

# Atmospheric Physics Exam 2011

## Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!** Note that the number of points available are stated at the start of each question.

## Formulae

Most formulae required are given here:

**Thermodynamics** Ideal gas law:

$$p = \rho R_m T. \quad (1)$$

First Law:

$$dq = c_p dT - v dp, \quad (2)$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T} \left( \frac{dT}{dz} + \frac{g}{c_p} \right). \quad (3)$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \quad (4)$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \quad (5)$$

Potential temperature:

$$\theta = T \left( \frac{p_0}{p} \right)^{\frac{R_d}{c_p}}. \quad (6)$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g \left( \frac{\theta - \theta_{env}}{\theta_{env}} \right) \quad (7)$$

where  $\theta$  is potential temperature and  $env$  refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio  $r_s$  ( $\text{kg kg}^{-1}$ ) as a function of pressure  $p$  (in Pa) and temperature  $T$  (measured in Kelvin):

$$r_s(T) = \frac{380}{p} \exp\left(17.5 \frac{(T - 273.16)}{(T - 32.19)}\right) \quad (8)$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s \frac{4217}{(T - 32.19)^2} \quad (9)$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \quad (10)$$

### Microphysics

Approximate diffusion equation for radius  $r > 1 \mu\text{m}$  droplets:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T} (S - 1) \quad (11)$$

### Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \quad (12)$$

Optical Thickness/Depth:

$$\delta_\lambda = \int_{z_1}^{z_2} k_\lambda^e \rho \sec\theta dz. \quad (13)$$

Transmittance  $\tau$  is related to optical depth by

$$\tau_\lambda = e^{-\delta_\lambda} \quad (14)$$

solid angle

$$\Omega = \frac{A}{r^2} \quad (15)$$

Gas constant for dry air	$R_d$	287.06	$\text{J kg}^{-1} \text{K}^{-1}$
Gas constant for vapour	$R_v$	461.5	$\text{J kg}^{-1} \text{K}^{-1}$
Density of liquid water	$\rho_l$	1000	$\text{kg m}^{-3}$
Universal Gas Constant	$R$	8.314	$\text{J K}^{-1} \text{mol}^{-1}$
Specific heat capacity at constant pressure for dry air	$c_p$	1005	$\text{J kg}^{-1} \text{K}^{-1}$
Specific heat capacity at constant volume for dry air	$c_v$	718	$\text{J kg}^{-1} \text{K}^{-1}$
Latent heat of vaporization	$L_v$	$2.5 \times 10^6$	$\text{J kg}^{-1}$
Vapour diffusion coefficient	$D$	$\approx 2.2 \times 10^{-5}$	$\text{m}^2 \text{s}^{-1}$
Stefan Boltzmann constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{Wm}^{-2} \text{K}^{-4}$
radius of the earth	$r_e$	6340	km
radius of the sun	$r_s$	$0.7 \times 10^6$	km
distance between earth and sun	$r_d$	$149.6 \times 10^6$	km
Solar Constant	$S_0$	1370	$\text{W m}^{-2}$
Planetary albedo	$\alpha_p$	0.3	
Planck Constant	$h$	$6.625 \times 10^{-34}$	J s
Constant $c_1$ in Planck's Law	$c_1$	$3.74 \times 10^{-16}$	$\text{W m}^{-2}$
Constant $c_2$ in Planck's Law	$c_2$	$1.45 \times 10^{-2}$	m K

Table 1: Constants

## Questions

### 1. Radiation: Earth's energy balance (12pt)

- i (2pt) The average irradiance per unit area ( $\text{W m}^{-2}$ ) at the top of a planet's atmosphere is known as the *Solar Constant* and for earth the table gives  $S_0=1370 \text{ Wm}^{-2}$ .

We assume that the absorbed solar radiation is balanced by radiation emitted by the earth/atmosphere system, treated as a black body with an effective emitting temperature  $T_e$ .

Derive the relationship (show a sketch and show your working) for  $T_e$  in terms of  $S_0$ ,  $r_e$  the radius of the Earth and the albedo  $\alpha_p$  and then give  $T_e$  assuming  $\alpha_p=0.3$ .

