1 Instructions

Do your best to answer all questions in the time allowed. **ALWAYS remember to state the UNITS!!!**

Note that not all questions have the same points, and the total exceeds 100 pts, as some of the harder questions parts are bonus questions.

Q1 = 18 pts, Q2 = 22 pts, Q3 = 17 pts, Q4 = 23 pts, Q5 = 30 pts.

2 Formulae

Most formulae required are given here:

Thermodynamics Ideal gas law:

$$p = \rho R_m T. \tag{1}$$

First Law:

$$dq = c_p dT - v dp, \tag{2}$$

Potential temperature Lapse Rate:

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{3}$$

Hydrostatic balance:

$$\frac{dp}{dz} = -\rho g \tag{4}$$

Clausius Clapeyron Equation:

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2} \tag{5}$$

Potential temperature:

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}. \tag{6}$$

Vertical momentum equation relating the vertical acceleration to the buoyancy force:

$$\frac{dw}{dt} = F_B = g\left(\frac{\theta - \theta_{env}}{\theta_{env}}\right) \tag{7}$$

where θ is potential temperature and env refers to the environment of the parcel.

Microphysics

Approximate diffusion equation for $r > 1\ \mu m$ droplets:

$$\frac{dr}{dt} \simeq \frac{D e_s^{Clausius}}{\rho_L r R_v T}(S - 1) \tag{8}$$

Bolton's empirical formula for the saturation vapour pressure as a function of T in ^{o}C:

$$e_s^{Clausius}(T) = 611.2exp\left(\frac{17.67T}{T+243.5}\right) \tag{9}$$

Radiation

Stephan-Boltzmann Law for black body emission :

$$E = \sigma T^4 \tag{10}$$

Gas constant for dry air	R_d	287.06	J kg^{-1} K^{-1}
Gas constant for vapour	R_v	461.5	J kg^{-1} K^{-1}
Density of liquid water	ρ_l	1000	kg m^{-3}
Universal Gas Constant	R	8.314	J K^{-1} mol^{-1}
Liquid water density	ρ_L	1000	kg m^{-3}
Specific heat capacity at constant pressure for dry air	c_p	1005	J kg^{-1} K^{-1}
Specific heat capacity at constant volume for dry air	c_v	718	J kg^{-1} K^{-1}
Latent heat of vaporization	L_v	2.5×10^6	J kg^{-1}
Vapour diffusion coefficient	D	$\approx 2.2 \times 10^{-5}$	m$^2 s^{-1}$
Stefan Boltzmann constant	σ	5.67×10^{-8}	Wm^{-2} K^{-4}
radius of the earth	r_e	6340	km
radius of the sun	r_s	0.7 $\times 10^6$	km
distance between earth and sun	r_d	149.6 $\times 10^6$	km
Solar Constant	S_0	1370	W m^{-2}
Planck Constant	h	6.625×10^{-34}	J s
Constant c_1 in Planck's Law	c_1	3.74 $\times 10^{-16}$	W m^{-2}
Constant c_2 in Planck's Law	c_2	1.45 $\times 10^{-2}$	m K

Table 1: Constants

3 Questions

Question 1: Thermodynamics

Take a look at the tephigram attached to the exam sheet, where the red line represents temperature and the blue dashed line humidity. In the following answer to TWO-SIGNIFICANT figures only are required, e.g. 660 hPa, or 32 C.

i) Is the boundary layer between 1000 and 950 hPa well-mixed? (1pt)

ii) If this parcel of air originating from the surface at 1000 hPa is forcibly lifted by rising over a mountain. What would the height of the mountain (give the pressure at the mountain top) in order to cause deep convection? (1pt)

iii) Instead deep convection is triggered by the sun warming the surface during the day, (without changing the boundary layer humidity) until it reaches the trigger temperature, what is this temperature? (1pt)

iv) What would be the cloud top height in hPa in this case? (1pt)

v) In the deep convective cloud, what would be the temperature excess with respect to the cloud's environment at p=600 hPa? (1pt)

vi) Do you think this estimated value from the tephigram is larger or smaller than it would be in reality? (1pt)

vii) Why? State two processes that are neglected.(2pt)

viii) What is the **potential temperature** lapse rate $d\theta_{env}/dp$ of the layer between 320hPa and 420 hPa (in units of K Pa^{-1})? (1pt)

ix) Is this layer stable, neutral, or unstable for ascent of a environmental parcel originating at 420 hPa? (1pt)

x) Assuming hydrostatic balance and the ideal gas law, and using the mid-layer values of temperature and pressure, what is the **potential temperature** lapse rate $d\theta_{env}/dz$ of the layer between 320hPa and 420 hPa? (in units of K km^{-1})? (2pt)

xi) The acceleration of a parcel of air starting at 420 hPa is related to the buoyancy force. In the following question use an approximate form, where the denominator is replaced by the parcel's temperature θ_p.

$$\frac{dw}{dt} \approx g\left(\frac{\theta_p - \theta_{env}(z)}{\theta_p}\right) \tag{11}$$

