TASI Lectures on Resonances

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Abstract

These TASI lectures were part of the summer school in 2008 and cover the collider signal associated with resonances in models of physics beyond the Standard Model. I begin with a review of the Z boson, one of the best-studied resonances in particle physics, and review how the Breit-Wigner form of the propagator emerges in perturbation theory and discuss the narrow width approximation. I review how the LEP and SLAC experiments could use the kinematics of Z events to learn about fermion couplings to the Z. I then make a brief survey of models of physics beyond the Standard Model which predict resonances, and discuss some of the LHC observables which we can use to discover and identify the nature of the BSM physics. I finish up with a discussion of the linear moose that one can use for an effective theory description of a massive color octet vector particle.

1 Introduction: The Z Boson

To begin with, let's look at the ordinary Z boson of the Standard Model. I am a big fan of using the Standard Model as a vehicle toward understanding new

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physics, and indeed, the Z boson is a perfect example of a vector resonance, one that illustrates almost any phenomena one could expect to encounter in the resonances of more exotic theories. We will discuss resonances with different spins when we turn to theories of physics beyond the Standard model further on. This review of the Z comes with two warnings: 1) it will by no means be complete (see the LEP EWWG report[1] for more details) and lacks any attempt at proper referencing, and 2) I made no real effort to match conventions with the rest of the world. Since I derived everything from scratch, it should be mostly self-consistent, but be careful when consulting another reference in tandem.

1.1 $e^+e^- \to f\bar{f}$

To begin with, we consider e^+e^- scattering into a pair of fermions, $f\bar{f}$. Some slight care is needed when the f's are themselves electrons, so we implicitly assume for now that $f \neq e$. In the Standard Model, this scattering is described at leading order in perturbation theory by two Feynman diagrams, one with an s-channel photon, and one with an s-channel Z. An example of one of these graphs is shown in Figure 1, where the labels indicate the incoming momenta p_a and p_b and outgoing momenta p_1 and p_2. The matrix element is given (in the unitary gauge) by,

$$\begin{aligned}\mathcal{M} &= e\left[\bar{v}_b\gamma^\mu u_a\right]\frac{-g_{\mu\nu}}{s+i\epsilon}Q_f e\left[\bar{u}_1\gamma^\nu v_2\right] + \\ &\quad \left[\bar{v}_b\gamma^\mu(g_R^e P_R + g_L^e P_L)u_a\right]\frac{-g_{\mu\nu}+p_\mu^Z p_\nu^Z/M_Z^2}{s-M_Z^2+i\epsilon}\left[\bar{u}_1\gamma^\nu(g_R^f P_R + g_L^f P_L)v_2\right]\end{aligned} \tag{1}$$

where $P_{L/R}$ are chiral projectors, the labels on the four-spinors u and v remind us which momentum they take as their arguments, $s \equiv (p_a+p_b)^2$ is the usual Mandelstam variable, e is the electromagnetic coupling, Q_f the charge of f, and $g^i_{L/R}$ are the (chiral) Z boson couplings to fermion i,

$$g^i = \frac{e}{\sin\theta_W\cos\theta_W}\left(T_3^i - Q^i\sin^2\theta_W\right) \tag{2}$$

where T_3 is the third component of weak iso-spin and θ_W the weak mixing angle.

For now, let's consider unpolarized scattering, summing over the final state spins and averaging over the initial spins. For simplicity, I will assume $s \gg m_f$ and drop m_f in the calculation. For LEP, this was a good

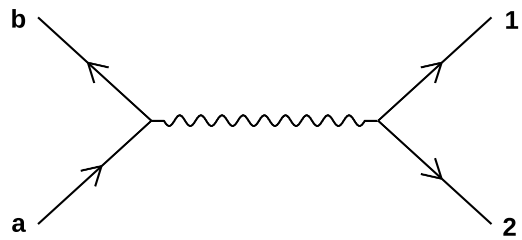


Figure 1: Feynman diagram showing $e^+e^- \to f\bar{f}$ through an intermediate photon or Z boson.

approximation for any fermion we had enough energy to produce anyway.

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= \frac{1}{4}\left\{\frac{e^4 Q_f^2}{s^2}\mathrm{Tr}\left[\not{p}_b\gamma^\mu \ \not{p}_a\gamma^\nu\right]\mathrm{Tr}\left[\not{p}_1\gamma^\mu \ \not{p}_2\gamma^\nu\right]\right. \qquad (3)\\
&+\frac{e^2 Q_f}{s(s-M_Z^2)}2\mathrm{Re}\mathrm{Tr}\left[\not{p}_b\gamma^\mu \ \not{p}_a\gamma^\nu(g_R^e P_R + g_L^e P_L)\right]\mathrm{Tr}\left[\not{p}_1\gamma^\mu \ \not{p}_2\gamma^\nu(g_R^f P_R + g_L^f P_L)\right]\\
&\left.+\frac{1}{(s-M_Z^2)^2}\mathrm{Tr}\left[\not{p}_b\gamma^\mu \ \not{p}_a\gamma^\nu(g_R^{e2} P_R + g_L^{e2} P_L)\right]\mathrm{Tr}\left[\not{p}_1\gamma^\mu \ \not{p}_2\gamma^\nu(g_R^{f2} P_R + g_L^{f2} P_L)\right]\right\}
\end{aligned}$$

where I have dropped the $+i\epsilon$ terms in the Feynman propagator, which I do not need here. If f is a quark, I will also need to sum over their (mutual) colors, which will produce a factor of $N_c = 3$. Performing the traces leads to,

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= \frac{1}{4}\left\{4e^4 Q_f^2(1+\cos^2\theta)\right. \qquad (4)\\
&+\frac{e^2 Q_f s}{(s-M_Z^2)}\left[(g_L^e + g_R^e)(g_L^f + g_R^f)(1+\cos^2\theta) + 2(g_L^e - g_R^e)(g_L^f - g_R^f)\cos\theta\right]\\
&\left.+\frac{s^2}{(s-M_Z^2)^2}\left[(g_L^{e2} + g_R^{e2})(g_L^{f2} + g_R^{f2})(1+\cos^2\theta) + 2(g_L^{e2} - g_R^{e2})(g_L^{f2} - g_R^{f2})\cos\theta\right]\right\}
\end{aligned}$$

where θ is the scattering angle of f in the center-of-mass frame. We'll come back to this expression in more detail after we fix up one important feature.

This expression for $\overline{|\mathcal{M}|^2}$ is very simply related to the differential cross section,

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{64\pi s}\overline{|\mathcal{M}|^2}\,. \qquad (5)$$

Since $\overline{|\mathcal{M}|^2}$ approaches a constant as $s \to 0$ or $s \to \infty$, we can see that the cross section is enhanced if we choose small center-of-mass energies (but note that this does not continue arbitrarily, since at some point we won't have enough energy to produce the $f\bar{f}$ final state and the process will switch off), and suppressed at very high energies.

The last term of Eq. 4 shows that the cross section is enhanced if we choose $s \simeq M_Z^2$. This is the resonant behavior we're looking for, but it also shows that the leading order to which we have been working is not cutting it. The cross section may be enhanced around the Z mass, but clearly this result, which says it should diverge, is unphysical[1]. To do better, we must consider improving our calculation with some higher order effects in perturbation theory.

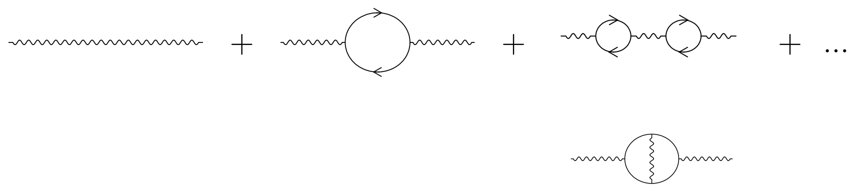
1.2 Resummed Propagator

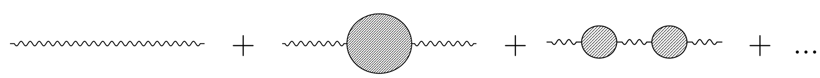
To simplify the discussion, but without dropping any important details, in my discussion of higher order effects I will neglect the fact that the Z boson has spin, and treat it like a scalar. I will also restrict myself to the fermion sector, and ignore their spins too. So as far as the loop calculation goes, I am looking at the correction induced to a *scalar* Z boson by its coupling to a pair of *scalar* quarks or leptons. The full case, carrying around the vector and spin indices, is left as an exercise for the reader. It's not hard, just a little more messy. Seriously.