- ii (1pt) The annual mean surface temperature of the earth  $T_s$  is roughly 288 K. Is your derived  $T_e$  higher or lower than this value and state why in ONE sentence.
- iii (2pt) What fraction of the radiative flux emitted by the sun does the earth intercept?
- iv (2pt) If we treat the atmosphere as a single slab (Fig. 1) that emits as a grey body in the infra-red with temperature  $T_a$  and fractional emittance  $\epsilon_{ir}$ . In the solar spectrum we assume it has an absorptance of  $\epsilon_{sol}$ . The atmosphere overlies the earth's surface with temperature  $T_s$ . Assuming the solar flux through the interface at the top of the atmosphere is given by your equation above -

**Derive the energy balance equation for the interface at top of the atmosphere**

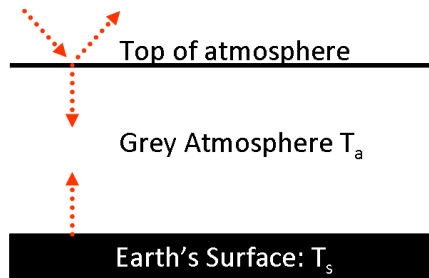


Figure 1: Sketch of the greenhouse slab model

- v (2pt) **Derive the energy balance equation for the atmospheric slab**
  - vi (2pt) Using these two expressions, give the value for  $T_s$  assuming a value of  $\epsilon_{ir}=0.6$  and  $\epsilon_{sol}=0.1$
  - vii (1pt) In class we derived the atmospheric and surface temperatures solving the surface and atmospheric balance equations. We could have taken the calculation further and divided the atmosphere up vertically into many slices to calculate the atmospheric temperature profile in a state of “radiative equilibrium”. Why would this be a poor representation of the true atmospheric vertical temperature profile?
- i (2pt)  $a=\epsilon$  and  $\tau=1 - \epsilon$
  - ii atmosphere slab:

$$\epsilon_{sol} \frac{S_0}{4} (1 - \alpha_p) + \epsilon_{ir} \sigma T_s^4 = 2\epsilon_{ir} \sigma T_a^4 \quad (16)$$

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- iii TOA:

$$\frac{S_0}{4} (1 - \alpha_p) = \sigma T_s^4 (1 - \epsilon_{ir}) + \epsilon_{ir} \sigma T_a^4 \quad (17)$$

- iv

$$T_a^4 = \frac{\epsilon_{sol} \frac{S_0}{4} (1 - \alpha_p) + \epsilon_{ir} \sigma T_s^4}{2\epsilon_{ir} \sigma} \quad (18)$$

substituted into TOA equation gives

$$T_s = \sqrt[4]{\frac{S_0(1 - \alpha_p)(1 - \frac{\epsilon_{sol}}{2})}{2\sigma(2 - \epsilon_{ir})}} \quad (19)$$

the values for  $\epsilon$  give  $T_s=275.2$  K

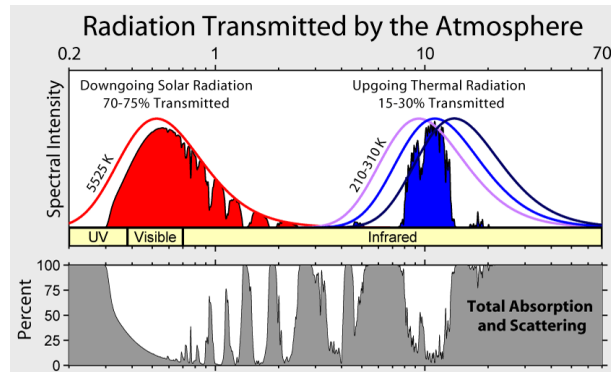


Figure 2: Upper panel shows the transmitted radiation in comparison to theoretical blackbody curves. Lower panels show the percentage absorptance

