which one goes up more quickly?

7. (1pt) What atmospheric process is the dew point temperature T_d relevant for ?

8. (2pt) A parcel of air at time t=0 has a temperature T_0 of 25^{o}C and a relative humidity RH_0 of 0.1. The environment has a pressure of 1000 hPa. What is the mixing ratio r_{v0}?

9. (4pt) The parcel is cooled and moistened isobarically by the evaporation of precipitation to reach a final temperature of temperature of T_1 and a final mixing ratio of r_{v1}. If we ignore the humidity/precipitation in the parcel's heat capacity (i.e. heat capacity is c_p as for dry air)

and treat the latent heat of vaporization as a constant L_v that is independent of temperature, then we can relate the change in temperature to the change in mixing ratio as follows:

$c_p(T_1 - T_0) = -L_v(r_{v1} - r_{v0})$

If the final relative humidity is $RH_1 = r_{v1}/r_s(T_1) = 0.7$, what is the final temperature T_1 and final mixing ratio r_1?

(HINT: To solve this, you will need to linearize the saturation mixing ratio curve around temperature T_0, i.e. at a temperature T_1, $r_s(T_1) = r_s(T_0) + (T_1 - T_0)\frac{dr_s}{dT}|_{T_0}$ where $\frac{dr_s}{dT}|_{T_0}$ means $\frac{dr_s}{dT}$ calculated at temperature T_0.)

10. (2pt) If RH_1 were instead 1.0, what is the temperature T_1 commonly known as? What process in clouds is it commonly associated with?

11. (2pt) Explain in words why you can sometimes see your breath in winter when there are cold, damp conditions?

12. (1pt) Using the tephigram, what is the lifting condensation level (LCL) of air originating from 1000 hPa?

13. (1pt) and what is the Level of free convection (LFC)?

14. (2pts) The updraught cloudy air then mixing with the environment at 700 hPa, with an equal mass of cloudy air mixing with an equal mass of updraught air. The parcel is brought to saturation by the evaporation of precipitation. Can the final mixed, saturated parcel form a downdraught?

15. (1pt) Using the tephigram, what is the trigger temperature for convection?