I actually need the first non-trivial correction to the propagator, the one that arises at one loop. However, I need to resum the important parts of it at all orders. Let's see how this works. The full correction to the propagator contains diagrams such as:

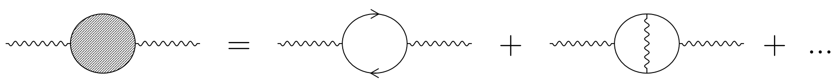
In addition to the genuine two loop and higher order corrections, we also have a two-loop term that is just the square of the one-loop correction itself. At every order n, there is a term that looks like the one-loop correction raised to the nth power. In fact, if I reorder my perturbative series so that instead of being organized by powers of the coupling, it is instead organized by the number of internal Z boson propagators, I can write this series as,

[1]If you thought the Feynman $+i\epsilon$'s could save us, note that since $\epsilon \to 0$ they are just a temporary regulation of the problem.





where the blob represents the *one-particle irreducible* correction to the Z boson propagation:



which I will call $i\Sigma(s)$ and will eventually compute, in our scalarized theory, to one loop in perturbation theory[2].

Organized this way, the propagator looks like a geometric series,

$$\begin{aligned} & \frac{i}{s-M_Z^2} + \frac{i}{s-M_Z^2} i\Sigma(s) \frac{i}{s-M_Z^2} \\ & \quad + \frac{i}{s-M_Z^2} i\Sigma(s) \frac{i}{s-M_Z^2} i\Sigma(s) \frac{i}{s-M_Z^2} + ... \\ = \;& \frac{i}{s-M_Z^2} \left\{ 1 + i\Sigma(s)\frac{i}{s-M_Z^2} + \left(i\Sigma(s) \frac{i}{s-M_Z^2} \right)^2 + ... \right\} \\ = \;& \frac{i}{s-M_Z^2} \left(\frac{1}{1+\frac{\Sigma(s)}{s-M_Z^2}} \right) = \frac{i}{s-M_Z^2+\Sigma(s)} \end{aligned} \tag{6}$$

By computing $\Sigma(s)$ to whatever order in perturbation theory I like, and then using this modified propagator, I am capturing *some* effects at all orders in

[2]Note that Σ, as a Lorentz scalar, can at most be a function of s. It was precisely to avoid dealing with its tensor structure that I went to a scalarized toy example.

perturbation theory. This is great, but it does have some dangers, because while I get some effects at all orders, I am also missing some effects at *every* order[3]. In general, I do need to be careful with this trick, because important features like gauge invariance hold order by order in perturbation theory. By keeping parts of a given order, I am potentially jeopardizing important cancellations and so forth. This issue will not bite us here, but it is good to keep the subtleties clearly in (the back of one's) mind even while not being rigorous.

We can write the most general expression for $\Sigma(s)$,

$$\Sigma(s) \quad = \quad Z^{-1}(s)s + B(s) + i\gamma(s) \tag{7}$$

or in other words, there is a dimensionless complex coefficient $Z^{-1}(s)$ (the wave function renormalization) which multiplies s itself, and two real functions B and γ which have dimensions of mass^2 , but whose dimensions are not made up from s itself. In our problem, they both must be proportional to M_Z^2, since there is no other mass scale (having assumed $m_f \to 0$ at hand.

The functions $Z^{-1}(s)$ and $B(s)$ turn out to be divergent. They also are not going to help with our problem at $s \simeq M_Z^2$, because Z^{-1} just rescales the entire amplitude, and $B(s)$ just shifts the place where it occurs by some amount. The divergent functions need to be defined by a renormalization scheme; the obvious one for this problem is the *on-shell* scheme, for which,

$$Z^{-1}(M_Z^2) \equiv 1 \qquad\qquad B(M_Z^2) \equiv 0 \tag{8}$$

which allows us to ignore both $Z^{-1}(s)$ and $B(s)$ for $s \simeq M_Z^2$. These really just amount to requiring that the parameter M_Z in the Lagrangian is defined to be the center of the Z boson resonance, and that the field is canonically normalized there.

The $\gamma(s)$ term is the one we are looking for. It regulates the divergence, resulting in the most divergent term (the third term of Eq. 4) becoming,

$$\overline{|\mathcal{M}|^2} \;\; \propto \;\; \frac{1}{\left(s - M_Z^2\right)^2 + \gamma^2} \tag{9}$$

As $s \to M_Z^2$, the cross section no longer goes to infinity, but instead is proportional to $1/\gamma^2$ (with a Breit-Wigner shape for s close, but not equal

[3]This is a general feature of resummations, and is i.e., part of the headache we get when we try to improve a parton shower at higher orders.

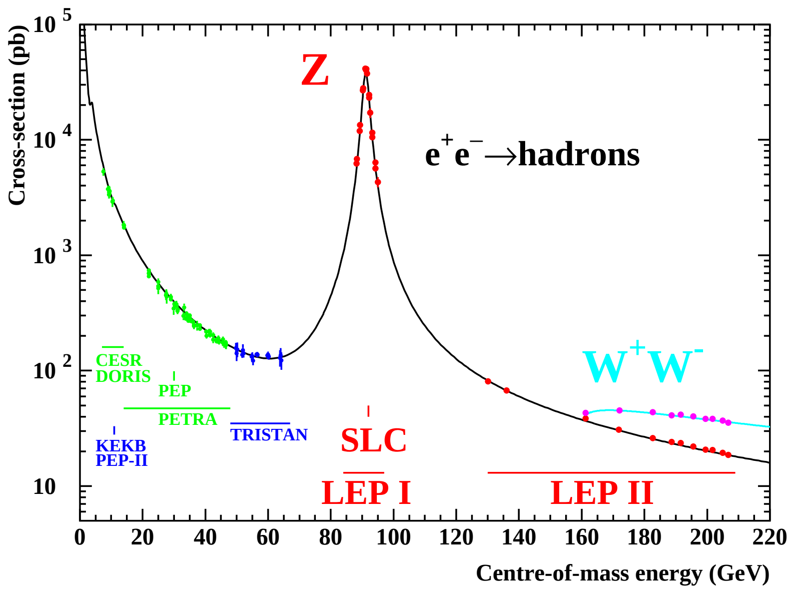


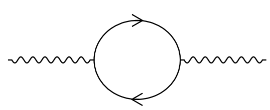
Figure 2: The cross section as a function of E_{CM} after resumming the propagator (from the LEP EWWG[1]).

to M_Z^2). The parameter $\gamma(M_Z^2)$ controls both the location of the peak in the distribution, and also the shape as the cross section falls off, slightly away from the peak position. In Figure 2, we show the result with the non-zero γ included, illustrating how γ regulates the behavior for $s \simeq M_Z^2$.

1.3 γ in the Toy Theory

Now let's compute γ in the scalarized theory. If you are very comfortable doing such calculations, I suggest you skip down to the next subsection, where we will interpret the results.

