Atmospheric Thermodynamics Exam 2011

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!** Note that the number of points available are stated at the start of each question.

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$p = \rho R T. \tag{1}$$

First Law:

$$dq = c_p dT - v dp, \tag{2}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{3}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{4}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{5}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{6}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta_v - \theta_{v-env}}{\theta_{v-env}}\right) \tag{7}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s (kg kg^{-1})as a function of pressure p (in Pa)and temperature T (measured in Kelvin):

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{8}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T - 32.19)^2} \tag{9}$$

Mixing Ratio r_v

$$r_v = \frac{\rho_v}{\rho_d} \tag{10}$$

specific humidity

$$q_v = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v} \tag{11}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{12}$$

Virtual Temperature

$$T_v = T(1 + \frac{1 - \epsilon}{\epsilon} r_v) \tag{13}$$

Virtual potential Temperature

$$\theta_v \equiv T_v \left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{14}$$

Isobaric Equivalent temperature T_{ie}

$$T_{ie} = T + \frac{L_v}{c_p} r_v \tag{15}$$

Adiabatic Equivalent temperature T_e

$$T_e = T\exp\left(\frac{L_v r_v}{c_p T}\right). \tag{16}$$

surface heat fluxes

$$H = \rho C_p C_d \Delta\theta \Delta V \tag{17}$$

$$LE = \rho L C_d \Delta r_v \Delta V \tag{18}$$

Microphysics

Approximate diffusion equation for radius $r > 1\ \mu m$ droplets:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S-1) \tag{19}$$

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{20}$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho sec\theta dz. \tag{21}$$

Transmittance τ is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \tag{22}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{23}$$

Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Ratio of gas constants	$\epsilon = \frac{R_d}{R_v} = \frac{m_v}{m_d}$	0.622	
Density of liquid water	ρ_l	1000	kg m^{-3}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2s^{-1}$
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between earth and sun	r_d	149.6×10^6	km
Solar Constant	S_0	1370	W m^{-2}
Planetary albedo	α_p	0.3	
Planck Constant	h	6.625×10^{-34}	J s
Constant c_1 in Planck's Law	c_1	3.74×10^{-16}	W m^{-2}
Constant c_2 in Planck's Law	c_2	1.45×10^{-2}	m K

Table 1: Constants

Questions

1. Buoyancy and stability

   i (3pt) Our by-now familiar air parcel of moist (but non-cloudy) air has a temperature of 24.9^{o}C. It is neutrally buoyant sitting in an environment that has a temperature of 25.0 ^{o}C and a mixing ratio of 10 g kg^{-1}. What is the mixing ratio in the air parcel?

   ii (2pt) the temperature of the air parcel is increased by 1 ^{o}C, so that it is no longer neutrally buoyant. Assuming the resulting acceleration is maintained (i.e. constant) for 10 seconds, what is the final parcel velocity?

   iii (2 pt) Dry air at 1000 hPa is measured to have a temperature of 27^{o}C, while at 900 hPa the temperature is 22^{o}C, is this layer of the atmosphere dry stable, neutral or unstable? Show your calculation.

2. Fog

   In this question, for simplicity, assume the definition of relative humidity defined in terms of mixing ratio: $RH = \frac{r_v}{r_s}$.

   i (2pt) The environmental air with temperature of 25.0 ^{o}C and a mixing ratio of 10 g kg^{-1} is located near the surface at a pressure of 1000hPa. What is the relative humidity?

   ii (1pt) The airstarts to cool isobarically at nightnight until eventually a fog forms. Assuming the pressure is 1000hPa. Which is the temperature at which the fogs occurs? (Use the tephigram to answer this, please mark the answer on the tephigram)

   iii (2pt) Derive this same temperature by inverting Teton's formula, showing your working. (Out of interest note how close the answer is to your tephigram result).

   iv (1pt) What is the common name given to this temperature?

3. Relative humidity

   In this question, for simplicity, assume the definition of relative humidity defined in terms of mixing ratio: $RH = \frac{r_v}{r_s}$.

   i (2pt) A parcel of air at time t=0 has a temperature T_0 of 25^{o}C and a relative humidity RH_0 of 0.1. What is the mixing ratio r_{v0}?

   ii (4pt) The parcel is cooled and moistened isobarically by the evaporation of precipitation to reach a final temperature of temperature of T_1 and a final mixing ratio of r_{v1}. If we ignore the humidity/precipitation in the parcel's heat capacity (i.e. heat capacity is c_p as for dry air) and treat the latent heat of vaporization as a constant L_v that is independent of temperature, then we can relate the change in temperature to the change in mixing ratio as follows:

$c_p(T_1 - T_0) = -L_v(r_{v1} - r_{v0})$

If the final relative humidity is $RH_1 = r_{v1}/r_s(T_1) = 0.6$, what is the final temperature T_1 and final mixing ratio r_1?

(HINT: To solve this, you will need to linearize the saturation mixing ratio curve around temperature T_0, i.e. at a temperature T_1, $r_s(T_1) = r_s(T_0) + (T_1 - T_0)\frac{dr_s}{dT}|_{T_0}$ where $\frac{dr_s}{dT}|_{T_0}$ means $\frac{dr_s}{dT}$ calculated at temperature T_0.)

iii (1pt) If RH_1 were instead 1.0, what is the temperature T_1 commonly known as?

4. Tephigram

In this question we use the attached tephigram with the thermodynamic profiles

i (1pt) What is the lifting condensation level (LCL) of air originating from 700 hPa?

ii (2pt) Does the wet bulb potential temperature θ_w increase or decrease between 700 and 600 hPa?

iii (1 pt) Given the answer to the previous question, you can now state whether the equivalent potential temperature is higher at 600 or 700 hPa, why?

iv (3 pts) Air at 700 hPa is brought to saturation by the evaporation of rainfall. It then decends to 1000 hPa, and during this descent is subject to further rainfall evaporation such that it arrives at 1000hPa still saturated. What is the final temperature?

5. Boundary Layer

In this question, for simplicity, assume the definition of relative humidity defined in terms of mixing ratio: $RH = \frac{r_v}{r_s}$.

i (2pt) why is the temperature diurnal cycle greater for land compared with the sea

ii (1pt) What is the definition of the Bowen ratio?

iii (4 pt) Somewhere nice in the Tropics, boundary layer air with a temperature of 25^{o}C and a relative humidity of 80% is blowing over an ocean surface which has a temperature of 27^{o}C. Assuming the surface pressure is 1000hPa, what is the Bowen ratio of the surface fluxes?