2. Radiation: general concepts and definitions (7pt)

- i (3pt) Take a look at Figure 2. Approximately 75% of the solar radiation is transmitted by the atmosphere. Give the THREE major contributions to the 25% extinction by the atmosphere.
- ii (1pt) A parallel beam radiation of wavelength  $\lambda$  is passing vertically through a layer of 1 km in thickness, containing a homogeneous gas with a density of  $0.01 \text{ kg m}^{-3}$  with an absorption coefficient  $k_{\lambda}^e = 0.1 \text{ m}^2 \text{ kg}^{-1}$  and a zero scattering coefficient. What is the optical thickness of the layer?
- iii (1pt) What is the transmissivity  $\tau$  of the layer?
- iv (2pt) How thick would the layer have to be to absorb half of the incidence radiation beam?
  - i Rayleigh scattering by NO<sub>2</sub>, absorption by ozone/oxygen and water vapour
  - ii optical thickness  $\delta_{\lambda} = \int_{z_1}^{z_2} k_{\lambda}^e \rho \sec\theta dz$  thus  $1000 \times 0.01 \times 0.1 = 1$  (no units)
  - iii transmission =  $\exp(-d) = 0.367$
  - iv  $\exp(-k \cdot \rho \cdot dz) = 0.5 \Rightarrow dz = 693 \text{ m}$
  - v  $P$  is independent of direction
  - vi no since the wavelength of the radiation is much smaller than the scattering droplet size

3. Thermodynamics (7pt)

In this question, for simplicity, assume the definition of relative humidity defined in terms of mixing ratio:  $RH = \frac{r_v}{r_s}$ .

i (2pt) A parcel of air at time  $t=0$  has a temperature  $T_0$  of  $25^\circ\text{C}$  and a pressure of 1000 hPa and a relative humidity  $RH_0$  of 0.1. What is the mixing ratio  $r_{v0}$ ?

ii (4pt) The parcel is cooled and moistened isobarically by the evaporation of precipitation to reach a final temperature of temperature of  $T_1$  and a final mixing ratio of  $r_{v1}$ . If we ignore the humidity/precipitation in the parcel's heat capacity (i.e. heat capacity is  $c_p$  as for dry air) and treat the latent heat of vaporization as a constant  $L_v$  that is independent of temperature, then we can relate the change in temperature to the change in mixing ratio as follows:

$$c_p(T_1 - T_0) = -L_v(r_{v1} - r_{v0})$$

If the final relative humidity is  $RH_1 = r_{v1}/r_s(T_1) = 0.6$ , what is the final temperature  $T_1$  and final mixing ratio  $r_1$ ?

(HINT: To solve this, you will need to linearize the saturation mixing ratio curve around temperature  $T_0$ , i.e. at a temperature  $T_1$ ,  $r_s(T_1) = r_s(T_0) + (T_1 - T_0)\frac{dr_s}{dT}|_{T_0}$  where  $\frac{dr_s}{dT}|_{T_0}$  means  $\frac{dr_s}{dT}$  calculated at temperature  $T_0$ .)

iii (1pt) If  $RH_1$  were instead 1.0, what is the temperature  $T_1$  commonly known as?

$$c_p(T_1 - T_0) = -L_v(RH_1 r_s(T_1) - r_{v0})$$

$$c_p(T_1 - T_0) = -L_v(RH_1(c_p(T_1 - T_0) = -L_v(RH_1 r_s(T_1) - r_{v0})) - r_{v0})$$

thus

$$T_1 = \frac{-\frac{L_v}{c_p}(RH_1 - RH_0)r_s(T_0)}{1 + RH_1 \frac{L_v}{c_p} \frac{dr_s}{dT}|_{T_0}}$$

#### 4. Microphysics (7 pt)

i (2pt) In order to form a stable liquid cloud droplet from the chance collisions of water vapour molecules, very high relative humidities are required of several hundred percent. Explain why such high relative humidities are **not** observed in the atmosphere.

ii (2pt) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water (of infinite radius, so we write  $e_s(\infty)$ ). But in fact the saturation vapour over a water droplet of radius  $r$  and with a mass  $M$  of dissolved impurity is modified by two factors  $(1 - \frac{b}{r^3})$  and  $\exp(\frac{a}{rT})$ , thus:

$$e_s = e_s(\infty) \left(1 - \frac{b}{r^3}\right) \exp\left(\frac{a}{rT}\right)$$

where  $a$  and  $b$  are constants. In a few sentences, explain what the two correction factors refer to (use a sketch if helpful).

iii (3pt)

Assuming these constants  $a = 3.3 \times 10^{-7} \text{ m K}$  and  $b = 1.47 \times 10^{-23} \text{ m}^3$ , a solution droplet with a radius of 0.1 micron ( $\mu\text{m}$ ) exists in an ambient environment with a temperature of  $T = 273\text{K}$  and supersaturation *over a planar surface* of 0.5% (i.e. the saturation ratio  $S = \frac{e}{e_s(\infty)} = 1.005$ ). Will the droplet grow or evaporate (show your working)?