At leading order in perturbation theory, we need the graph,



with (scalarized) leptons $\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau$ and quarks u, d, s, c, b, t running inside the loops. For a single one of these graphs, our loop correction is,

$$g^2 \int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\ell^2 - m_f^2 + i\epsilon} \frac{1}{(\ell+p)^2 - m_f^2 + i\epsilon} \tag{10}$$

(remember that this loop is divergent, but that won't hurt us since ultimately we are after γ which is not). p_μ is the incoming/outgoing external momentum; p^2 is s in our previous discussion. We can rewrite this using the Feynman parameterization,

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{\left[xA + (1-x)B\right]^2} \ , \tag{11}$$

which after some algebra results in,

$$g^2 \int \frac{d^4\ell}{(2\pi)^4} dx \left[\ell^2 + 2(1-x)\ell \cdot p + (1-x)p^2 - m_f^2 + i\epsilon\right]^{-2} \ . \tag{12}$$

Adding and subtracting $(1-x)^2 p^2$ inside the square brackets allows us to complete a square, and then changing integration variables to $\ell'_\mu \equiv \ell_\mu + (1-x)p_\mu$ yields,

$$g^2 \int \frac{d^4\ell'}{(2\pi)^4} dx \frac{1}{\left[\ell'^2 - \Delta + i\epsilon\right]^2} \tag{13}$$

where

$$\Delta \equiv -x(1-x)p^2 + m_f^2 \ . \tag{14}$$

To perform the integration over ℓ', we move into Euclidean space, taking $\ell_E^0 = i\ell'_0$, $\vec{\ell}_E = \vec{\ell'}$ (so $\ell'^2 \to -\ell_E^2$). The $+i\epsilon$'s allow us to deform the path integration back to the real ℓ_E^0 axis. Having kept them around long enough to keep us honest at this step, we can now afford to ignore them. The loop integral becomes,

$$g^2 \int \frac{d^4\ell_E}{(2\pi)^4} dx \frac{1}{\left[\ell_E^2 + \Delta\right]^2} = g^2 \frac{2\pi^2}{(2\pi)^4} \int_0^\infty d\ell_E^2 \int_0^1 dx \ \frac{\ell_E^2}{\left[\ell_E^2 + \Delta\right]^2} \ . \tag{15}$$

We now shift ℓ_E^2 by Δ and cut the divergent integral off at some large scale Λ, resulting in,

$$\begin{aligned} & \frac{g^2}{8\pi^2} \int_\Delta^{\Lambda^2} d\ell_E^2 \int_0^1 dx \left\{ \frac{1}{\ell_E^2} - \frac{\Delta}{\ell_E^4} \right\} \\ = \ & \frac{g^2}{8\pi^2} \int_0^1 dx \left\{ \log\left(\frac{\Lambda^2}{\Delta}\right) - 1 + \frac{\Delta}{\Lambda^2} \right\} \end{aligned} \tag{16}$$

(where we note in passing that there is no term proportional to p^2, implying that Z^{-1} vanishes at the one loop order in our toy theory). The last term vanishes as we take the cut-off Λ to infinity. The middle term clearly does not contain an imaginary part. So to get a non-zero result for γ, we need the log to develop a branch cut, which will happen if $\Delta < 0$,

$$\log\left(\frac{\Lambda^2}{\Delta}\right) \to i\pi + \log\left(\frac{\Lambda^2}{|\Delta|}\right) \qquad (\Delta < 0) \tag{17}$$

Provided this happens for some values of x in the range of its integration, the imaginary part of the self-energy correction is given by,

$$\gamma(p^2) = \frac{g^2}{8\pi}\int_{\Delta<0} dx = \frac{g^2}{8\pi}\left(x^+ - x^-\right) \tag{18}$$

where $x^{\pm}$ is the largest (smallest) value of x for which $\Delta < 0$. Note that the dependence on the regulator Λ has vanished for the imaginary part, justifying my initial statement that the imaginary part of Σ was not divergent.

From the definition of Δ, Eq. 14, and the fact that for $0 \leq x \leq 1$, $x(1-x)$ is maximal at $x = 1/2$, we see that to have a region with $\Delta < 0$, we need a large enough p^2 such that,

$$p^2 \geq 4m_f^2 \ . \tag{19}$$

This implies that we can ignore any particle in the loop whose mass is less than half of the center-of-mass energy – such particles induce contributions to $Z^{-1}(p^2)$ and $B(p^2)$, but they do not contribute to the imaginary part of the self-energy. At LEP energies, this means we didn't actually need the top quark in the loop.

Provided $p^2 \geq 4m_f^2$,

$$x^{\pm} = \frac{1}{2}\left(-1 \pm \sqrt{1 - \frac{4m_f^2}{p^2}}\right) \ , \tag{20}$$

for which,

$$\gamma(p^2) = \frac{g^2}{8\pi}\sqrt{1 - \frac{4m_f^2}{p^2}} \ . \tag{21}$$

1.4 Back to $e^+e^- \to f\bar{f}$

For the region that sparked our interest in γ to begin with, $p^2 \simeq M_Z^2$, we can further recognize,

$$\gamma(M_Z^2) \;\; = \;\; M_Z \Gamma(Z \to f\bar{f}) \; . \tag{22}$$

We thus see at for energies close to the mass of the Z, the imaginary part of the one-loop contribution has produced the leading order decay width. Summing over all fermions for which $p^2 \geq 4m_f^2$, the imaginary parts of each of the one-loop amplitudes simply adds, and $\gamma(M_Z^2) = M_Z \Gamma_Z$ reproduces the inclusive decay width.

The fact that the width has appeared is not an accident – it is a general feature of quantum field theory, guaranteed by the optical theorem [2] as applied to the propagator. The optical theorem relates the imaginary part of the scattering $a \to b$ to the amplitudes for all possible final states $a \to F$ and $b \to F$,

$$2 \; \mathrm{Im} \; \mathcal{M}(a \to b) \;\; = \;\; \sum_F \int d\Pi_F \mathcal{M}^*(b \to F) \mathcal{M}(a \to F) \tag{23}$$

where $d\Pi_F$ is the integral over the phase space of F. Applied to the self-energy for the Z boson, the left-hand side of the equation (at one loop) becomes $2 \; \mathrm{Im} \; \mathcal{M}(Z \to Z) = 2\gamma(M_Z^2)$ and we have,

$$\gamma(M_Z^2) \;\; = \;\; \frac{1}{2} \sum_F \int d\Pi_F |\mathcal{M}(Z \to F)|^2 \; . \equiv M_Z \Gamma_Z \tag{24}$$

Since nothing in our argument made explicit reference to the Z itself, it is clear that this will work for any particle which has decay amplitudes into on-shell states. In practice, this is how we compute γ in terms of the the width using only tree-level Feynman diagrams.

What we've learned is that the Z boson propagator has an imaginary part which receives contributions from every particle into which the Z can decay. At very low and very high energies, we don't notice the width because it is small, arising from higher order in perturbation theory than the zeroth order propagator. Around the Z mass, the width is crucial, because the zeroth order propagator vanishes, and the higher order effects are leading.

1.5 The Narrow Width Approximation

In the limit $\Gamma/M \to 0$, the Breit-Wigner becomes a δ-distribution:

$$\frac{1}{(s-M^2)^2+M^2\Gamma^2} \quad \to \quad \frac{\pi}{M\Gamma}\delta(s-M^2) \qquad\qquad (\Gamma/M \to 0) \tag{25}$$

In this limit we can only produce the Z boson on-shell. The δ-function puts the intermediate boson on-shell, and at leading order factorizes the production from the decay. The cross section becomes,

$$\begin{aligned}
&\sigma(e^+e^- \to f\bar{f}) \\
&\quad \to \frac{1}{32\pi s}\int d\Pi_f d\Pi_{\bar{f}} |\mathcal{M}(e^+e^- \to Z|^2 \frac{\pi}{M\Gamma}\delta(s-M^2)|\mathcal{M}(Z \to f\bar{f}|^2 \\
&\quad = \sigma(e^+e^- \to Z)\frac{\Gamma(Z \to f\bar{f})}{\Gamma} = \sigma(e^+e^- \to Z)\mathrm{BR}(Z \to f\bar{f})
\end{aligned} \tag{26}$$

the expected product of on-shell production times the branching ratio into $f\bar{f}$. For most weakly coupled resonances, this approximation holds very well.

In the narrow width limit, the unpolarized cross section for $e^+e^- \to f\bar{f}$ "on the pole" becomes,

$$\begin{aligned}
\frac{d\sigma}{d\cos\theta} \;&=\; \frac{1}{32}\frac{M}{\Gamma}\delta(s-M^2) \times \\
&\left\{2(g_L^{e2}-g_R^{e2})(g_L^{f2}-g_R^{f2})\cos\theta + (g_L^{e2}+g_R^{e2})(g_L^{2f}+g_R^{f2})(1+\cos^2\theta)\right\}
\end{aligned} \tag{27}$$

which reveals how we can start to learn about the couplings to the Z. When computing the inclusive cross section, integrated over all θ, the first term in the curly braces integrates to zero. So the inclusive cross section is proportional to the sum of the left- and right-handed couplings squared.