We shall consider a *small* displacement of the parcel, so that we can approximate the change in environment temperature using a linearization (first order Taylor series expansion) about the original level:

$$\theta_{env}(z) = \theta_{env}(0) + \frac{d\theta_{env}}{dz} z = \theta_{env}(0) + \Gamma z \tag{12}$$

where $\Gamma = \frac{d\theta_{env}}{dz}$ is the environmental lapse rate. Using this vertical momentum equation and assuming $\frac{dw}{dt} = w\frac{dw}{dz}$, derive and write down the expression for the parcel vertical velocity w in terms of height z. [4 pts]

xii) If an environmental air parcel at 420 hPa has an initial vertical velocity of 1 m s^{-1}, calculate the maximum vertical displacement of the parcel (in metres). [2pts]

Question 2: Thermodynamics

For the following, use this approximate empirical formula for the saturation mixing ratio r as a function of pressure p and temperature T:

$$r_s(T) = \frac{380}{p} exp\left(17.5\frac{(T - 273.16)}{(T - 32.19)}\right) \tag{13}$$

which can be differentiated to give:

$$\frac{dr_s(T)}{dT} = r_s\frac{4217}{(T - 32.19)^2} \tag{14}$$

You will also assume the approximate version of the First Law that accounts for latent heating $dq = c_p dT - vdp + L_v dr_s$.

A non-saturated air parcel at pressure p_0=800 hPa has temperature T_0=280 K and mixing ratio r_0=7 g kg^{-1}.

i) Using eqn. 13, calculate the dew point temperature showing your method. (Clue: note that $\frac{T-a}{T-b} = \frac{T-b+b-a}{T-b} = 1 + \frac{b-a}{T-b}$) (3pts)

ii) Through an isobaric process, humidity is added to the parcel, and it becomes saturated. Compute the saturation mixing ratio at the point the parcel becomes saturated [2 pts]

iii) Further humidity is added to the parcel isobarically. Considering the temperature of the parcel, what happens to this humidity? (1 pt)

iv) The total amount of humidity added to the parcel is 3 g kg^{-1}, thus implying a final *total water* mixing ratio of r_t = 10 g kg^{-1}. Write down an expression for the final temperature T in terms of r_t, the initial temperature T_0, and the final saturation mixing ratio $r_s(T)$. (2 pts) v) Calculate the final equilibrium temperature T. Clue: In order to do this you need to know $r_s(T)$. To simplify the calculation, define $r_s(T)$ using a linearization of the saturation specific humidity around T_0 (4 pts)

vi) What is the final liquid water mixing ratio ? (2 pt)

vii) What is the dry adiabatic lapse rate in pressure coordinates, $\frac{dT}{dp}$? Use the chain rule and assume the atmosphere is in hydrostatic balance and use the ideal gas law. (2pts)

viii) Now consider that our parcel with temperature T_0 and moisture r_0 is rising adiabatically from p=800hPa to a final pressure p=750hPa while undergoing condensation. What is the temperature at 750 hPa? In order to calculate this imagine a two stage process. Stage 1: the parcel rises dry adiabatically to 750 hPa neglecting the condensation process. Stage 2: At p=750 hPa, the condensation process occurs isobarically as in the earlier example (6 pts)

Question 3: Microphysics

i) In one sentence, explain the process of heterogeneous nucleation of liquid water droplets (1pt).

ii) The Clausius Clapeyron equation gives a relationship for the saturation vapour pressure over a planar body of liquid water, which we will write $e_s^{Clausius}$. But in fact the saturation vapour over a water droplet of radius r and with a mass M of dissolved impurity is modified by two factors:

$$e_s = e_s^{Clausius}\left(1 - \frac{b}{r^3}\right)\exp\left(\frac{a}{rT}\right) \tag{15}$$

where $a = 3.3 \times 10^{-7}$ [units are m K]and $b = 1.47 \times 10^{-4}M$[units are m^3] for sodium chloride NaCl. In a few sentences, explain what the two correction factors refer to (4 pts).

iii) A sample of air is supersaturated by 1% *over a planar surface* (i.e. the saturation ratio $S = \frac{e}{e_s(\infty)} = 1.01$). Calculate the saturation ratio over a droplet with $M = 10^{-19}$ kg of NaCl and a radius of 0.1 μm at a temperature of $T = 273K$. Will the droplet grow or evaporate? (4pt)

iv)With the help of a sketch, explain what it means for a droplet to become *activated.* (4 pts)

v)A cloud condensation nuclei (CCN) can be *soluble* or *wettable.* Write down the equation for the critical radius at which a droplet becomes activated in terms of the saturation ratio S when formed on a *wettable* CCN. What is the critical radius for activation if S = 1.01, again at T=273K? (4pts)

Question 4: Microphysics

i) Show how long an activated droplet of radius 0.5 μm in an environment of $S = 1.002$ and $T = 284K$ takes to grow by the process of diffusion to a size of 10 μm and 1000 μm respectively. Assume that this process does not alter S (4 pts).