5. Microphysics (9 pt)

- i (2 pt) Show how long an activated droplet of radius  $0.5 \mu\text{m}$  in an environment of  $S = 1.002$  and  $T = 284\text{K}$  (which gives  $e_s(\infty) = 1300\text{Pa}$ ), takes to grow by the process of diffusion to a size of  $1000 \mu\text{m}$  respectively. (Assume that this process does not alter  $S$ ).
- ii (2 pt) Explain in a sentence or two what this tells you about the role of diffusion in creating raindrops
- iii (2pt) In an air mass with a CCN concentration of  $N$ , a mass  $L$  of water condenses and forms a cloud. Assuming all the condensed water is shared equally among the CCN, write down a relationship for the droplet radius in terms of  $L$ ,  $N$  and the density of liquid water  $\rho_L$ .
- iv (2pt) In two clouds a mass  $L = 5 \times 10^{-4} \text{kgm}^{-3}$  of water condenses. In the first cloud the CCN concentration is  $N = 100 \text{cm}^{-3}$  and in the second the concentration is  $N = 1000 \text{cm}^{-3}$ . What is the droplet radius in each cloud?
- v (1pt) What might the different concentrations of CCN indicate about the locations of the clouds?

6. Microphysics (8 pt)

- i (2pt) Another process important in clouds is collision and coalescence where we imagine a large raindrop of radius  $R$  falling at its terminal velocity  $V_t$  collecting other smaller, slower moving droplets. Give the relationship for the volume of air swept out by the larger droplet in time  $t$ .
- ii (2pt) We assume the cloud droplets of radius  $r$  are much smaller than the raindrop,  $r \ll R$ , and that their terminal velocities are also negligibly small (we assume they are at rest), What is the rate of change of mass  $dM/Dt$  of the raindrop for a cloud of absolute liquid water density  $L$  ( $\text{kg m}^{-3}$ )?

- iii (2pt) The rate of change of mass is related to the raindrop radius by  $\frac{dM}{dt} = 4\pi R^2 \rho_L \frac{dr}{dt}$ . Assuming a terminal fallspeed of  $V_t = X_1 r^2$  where  $X_1 = 1.2 \times 10^8 \text{s}^{-1} \text{m}^{-1}$ , calculate the time taken for a droplet to grow from  $20 \mu\text{m}$  to  $30 \mu\text{m}$  by this process in a cloud with mean absolute liquid water density of  $L = 1.0 \times 10^{-3} \text{kg m}^{-3}$ .
- iv (2pt) Growth rates from this process are lower than we calculated here. Apart from the stated assumption  $r \ll R$ , state two further approximations that have been made in this calculation, that if corrected for, lead to the lower growth rates.

7. Microphysics (6 pt)

- i (2pt) Describe the two nucleation mechanisms by which ice crystals can form in the atmosphere in one sentence.
- ii (2pt) which of these mechanisms creates the high ice crystal number concentration ( $\text{m}^{-3}$ ) and why?
- iii (2pt) Does the *riming* process increase or decrease the terminal fall speed of ice crystals and why?

8. Convection (6 pt)

Use the tephigram to graphically this question and write your answers on the tephigram sheet- please hand it in with your name clearly marked on the top with your answers.

- i (3pt) The surface parcel of this profile has properties of a temperature of  $T=25 \text{C}$  and a mixing ratio of  $16 \text{g/kg}$  (marked with the red and blue circles). The parcel undergoes forced ascent such that it reaches its level of free convection and forms a convective cloud. Plot the parcel ascent and **give the pressure (hPa) of the (i) LCL lifting condensation level (ii) level of free convection and (iii) level of neutral buoyancy (LNB)**
- ii (3pt) The saturated updraught air at  $850 \text{hPa}$  undergoes a mixing event with its environment, such that air is formed of  $50\%$  updraught air and  $50\%$  environmental air. This mixed air parcel is then brought to saturation by the evaporation of precipitation falling from above. **(i) What is this final temperature and (ii) can it form a saturated downdraught?**