Atmospheric Physics Exam 2010

Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to check the UNITS in the question and state the UNITS in your answer!!!** Note that the number of points available are stated at the start of each question.

Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$p = \rho R_m T. \tag{1}$$

First Law:

$$dq = c_p dT - v dp, \tag{2}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{3}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{4}$$

Clausius Clapeyron Equation for saturation vapour pressure over a planar water surface:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{5}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{6}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{7}$$

where θ is potential temperature and *env* refers to the environment of the parcel.

Teton's formula for the saturation mixing ratio r_s as a function of pressure p and temperature T:

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T-273.16)}{(T-32.19)}\right) \tag{8}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T-32.19)^2} \tag{9}$$

Relative humidity

$$RH = \frac{e}{e_s} \approx \frac{r_v}{r_s}. \tag{10}$$

Microphysics

Approximate diffusion equation for radius $r > 1$ μm droplets:

$$\frac{dr}{dt} \simeq \frac{De_s(\infty)}{\rho_L r R_v T}(S-1) \tag{11}$$

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{12}$$

solid angle

$$\Omega = \frac{A}{r^2} \tag{13}$$

Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	$m^2 s^{-1}$
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7×10^6	km
distance between earth and sun	r_d	149.6×10^6	km
Solar Constant	S_0	1370	W m^{-2}
Planetary albedo	α_p	0.3	
Planck Constant	h	6.625×10^{-34}	J s
Constant c_1 in Planck's Law	c_1	3.74×10^{-16}	W m^{-2}
Constant c_2 in Planck's Law	c_2	1.45×10^{-2}	m K

Table 1: Constants

Questions

Points per question

1. 9
2. 11
3. 9
4. 11
5. 9
6. 11

1. Radiation: Earth's energy balance (9pt)

   i (1pt) State the definition of planetary albedo α_p.

   ii (3pt) The average irradiance per unit area (W m^{-2}) at the top of a planet's atmosphere is known as the *Solar Constant* and for earth the table gives S_0=1370 Wm^{-2}. We assume that the absorbed solar radiation is balanced by radiation emitted by the earth, treated as a black body with an effective emitting temperature T_e.

   Derive the relationship (show a sketch and show your working) between T_e, S_0, r_e and α_p, where r_e is the radius of the Earth. Then give T_e assuming α_p=0.3

   iii (1pt) The annual mean surface temperature of the earth is roughly 288 K. Is your derived T_e higher or lower than this value and state why in ONE sentence.

   iv (2pt) A grey body emits radiation according to

$E_{\text{grey}} = \epsilon\sigma T^4$, where ϵ is the fractional emittance ($\epsilon \leq 1$). According to Kirchoff's Law what is the body's Transmittance τ and Absorptance a? (2 pt)

v (1pt) If we treat the atmosphere as a single slab that emits as a grey body in the infra-red with temperature T_a and fractional emittance ϵ, and does not absorb solar radiation, overlying the earth's surface with temperature T_s. Derive the energy balance equation for the atmospheric slab.

vi (1pt) In class we derived the atmospheric and surface temperatures solving the surface and atmospheric balance equations. We could have taken the calculation further and divided the atmosphere up vertically into many slices to calculate the atmospheric temperature profile in a state of "radiative equilibrium". Why would this be a poor representation of the true atmospheric vertical temperature profile?

2. Radiation: general concepts and definitions (11pt)

   i (1pt) In ONE sentence, explain what is the scattering phase function $P(\theta', \phi', \theta, \phi)$ where θ and ϕ are azimuth and elevation angles?

   ii (1pt) What simplifying approximation we make about P in the Rayleigh scattering regime?

   iii (1pt) Can scattering of solar radiation by clouds be treated using the Rayleigh approximation (yes/no) and why?

   iv (2pt) A cloud contains droplets that scatter 99% of the incident light at visible wavelengths in the forward direction. Explain why clouds that are optically thick can have albedo exceeding 70% and are dark when viewed from below.

   v (2pt) Atmospheric gases absorb radiation at specific wavelengths which are not discrete, but are broadened by the line broadening mechanisms of pressure broadening, Doppler broadening and natural broadening. Which one dominates in the lower troposphere and which one dominates in the stratosphere?

   vi (2pt) With the aid of a sketch of absorbing bands (the x axis should be wavelength or wavenumber), explain what it means for an atmospheric gas to be absorbing in the "strong limit" and the "weak limit".

   vii (2pt) On your sketch, mark the wavelength (or wavenumber) where a satellite would measure in order to "retrieve" the amount of a gas, and state in ONE sentence why you chose this wavelength.