This is also a good place to remark on an interesting feature of the SM Z boson – because the weak mixing angle takes a particular value, $\sin^2\theta_W \simeq 1/4$, the electron couplings (see Eq. 2) approximately satisfy $g_L^e \simeq -g_R^e$. This implies that there is an approximate cancellation in the coefficient of the first term in the curly braces for the SM Z. For the real Z boson, that is fine – it's the physics we have been handed. However, it does imply that taking the SM Z as the most generic possible model for a resonance may lead us to conclude that any effect from that term is suppressed, and this may not be a generic feature of new physics. So when you hear the words "Sequential Standard Model Z'" in an experimental search, don't be fooled.

Moving back to measuring the couplings, we saw that the production cross section is sensitive to the sum of the squares of the left- and right-handed couplings. What we would like to do next is to unravel the relative amounts of right- versus left-handed couplings. One way to do that is to follow the SLC route and polarize the incoming beams. For example, we can just count events when the electron is left-handed compared to the number of events when it is right-handed, and form the ratio,

$$A_{LR}\ (= A_e) \quad \equiv \quad \frac{N_L - N_R}{N_L + N_R} = \frac{g_L^{e2} - g_R^{e2}}{g_L^{e2} + g_R^{e2}} \tag{28}$$

the information about the final state cancels out in the ratio.

The other way to disentangle right- from left-handed couplings is to measure forward-backward asymmetries. From the differential cross section on the Z pole, it is clear that the way to do that is to pick up sensitivity to the $\cos\theta$ term in the differential cross section. θ is the angle between the incoming electron beam (which we know) and the out-going particle, f. Provided we can tell f from the anti-particle $\bar{f}$ in a given event[4], we can measure θ. If we define a forward particle to lie in the region $0 \le \cos\theta \le 1$ and a backward particle to be in the region $-1 \le \cos\theta \le 0$, we arrive at,

$$A_{FB}^f \quad \equiv \quad \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\left(\frac{g_L^{e2} - g_R^{e2}}{g_L^{e2} + g_R^{e2}}\right)\left(\frac{g_L^{f2} - g_R^{f2}}{g_L^{f2} + g_R^{f2}}\right) = \frac{3}{4}A_e A_f \tag{29}$$

Since the cross section on the Z pole depends only on the square of couplings, we can't actually get information about the absolute or relative signs of the couplings from polarized or forward-backward measurements. This is where the photon actually becomes a help instead of just a nuisance. From Eq. 4, we see that the interference terms contain single powers of Z couplings and single powers of electromagnetic couplings, and are sensitive to the relative signs. This is illustrated in Figure 3, which shows A_{FB}^b as a function of center of mass energy, for the four possible sign combinations with the magnitudes of g_L^b and g_R^b set as in the SM. The four curves meet at the Z mass, and diverge at higher and lower energies, driven by the different admixture of Z-γ interference.

[4] It's not always easy. For charged leptons it is moderately simple, and for heavy quarks with semi-leptonic decays, it can be done. For light quarks, it is probably hopeless.

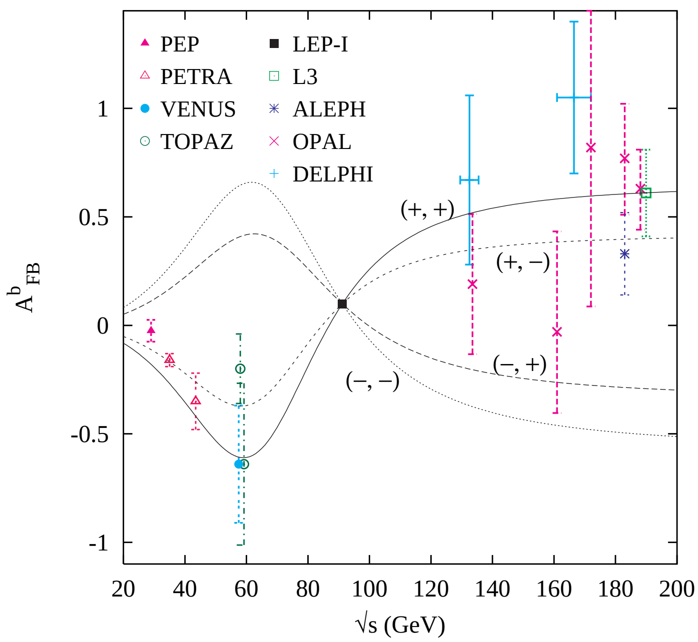


Figure 3: The forward-backward asymmetry of the bottom quark as a function of center-of-mass energy, showing data and the four possible sign combinations with magnitudes set by the SM (from Ref[3]).

1.6 The Lone Resonance: $\nu\bar{\nu} \to f\bar{f}$

To better understand the Z resonance without the complication of the photon exchange, let's consider a process which we can't really do realistically in the lab. If we consider inelastic neutrino scattering $\nu\bar{\nu} \to f\bar{f}$, there is a single Feynman diagram, Fig 1 with only the Z boson exchanged. Since right-handed neutrinos (if they exist at all...) don't couple to the Z, the inclusive cross section simplifies to,

$$\sigma(\nu\bar{\nu} \to f\bar{f}) \;=\; \frac{g_L^{\nu 2}}{96\pi} \frac{s}{(s-M^2)^2 + M^2\Gamma^2} (g_L^{f2} + g_R^{f2}) \ . \qquad (30)$$

We've already discussed ad nauseum the Breit-Wigner behavior around $s \simeq M^2$. For $s \ll M^2$, we can Taylor-expand the propagator and find a cross section which grows with energy, but is suppressed by the large mass. For $s \gg M^2$, the cross section falls as $1/s$, consistent with the requirements of unitarity. To summarize,

$$\sigma(s) \;\sim\; \begin{cases} s/M^4 & (s \ll M^2) \\ s/[(s-M^2)^2 + M^2\Gamma^2] & (s \sim M^2) \\ 1/s & (s \gg M^2) \end{cases} \tag{31}$$

When interference is not important, these three regimes summarize the most important behavior of an exchanged "resonance". When $s \ll M^2$, the resonance is effective integrated out, and the cross section represents the four-fermion interaction it leaves behind in the low energy theory.

2 Models with Resonances

A new resonance can be classified by its basic properties such as spin and transformation under the $SU(3)_c \times SU(2)_W \times U(1)_Y$ SM gauge symmetries. Below, I will run through some of our favorite models which lead to such objects, with some of their known LHC phenomenology interleaved as appropriate.

2.1 Z'

Experimentalists sometimes call any resonance (usually decaying into leptons) a Z', which is a useful classification for them because it organizes many models with similar signatures together. For theorists, the standard definition is a color singlet, electrically neutral, vector particle. As a vector particle, a Z' is a typical signal when the SM gauge groups are extended. That could occur in a simple way, such as promoting the gauge sector to $SU(3)_c \times SU(2)_W \times U(1)_Y \times U(1)'$, or we could enlarge either $SU(2)_L$ or $U(1)_Y$ (or both), or unify the entire SM gauge structure into a higher rank GUT such as $SO(10)$. In any of these options, extra massive gauge fields result when we break these larger symmetries down to the SM itself.

If we take a $U(1)'$ extension of the SM as a prototype, we have a well defined framework with well defined parameters. We can add terms to describe the propagation of the new vector (V_μ),

$$\mathcal{L} = -\frac{1}{4}(F'_{\mu\nu})^2 + \frac{\zeta}{4}F'_{\mu\nu}F_Y^{\mu\nu} + \frac{1}{2}M^2 V_\mu V^\mu \tag{32}$$

The first term is a usual kinetic term for a vector particle. The second term induces kinetic mixing between the Z' and hyper-charge, which causes the Z'

to pick up some fraction of coupling to all SM fields proportional to their SM hypercharges[4]. The final term is actually a stand-in for an entire new Higgs sector which gives mass to the Z'. For colliders, this new Higgs sector usually boils down to just the mass of the Z', as we have written here. However, one may choose the mass of this new scalar to be light enough it can be produced at colliders, and couple it, say, to the usual SM Higgs, affecting Higgs physics.