ii) Explain in a sentence or two what this tells you about the role of diffusion in creating raindrops (2pts)

iii) What other factors are neglected in this calculation, apart from the assumption that S is constant? (1pt)

iv) In an air mass with a CCN concentration of N, a mass L of water condenses and forms a cloud. Assuming all the condensed water is shared equally among the CCN, write down a relationship for the droplet radius in terms of L, N and the density of liquid water ρ_L. (2pts)

v) In two clouds a mass $L = 5 \times 10^{-4} kgm^{-3}$ of water condenses. In the first cloud the CCN concentration is N=100 cm^{-3} and in the second the concentration is N =1000 cm^{-3}. What is the droplet size in each cloud and what might the different concentrations of CCN indicate about the locations of the clouds? (3pts)

vi) An ice crystal crystal is introduced into a cloud of supercooled liquid water droplets. In a few sentences explain how the Bergeron effect implies accelerated growth rates (3 pts)

vii) Another process important in clouds is collision and coalescence where we imagine a large raindrop of radius R falling at its terminal velocity V_t collecting other smaller, slower moving droplets. Give the relationship for the volume of air swept out by the larger droplet in time t. (2pt)

viii) We assume the cloud droplets of radius r are much smaller than the raindrop, $r << R$, and that their terminal velocities are also negligibly small (they are at rest), What is the rate of change of mass dM/Dt of the raindrop for a cloud of absolute liquid water density L (kg m^{-3})? (2pts)

ix) The rate of change of mass is related to the raindrop radius by $\frac{dM}{dt} = 4\pi R^2 \rho_L \frac{dr}{dt}$. Assuming a terminal fallspeed of $V_t = X_1 r^2$ where $X_1 = 1.2 \times 10^8$s^{-1} m^{-1}, calculate the time taken for a droplet to grow from 20μm to 30 μm by this process in a cloud with mean absolute liquid water density of $L = 1.0 \times 10^{-3}$ kg m^{-3}. (2pts) x) Apart from the stated assumption $r << R$, give two further approximations that have been made in this calculated that would imply *lower* growth rates. (2 pts)

Question 5: Radiation

i) The average irradiance per unit area (W m^{-2}) at the top of a planet's atmosphere is known as the *Solar Constant*. Assuming that the sun emits as a black body of temperature 5800 K, give the solar constant of the planet Venus. The average distance between Venus and the Sun is 108 million km. (Clue: inverse square law!) (2 pts)

ii) The solar constant for Earth is given in the table as S_0=1370 Wm^{-2}. For a moment we neglect the presence of the atmosphere; in this case what is the E_e, the average incident solar radiation on the earth's surface? (1 pt)

iii) State the definition of planetary albedo (1 pt)

iv) We denote planetary albedo α_p and we assume that the absorbed solar radiation is balanced by radiation emitted by the earth, treated as a black body with an effective emitting temperature T_e. Derive the relationship between T_e, S_0, r_e and α_p, where r_e is the radius of the Earth. Then give T_e assuming α_p=0.3 (2pts)

v) The annual mean surface temperature of the earth is roughly 288 K. Is T_e higher or lower than this value? State why in one sentence. (2 pts).

vi) A grey body emits radiation according to

$$E_{\text{grey}} = \epsilon \sigma T^4, \tag{16}$$

where ϵ is the fractional emittance ($\epsilon \leq 1$). According to Kirchoff's Law what is the body's Transmittance τ and Absorptance a? (2 pts)

vii) If we treat the atmosphere as a single slab that emits as a grey body with temperature T_a and fractional emittance ϵ, and does not absorb solar radiation. Assuming the planet is in radiative equilibrium, derive and write down the energy balance equations at the top of the atmosphere, for the atmospheric slab and the earth's surface (that has a temperature T_s). (6 pts)

viii) We have three equations with two unknowns T_s and T_a. Assuming ϵ=0.6, derive T_s and T_a.(2 pts)

ix) Is T_s larger or smaller than T_a and what is this effect known as? (2 pts) x) We could repeat this calculation for a atmosphere divided into a number of slabs in the vertical to obtain the vertical temperature profile of the atmosphere in radiative equilibrium. Give three reasons why this profile would be unrealistic (3 pts).

xi) If a energy flux δE (W m^{-2}) is absorbed by a body of mass M and heat capacity c_p then the heating rate is given by

$$\frac{dT}{dt} = \frac{\Delta E}{c_p M}. \tag{17}$$

If we assume that the sun is exactly overhead at local midday on the equator at the spring equinox, and the surface pressure is 1000 hPa, and that the slab atmosphere absorbs 15% of the incoming solar radiation. Calculate the instantaneous solar heating rate. (3 pts)

xii) Repeat the calculation for a solar zenith angle of 60°. (2pt)

xiii) Clouds of sufficient optical thickness scattering solar radiation, while they act as black bodies in the infra-red. Do stratocumulus clouds warm or cool the underlying surface and why? (2 pts)