3. Basic Thermodynamics (9pt)

   i (4 pt) Dry air at 1000 hPa is measured to have a temperature of 300K, while at 900 hPa the temperature is 295K, is this layer of the atmosphere dry stable, neutral or unstable and why? (show your reasoning).

   ii (2pt) name of all the following quantities that are conserved in pseudoadiabatic *saturated* ascent: mixing ratio r_v, pressure p, wet bulb potential temperature θ_w, (dry bulb) temperature T, potential temperature θ, dew point temperature T_d, total water mixing ratio r_t, vapour pressure e, wet bulb temperature T_w, adiabatic equivalent temperature T_e, equivalent potential temperature θ_e?

   iii (1pt) What atmospheric process is the dew point temperature T_d relevant for ?

iv (2pt) If a parcel of air at 1000 hPa has a temperature T=300K and dew point temperature of T_d=293K, what is the relative humidity? (use the approximate form for relative humidity: $RH \approx \frac{r_v}{r_s}$).

4. Thermodynamics: The wet bulb process (11 pt)

   i (1pt) Name an atmospheric process that the wet bulb temperature T_w is relevant for.

   ii (1pt) An air parcel with initial temperature T_0 and mixing ratio r_0 is cooled by a wet bulb process to reach it's wet bulb temperature of T_w, when it has a final mixing ratio of r_1. If we ignore the humidity adjustment to the heat capacity and treat the latent heat as constant then we can relate the change in temperature to the change in mixing ratio as follows:
   $c_p(T_0 - T_w) = L_v(r_1 - r_0)$
   How is r_1 related to T_w?

   iii (4pt) To calculate T_w we can linearize the saturation mixing ratio curve, i.e. at a temperature T, $r_s(T) = r_s(T_0) + (T - T_0)\frac{dr_s}{dT}|_{T_0}$ where $\frac{dr_s}{dT}|_{T_0}$ means $\frac{dr_s}{dT}$ calculated at temperature T_0.
   Using this linearization, derive the expression for T_w and give the value for T_w that it provides.

   iv (2pt) What is the corresponding value for r_1?

   v (1pt) Calculate $r_s(T_w)$.

   vi (2pt) The above linearization is often used to model the wet bulb process in computer models, can you suggest a way in you could make the process more accurate (i.e. to bring your answer for r_1 closer to $r_s(T_w)$)?

5. Microphysics (9 pt)

   i (2pt) In order to form a stable cloud droplet from the chance collisions of water vapour molecules, very high relative humidities are required of several hundred percent. Explain why such high relative humidities are **not** observed in the atmosphere at the surface.

   ii (3pt) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water (of infinite radius, so we write $e_s(\infty)$). But in fact the saturation vapour over a water droplet of radius r and with a mass M of dissolved impurity is modified by two factors $\left(1 - \frac{b}{r^3}\right)$ and $\exp\left(\frac{a}{rT}\right)$, thus:
   $e_s = e_s(\infty)\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right)$

where a and b are constants. In a few sentences, explain what the two correction factors refer to (use a sketch if helpful).

iii (4pt) A sample of air is supersaturated by 1% *over a planar surface* (i.e. the saturation ratio $S = \frac{e}{e_s(\infty)} = 1.01$). Calculate the saturation ratio over a droplet with a radius of 0.1 micron (μm) at a temperature of $T = 273K$ with constants $a = 3.3 \times 10^{-7}$ m K and $b = 1.47 \times 10^{-23}$ m^3. Will the droplet grow or evaporate?

6. Ice Microphysics (11 pt)

i (2pt) Name two properties that an atmospheric aerosol should have to be an effective ice nuclei.

ii (1pt) Aerosols can become effective ice nuclei at supersaturations of order 10%, and yet often supersaturations **with respect to ice** in excess of 50% can be measured in the upper atmosphere at cold temperatures. In ONE sentence explain why this is the case.

iii (2pt) The observation of which kind of cloud *with which characteristic* provides evidence of a supersaturated upper troposphere?

iv (2pt) Homogeneous nucleation of ice from the vapour phase requires supersaturations with respect to ice of many thousands of percent, but measured supersaturations never exceed approximately 70%, explain in one or two sentences why this is the case.

v (2 pt) Two air parcels at very cold temperature ($< 200K$) are lifted until ice crystals form. The two parcels are identical (same starting pressure, temperature and vertical velocity) except that parcel A contains no ice nuclei, while parcel B contains some aerosols that act as ice nuclei. At the end of the process (i.e. when the ice crystals have grown by diffusion and the relative humidity with respect to ice has reduced back towards 100%), which parcel contains the most ice crystals and why?

vi (2pt) Now imagine two parcels of air at -15^oC containing an ice crystal, both with a supersaturation of 10% with respect to ice. Parcel A contains many liquid cloud droplets while parcel B has none. In which parcel does the ice crystal grow fastest and why?