A Standard Model matter field Ψ will couple to the Z' through its charge under $U(1)'$ in terms of an extended covariant derivative,

$$D_\mu \Psi \to D_\mu^{(SM)} \Psi - i g' z_\Psi V_\mu \Psi \tag{33}$$

where g' is the Z''s universal gauge coupling, which may be a free parameter or may be predicted if the Z' descends from a higher rank group in which the SM is also embedded. Every matter field may have its own charge z, which again may be predicted some models.

If we want the SM fermion masses to come from renormalizable interactions, we should choose the z's such that they permit us to write such terms down. For example, the top Yukawa coupling would require,

$$z_{t_R} - z_{Q_3} + z_H = 0 \ . \tag{34}$$

Note, however, that most of the SM fermion masses are counter-intuitively small, and by assigning charges which violate $U(1)'$, we can actually engineer their observed sizes to some degree - this is one particular UV realization of a Frogatt-Nielsen model of flavor[5].

Many models[6] choose the Higgs charge z_H to be zero. This choice removes the danger that the ordinary Higgs VEV will induce Z-Z' mixing. Given the success of the electroweak fit, such mixing is generically limited at the 10^{-3} level[1], though it is possible in a multi-Higgs model to play the VEVs against one another so that the mixing is small despite the charges being non-zero. That is how the canonical E_6 GUTs survive precision measurements[7]. Another set of bounds on the charges and mass come about from LEP-II, which studied the reaction $e^+e^- \to f\bar{f}$ above the Z-pole. We saw in the previous section that at energies far below the mass of the Z', its contribution to to the cross section goes like $\sim s/M^2$ (when the Z' is the only mediator of the reaction, the cross section goes like s/M^4, but in this case the interference with the SM photon and Z goes like the SM rate times

$\sim s/M^2$. As a result, LEP-II is able to derive limits on M/g' for different patterns of couplings to fermions [8, 6].

A further guide to the charges of the fermions is provided by anomaly cancellation. In order for the gauge symmetry to survive at the quantum level, a model with an extra gauge symmetry should not have gauge anomalies. However, the fact that the symmetry is spontaneously broken raises the possibility that the anomalies may cancelled by additional fermions which are chiral under $U(1)'$ (and are usually vector-like under the SM gauge symmetries in order to preserve the success of anomaly cancellation in the SM). If such fermions have masses around the mass of the Z' itself, it is unlikely they will have much effect on the collider phenomenology of the Z'.

A final concern for the fermion charges comes from flavor-changing neutral currents (FCNCs). If the Z' charges are not universal over the three families, we will generically have tree level FCNCs,which in the least will require the Z' to be heavy to avoid low energy bounds[9]. Let's see how this works using the right-handed down-quarks as an example. If the charges are not Universal, then before EWSB we will have interactions with the Z' such as

$$g' \left[\bar{d}_1 \ \bar{d}_2 \ \bar{d}_3\right] \begin{bmatrix} z_{d_1} & 0 & 0 \\ 0 & z_{d_2} & 0 \\ 0 & 0 & z_{d_3} \end{bmatrix} \not{Z}' P_R \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \equiv g' \bar{\mathcal{D}} \mathcal{Z}_d \ \not{Z}' P_R \mathcal{D} \tag{35}$$

where $\mathcal{D}$ is a vector in flavor space, and $\mathcal{Z}_d$ the 3×3 diagonal matrix of the couplings to right-handed down quarks. After EWSB, there will be a change of basis to take us from the quarks from the weak to the mass eigenstates,

$$\mathcal{Q}_d \equiv \begin{bmatrix} d \\ s \\ b \end{bmatrix} = U_{dR} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = U_{dR} \mathcal{D} \tag{36}$$

where $\mathcal{Q}_d$ is the vector of the mass eigenstates. Applying this rotation to the Z' interactions in Eq. 36, we arrive at the quark mass eigenstate interactions,

$$g' \bar{\mathcal{Q}}_d \ \left(U_{dR}^\dagger \mathcal{Z}_d U_{dR}\right) \ \not{Z}' P_R \mathcal{Q}_d \tag{37}$$

where we see that the matrix $(U_{dR}^\dagger \mathcal{Z}_d U_{dR})$ need not be diagonal, even though $\mathcal{Z}_d$ was. The easiest way to insure that it there are no FCNCs is to assign a common charge for all three right-handed down-type quarks. In that case, $\mathcal{Z}_d$ is proportional to the unit matrix, and the product of unitary transformations

$U_{dR}^{\dagger} U_{dR}$ result in the unit matrix. This is the same mechanism by which the SM Z was left without tree-level FCNC interactions.

Putting all of my personal prejudices together, we should have family universal couplings to fermions (to avoid FCNCs), no charge on the SM Higgs to avoid Z-Z' mixing, and I don't particularly care if the anomalies cancel with SM fermion content or not. When the dust has settled, I am left with parameters:

$$M_{Z'}, \Gamma_{Z'} \qquad z_Q, z_u, z_d, z_L, z_e \tag{38}$$

technically there is also the gauge coupling g', but unless I am embedding into a GUT or other extended structure, I can just absorb it into the normalization of the the z's without any confusion. I left the width of the Z' as a free input to allow for decays into non-SM channels, including perhaps right-handed neutrinos. Obviously, if there are important observable decays into non-SM modes, I would have to add to this list accordingly.

2.1.1 Z's at the LHC

At the Tevatron or LHC, the obvious signal for a Z' would be a bump in an invariant mass distribution, much the way the Z appears as a bump against the continuum photon-mediated processes. A final state with a pair of charged leptons[10, 11] is an obvious place to look, because they are easy to trigger on, have smaller backgrounds, and more precise reconstruction than jet final states. A Z' with mass below a few TeV and large branching ratio into leptons is a discovery that could be made with a relatively modest amount of LHC data.

We can transcribe much of our experience with the Z boson in the earlier chapter onto a Z'. At a hadron collider, we replace the e^+e^- initial state with a $q\bar{q}$ one. The rate at a hadron machine is estimated by taking this "partonic" reaction and convolving it with the parton distribution functions (PDFs). The rate of Z' production thus turns into a measurement of a sum of the quark charges (both right- and left-handed) with coefficients determined by the PDFs, and an over-all factor of the branching ratio into the final state of interest. Information about individual quark charges including right- versus left-handed couplings is a little more difficult, because the LHC, as a pp collider, makes it difficult to identify which direction along the beam is the direction of the quark, and which direction the anti-quark. However, using the difference of valence q versus $\bar{q}$ PDFs as well as clever observables can disentangle a lot of parameter space[12].

2.2 Topcolor

A second class of theory which predicts an interesting resonance is the family of topcolor models[13]. Broadly, these models predict a composite Higgs boson which is a bound state of top quarks, and thus couples strongly to top as a residual of the binding force[14]. In practice, topcolor generally has difficulty getting the right amount of EWSB and the right top mass by itself, leading to extensions in which either there is more electroweak breaking from a technicolor sector[15] or there is an additional fermion participating in the strong dynamics whose mixing with top dials in the correct top mass[16]. The specific details of those models are not very important for our purposes here – all of them lead to resonances with similar phenomenology. So I will describe the simplest model[13] here.

The theory extends the color sector of the Standard Model to $SU(3)_1 \times SU(3)_2$, usually called "topcolor" and "protocolor", respectively. The top quark (both Q_3 and t_R) are triplets under $SU(3)_1$ and the light quarks are triplets under $SU(3)_2$. At the scale of a few TeV, this structure breaks down through some usually unspecified dynamics, resulting in an unbroken $SU(3)_c$ to play the role of ordinary color and a massive color octet of "topgluons" (g^1). The couplings are usually chosen such that the topgluons are mostly the original $SU(3)_1$ bosons, and the ordinary gluons are mostly the $SU(3)_2$ gluons. This choice results in the topgluons coupling strongly to top quarks and weakly to the light quarks. The unbroken $SU(3)_c$ symmetry guarantees that the massless bosons couple universally to all of the quarks. If you find this discussion confusing, you can go down to Section 3 and you will find it worked out in more detail.

At sub-TeV energies, we are in the regime where the topgluons can be integrated out, and look like a contact interaction (see Figure 4). Their important effect in this low energy effective theory is to produce a coupling between right-handed and left-handed top quarks,

$$-\frac{g^2}{M^2}\left[\bar{t}_R\gamma^\mu T^a t_R\right]\left[\bar{Q}_L\gamma_\mu T^a Q_L\right] = -\frac{g^2}{M^2}\left[\bar{t}_R Q_L\right]\left[\bar{Q}_L t_R\right] \qquad (39)$$

where g is the g^1 coupling to top quarks and M is its mass, Q_L is the third family quark doublet and the last equality is easily understood as a Fierz

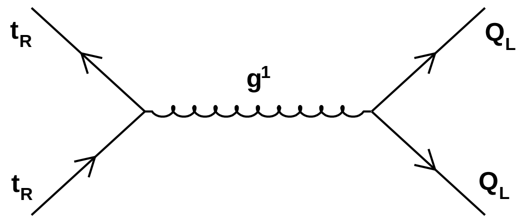


Figure 4: Feynman diagram showing how g^1 mediates a 4-top interaction at low energies.

transformation. Four fermion interactions of this kind (but involving the light quarks) have been extensively studied as a low energy model of QCD and chiral symmetry breaking, the Nambu-Jona Lasinio (NJL) model [17].

If g is large enough, this interaction results in scalar bound states. We can see how this works in more detail by taking our four-fermion interaction, and rewriting it in terms of an auxiliary field Φ,

$$\mathcal{L}(\Lambda) \;=\; \frac{1}{2} g_t \bar{Q}_L t_R \Phi + g_t^* \bar{t}_R Q_L \Phi^* - \Lambda^2 |\Phi|^2 \qquad (40)$$

where Φ transforms as a SM Higgs, but has no kinetic terms. Integrating it out is thus trivial, and reproduces the original four fermion interaction provided $g_t^2/\Lambda^2 = g^2/M^2$. An obvious identification would be $g_t = g$ and $\Lambda = M$ (and at tree level, it makes no difference), but the fact that nothing guarantees that identification is an indication that our effective theory is sensitive to the cut-off[5], and thus to the details of the UV dynamics responsible for binding Φ.

Our effective theory lives at the scale Λ. Below that scale, kinetic terms (and a quartic interaction) for Φ will be induced at one loop, and the parameters Λ^2 and g_t will be renormalized (see Figure 5). Just as in the case of the self-energy of the Z, these quantities are divergent and need to be renormalized. However, we know the effective theory at the scale Λ, and we can use this to set the renormalization constants,

$$Z(\Lambda) \;=\; 0 \qquad (41)$$

[5] Another way to appreciate this fact is to notice that we are taking a non-renormalizable interaction, Eq. 39 and writing it in renormalizable language. This arbitrariness is a representation of the non-renormalizability of the original theory as expressed in renormalizable language.

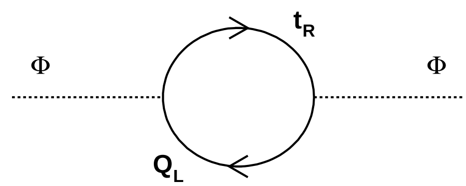


Figure 5: Feynman diagram showing how Φ develops kinetic terms at one loop.

$$m^2(\Lambda) \;=\; \Lambda^2 \tag{42}$$

The vanishing of the wave function renormalization at the compositeness scale is a generic feature of a composite field, because it implies that this field has no non-zero matrix elements between the vacuum and a single particle state [18]. At scales below Λ,

$$\begin{aligned}\mathcal{L}(\mu) \;&=\; Z(\mu)\,(D^\mu\Phi)^\dagger\,(D_\mu\Phi) - m^2(\mu)|\Phi|^2 - \lambda(\mu)|\Phi|^4 \\ &\quad + \frac{1}{2}g_t(\mu)\bar{Q}_L t_R\Phi + H.c.\end{aligned} \tag{43}$$

where,

$$Z(\mu) \;=\; N_c\frac{g_t^2}{16\pi^2}\log\left(\frac{\Lambda}{\mu}\right) + \ldots \tag{44}$$

$$m^2(\mu) \;=\; \Lambda^2 - N_c\frac{g_t^2}{8\pi^2}\Lambda^2 + \ldots \tag{45}$$

with similar expressions for $g_t(\mu)$ and $\lambda(\mu)$. Note that if

$$g_t^2 \;>\; \frac{8\pi^2}{N_c} \equiv g_{crit}^2 \tag{46}$$

$m^2(\mu)$ becomes negative, and Φ develops a VEV, breaking the electroweak symmetry.

To analyze the physics of Φ, we rescale the kinetic term for Φ to canonical form by taking $\Phi \to Z^{-1/2}\Phi$. In terms of the canonically normalized field, $m^2 \to Z^{-1}m^2$, $\lambda \to Z^{-2}\lambda$, and $g_t \to Z^{-1/2}g_t$. A useful approximation to determine the physics at the electroweak scale is to fix the parameters in the canonically normalized theory such that $m^2(\Lambda) \sim \infty$ and $g_t(\Lambda) \sim \lambda(\Lambda) \sim \infty$ (since all of these are finite in the unnormalized theory at Λ and going to the canonical normalization divides each by a positive power of $Z(\Lambda) \simeq 0$ at that scale), and then use the renormalization group to determine the parameters in the canonically normalized theory at scales $\mu \leq \Lambda$.

At energies much below Λ, our theory is nothing more than the Standard Model itself, but with the Higgs potential parameters m^2 and λ and the top Yukawa coupling g_t all predicted in terms of the original parameters Λ and g_t (provided $g_t > g_{crit}$ so that we have EWSB). The constraint required so that $m^2 < 0$ is the reason this theory has trouble fitting the right top mass – it turns out that for g_t large enough for $m^2 < 0$, we are in a regime where the top mass turns out to be too large.

2.2.1 Topcolor at the LHC

Since topcolor looks like the Standard Model at low energies, to distinguish it from the Standard Model, we need to turn to the high energy behavior of the theory. The generic feature of topcolor is the need to invoke the extended $SU(3)\times SU(3)$ symmetry in order to generate the four-top contact interaction at low energies, resulting in the composite Higgs. So a very generic feature at high energies is the existence of the massive color octet vector particle g^1 that couples strongly to top and weakly to light quarks.

At the LHC, g^1 can be produced by $q\bar{q}$ annihilation in the initial state. While the coupling of g^1 to light quarks is not huge, the fact that the light quark PDFs are sizeable usually renders this the dominant production mechanism. Once produced, the large coupling to top dictates that g^1 decays into a $t\bar{t}$ pair with a very high branching ratio. So the signature is a resonant structure in the invariant mass of $t\bar{t}$ pairs. While not as clean as a decay into leptons, decays into tops still have a lot of potential compared to backgrounds, because top decays produce jets enriched with bottom quarks which can be tagged and also a fair fraction of leptonic W decays. The primary challenge for high mass resonances is that they result in very boosted tops, whose decay products become highly collimated, which can be challenging to reconstruct properly as jets merge and leptons end up buried inside them[19] This is an interesting and active subject in collider phenomenology and analysis technique[20].

2.2.2 Topcolor versus Technicolor

I often get asked what the difference between topcolor and technicolor is. They get confounded in people's minds because both use strong dynamics to trigger electroweak symmetry-breaking and further confusion arises because both are families of theories, as opposed to single models. But the systematic differences are that topcolor is a model of a composite Higgs. Technicolor is a model of no Higgs.

One way to keep the definitive difference straight is to think about how each model explains the high energy perturbative unitarity of $WW \rightarrow WW$ scattering. Topcolor has a Higgs which does the job much the way the Higgs does it in the Standard Model. Technicolor has no Higgs, and perturbative unitarity is maintained by the existence of a weak triplet of vector particles (usually called "techni-rhos" in analogy with the massive vectors of QCD). They look like a Z' and a pair of W's with significant coupling to the SM Ws and Z (and perhaps weak coupling to fermions). So technicolor is another model predicting new resonances! We will talk about particles much like the technirhos below when we discuss topflavor.

This brings up an important point: the SM Higgs is an example of a resonance! But you have many lectures devoted just to the Higgs itself, so that is all I will say about it here.

2.3 Topflavor

Topflavor[21] is a similar construction to topcolor in many ways. The difference is that instead of taking the SM $SU(3)$ interaction and identifying it with the diagonal subgroup of two $SU(3)$'s, we take the SM $SU(2)$ interaction, and promote it to $SU(2)_1 \times SU(2)_2$. The left-handed fermions of the third generation are charged under $SU(2)_1$ and the light fermions under $SU(2)_2$. This arrangement automatically takes care of anomaly cancellation, because the $SU(2)$ anomalies cancel within an entire generation of SM fermions.

When the symmetry breaks, $SU(2) \times SU(2) \rightarrow SU(2)$, a massive Z' and pair of W's results, with enhanced coupling to the third generation. The unbroken $SU(2)$ is identified with the usual weak interaction, and has approximately universal coupling to all of the SM fermions. In its many incarnations topflavor has been helpful as a model of top mass generation on top of strong dynamics models[21], raised the light Higgs mass above the LEP-II bound in supersymmetric models[22] and its phase transition in the

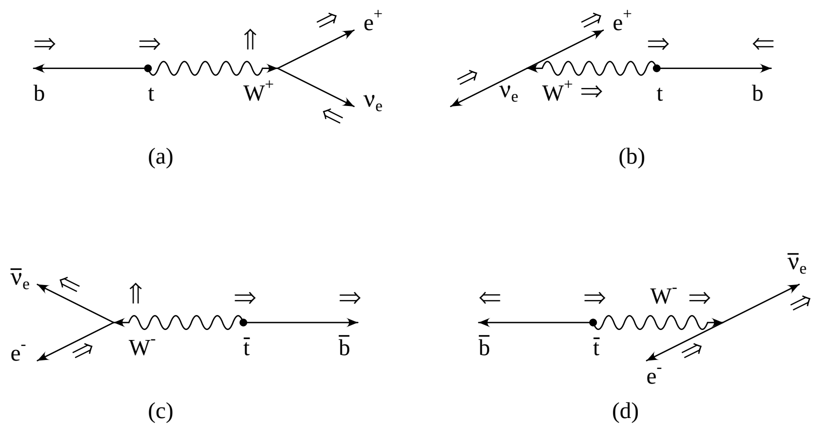


Figure 6: Cartoon explanation for why the charged lepton in a top (anti-top) decay tends to go in the same (opposite) direction as the top spin. The arrows above each particle indicate their spin. Figures (a) and (c) cover the longitudinally polarized W decay cases, and figures (b) and (d) cover the transverse W cases. From Ref[24]

early Universe may drive baryogenesis[23].

The Z' can be produced at the LHC by light quark annihilation. Once produced, it tends to decay into third family fermions: tops, bottoms, tau leptons, and their neutrinos. We've already covered top resonances. The resonance in $b\bar{b}$ is challenging to distinguish over backgrounds. The τ resonance is challenging to reconstruct because τ decays include missing energy and are usually into hadrons, but nonetheless it is an interesting and probably viable channel (with large statistics). The topflavor Z' will also decay into ordinary Ws. This decay is usually rare, but can be covered by high mass SM Higgs searches. Because of the chiral structure of topflavor, the decay into top quarks is mostly into left-handed tops. Top polarization can be reconstructed, for example by looking at the direction of the charged lepton from a semi-leptonic top decay (see Figure 6 for a cartoon explanation as to why the charged lepton tends to move in the direction the top spin was pointing).

The topflavor W' will decay largely into $t\bar{b}$ and $\tau\nu$. The decay into a top quark appears as a resonance against the continuum background of s-channel single top production[24, 25, 26]. Again, the coupling is left-handed and polarization observables can help distinguish the W' of topflavor from other theories.

2.4 $SU(2)_R$

$SU(2)_R$ models have the gauge structure $SU(2)_L \times SU(2)_R \times U(1)_X$ followed by a breakdown of $SU(2)_R \times U(1)_X \to U(1)_Y$ and were briefly covered in the lectures by David Kaplan. The fermion generations couple universally, with the left-handed doublets charged as usual under $SU(2)_L$ and the right-handed fermions assembled (together with right-handed neutrinos) into $SU(2)_R$ doublets. This gauge structure descends naturally from $SO(10)$ GUTs, and $SU(2)_R$ is often invoked in other contexts because it acts as a custodial symmetry which prevents large contributions to the Peskin-Takeuchi T parameter[27].

Because the W' and Z' couple in a family-universal manner, searches typically look for electrons and muons from their decays. While decays into tops are not particularly enhanced, they are still useful to measure the fact that the couplings are right-handed.

2.5 Little Higgs

Little Higgs theories are an interesting solution to the little hierarchy problem. There are too many versions and too many details that go into constructing such a model, so I restrict myself to a few remarks here. There are nice lectures from a previous TASI by Martin Schmaltz[30], and a good review article[31] which is more easily accessible online.

Little Higgs theories postulate that the Higgs is a pseudo-Nambu-Goldstone boson in order to protect its mass and solve the little hierarchy problem. From a nuts and bolts point of view, they invoke W's and Z's to cancel the one-loop quadratic divergence induced on the Higgs mass by the SM W and Z, and a heavy t' quark to cancel the divergence induced by the SM top. The W's and Z's have phenomena similar to the ones we have already described. The heavy quark will be dominantly produced singly through its mixing with the ordinary top[28] and decays into Wb, Zt, and ht, forming a (fermion) resonance in those channels.

Modern little Higgs theories often incorporate a symmetry (T-parity) to provide a dark matter candidate and lessen contraints from precision electroweak data[29] In this class of models, the signatures involve missing energy and were covered by Howie Baer.

2.6 Extra Dimensions

Bogdan Dobrescu provided nice lectures about extra dimensions, explaining how the Kaluza-Klein decomposition results in massive copies of any particles propagating in the extra dimension. You can also take a look at previous TASI lectures by Graham Kribs[32] or Csaba Csaki[33]. The heavy KK modes provide many potential new resonances: gravtons, gluons, weak bosons, Higgs, etc....

3 Effective Theory Descriptions

We can write the low energy physics of a wide class of models with extended gauge structures using effective field theory. A common theme among the models of the previous section was a structure like $G_1 \times G_2 \to G_{SM}$ where G_{SM} is any of the SM gauge groups, $SU(3)_c$ (as in topcolor), $SU(2)_W$ (as in topflavor, $SU(2)_R$), or $U(1)_Y$. This structure is also common in Little Higgs theories, and mocks up the lowest KK mode of an extra dimension through dimensional deconstruction[34].

In fact, it is more general than even those examples would indicate. If we have vector particles transforming as adjoints under a SM group, the combined requirements of gauge invariance under the SM symmetries with the need for a hidden gauge symmetry for the heavy particles to insure consistency, leads us to $G_1 \times G_2 \to G_{SM}$ in every case. Indeed $SU(2) \times SU(2)$ is the effective theory description of the technirho[35] and also of the ordinary rho meson of QCD[36].

So let's work out an example in detail. The specific example I will use is motivated by the topcolor model, with $SU(3)_1 \times SU(3)_2 \to SU(3)_c$, but one can pretty easily transcribe it into any $SU(N) \times SU(N) \to SU(N)$, and with minimal headaches into $U(1) \times U(1)$. A "moose" or "quiver" diagram[37] is shown in Figure 7, and specifies the gauge groups $(SU(3)_1 \times SU(3)_2)$ as the

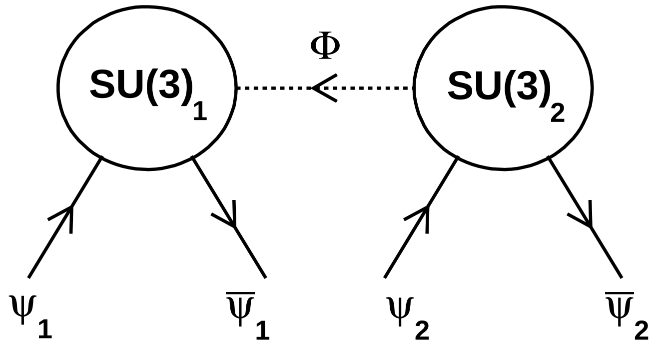


Figure 7: Moose diagram for the $SU(3) \times SU(3)$ model.

Table 1: Representations for the matter fields.

Field	Spin	$SU(3)_1$	$SU(3)_2$
ψ_1	1/2	3	1
$\bar{\psi}_1$	1/2	$\bar{3}$	1
ψ_2	1/2	1	3
$\bar{\psi}_2$	1/2	1	$\bar{3}$
Φ	0	3	$\bar{3}$

two circles in the diagram. Each group has its own gauge coupling, g_1 and g_2. I have chosen ψ_1 to be a left-handed fundamental of $SU(3)_1$, indicated by its arrow going into that group. $\bar{\psi}_1$ is a right-handed fundamental (or if you like, a left-handed anti-fundamental). ψ_2 and $\bar{\psi}_2$ form another vector-like pair, fundamental under $SU(3)_2$. The arrows make it very easy to keep track of anomaly cancellation. The dashed line Φ going between the two groups is a scalar field which is a fundamental under $SU(3)_1$ and an anti-fundamental under $SU(3)_2$. The gauge assignments are written out in Table 1, but it is worthwhile to take the time to learn how to read the moose diagram. It may seem awkward at first, but once you get used to it, it becomes second-nature.

The Lagrangian follows from the gauge structure. The gauge assignments dictate the kinetic terms,

$$-\frac{1}{4}\left(F^1_{\mu\nu}\right)^2 - \frac{1}{4}\left(F^2_{\mu\nu}\right)^2 + i\bar{\psi}_1\left(\not{D} - m_1\right)\psi_1 + i\bar{\psi}_2\left(\not{D} - m_2\right)\psi_2 \quad (47)$$

$$+\mathrm{Tr}\,(D^\mu\Phi)^\dagger\,(D^\mu\Phi) - V(\Phi) + y\Phi\bar{\psi}_2\psi_1 + H.c.$$

where the F's are the usual $SU(3)$ field strengths, built from the gauge fields A^1_μ and A^2_μ, and we have used the fact that we chose the ψ's to be vectorlike to pass immediately to four component language. If we had chosen a chiral theory, we would have just used 2-component language or put in projectors to end up where we wanted. Since I chose a vectorlike theory, I went ahead and wrote down masses for the fermions. If they had had chiral electroweak quantum numbers, as the SM fermions, the $SU(2)_W \times U(1)_Y$ gauge symmetries would forbid such masses. The Tr on the kinetic term for Φ is a way of tracing over the (double) gauge indices when I represent the field as a 3×3 matrix with the rows representing the $SU(3)_1$ index and the columns representing the $SU(3)_2$ index. The covariant derivatives are given by,

$$\begin{aligned} D^\mu\psi_1 &= \partial_\mu\psi_1 - ig_1 A^{1a}_\mu T^a\psi_1 & (48)\\ D^\mu\psi_2 &= \partial_\mu\psi_1 - ig_2 A^{2a}_\mu T^a\psi_2 & (49)\\ D^\mu\Phi &= \partial_\mu\Phi - ig_1 A^{1a}_\mu T^a\Phi + ig_2 A^{2a}_\mu \Phi T^a & (50) \end{aligned}$$

in terms of the two gauge couplings and the generators of $SU(3)$ in the fundamental representation, T^a. We have also written down a potential for Φ, and the Yukawa interaction between it and the fermions allowed by gauge invariance.

Our theory thus has parameters,

$$g_1, g_2 \qquad m_1, m_2, y \tag{51}$$

plus the parameters needed to describe the potential of Φ. The potential parameters will be important for determining the VEV of Φ, and also the masses of the Higgs bosons associated with it. These new Higgs bosons are interesting physics, but usually hard to access at near-future colliders, so in many cases we can ignore them when we talk about phenomena at the LHC. Since our theory is a *generic* description of a massive color octet vector particle, these parameters tell us what we are aiming for when we discover an octet vector and want to measure its properties. Once we pin them down, we can either try to fit them into a bigger framework for the UV physics, or we may see that the framework doesn't describe all of the phenomena we see – in which case we need to extend the effective theory itself.

We assume that the potential for Φ induces a VEV that is "diagonal" in $SU(3)_1$-$SU(3)_2$ color space,

$$\langle \Phi^i_\alpha \rangle \;=\; u\,\delta^i_\alpha \tag{52}$$

where u is the magnitude of the VEV and i is the gauge index of $SU(3)_1$ and α the index of $SU(3)_2$. In practice, it is not hard to write down a potential which results in the vacuum that we want. Inserting this VEV in the kinetic term for Φ yields,

$$\begin{aligned} & u^2 \mathrm{Tr}\left[\left(-g_1 A^{1a}_\mu T^a + g_2 A^{2a}_\mu T^a\right)\left(-g_1 A^\mu_{1a} T^a + g_2 A^\mu_{2a} T^a\right)\right] \\ = \; & \frac{1}{2} u^2 \left[g_1^2 A^1 \cdot A^1 - 2 g_1 g_2 A^1 \cdot A^2 + g_2^2 A^2 \cdot A^2\right] \end{aligned} \tag{53}$$

where $\cdot$ denotes contraction of the Lorentz and gauge indices. In analogy with the electroweak theory, we rewrite the two gauge couplings,

$$g_1 \equiv \frac{g}{\sin\phi} \quad , \qquad g_2 \equiv \frac{g}{\cos\phi} \ . \tag{54}$$

In terms of these new parameters, the mass terms for the gauge fields can be written,

$$\frac{1}{2}\frac{g^2}{\sin^2\phi\cos^2\phi} u^2 \ \left[A^{a\mu}_1 \ A^{a\mu}_2\right] \left[\begin{array}{cc} \cos^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \sin^2\phi \end{array}\right] \left[\begin{array}{c} A^a_{1\mu} \\ A^a_{2\mu} \end{array}\right] \tag{55}$$

The mass matrix is now easily diagonalized, and yields mass eigenstates,

$$\begin{aligned} g^a_\mu &= \sin\phi\ A^{1a}_\mu + \cos\phi\ A^{2a}_\mu \\ A^a_\mu &= -\cos\phi\ A^{1a}_\mu + \sin\phi\ A^{2a}_\mu \end{aligned} \tag{56}$$

where g^a is the massless mode, which we identify with the usual SM gluon, and A^a is the massive color octet with mass,

$$M^2 \;=\; \frac{g^2}{\sin^2\phi\cos^2\phi} u^2 \ . \tag{57}$$

The fermion couplings to g^a_μ are universal, as required by the unbroken residual gauge invariance,

$$g_1 A^{a1}_\mu \bar{\psi}_1 \gamma^\mu T^a \psi_1 \;\rightarrow\; g\left(-\cot\phi A^a_\mu + g^a_\mu\right)\bar{\psi}_1\gamma^\mu T^a\psi_1 \tag{58}$$

$$g_2 A^{a2}_\mu \bar{\psi}_2 \gamma^\mu T^a \psi_2 \;\rightarrow\; g\left(\tan\phi A^a_\mu + g^a_\mu\right)\bar{\psi}_2\gamma^\mu T^a\psi_2 \tag{59}$$

with g the QCD coupling. The fermions have different couplings to the massive octet, depending whether they were originally charged under $SU(3)_1$ or $SU(3)_2$. In topcolor, we could take ϕ to be a small angle, in which case ψ_1 could represent the top quark, and ψ_2 could represent the light quarks.

Through the Yukawa interactions, the symmetry-breaking can also induce mixing between ψ_1 and ψ_2. The mass matrix is,

$$\left[\bar{\psi}_1 \ \bar{\psi}_2\right] \left[\begin{array}{cc} m_1 & yu \\ y^* u & m_2 \end{array} \right] \left[\begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right] \tag{60}$$

The mass matrix is diagonalized (in general) by a separate rotation of the left-handed and right-handed fields, which can be found by considering the matrices $M^\dagger M$ and $M M^\dagger$. The mass eigenstates will continue to have universal couplings to the zero mass vector, but will have a mixture of ψ_1 and ψ_2 couplings to the massive vector.

4 Closing Thoughts

Theories of physics beyond the Standard Model can show resonances in almost any pair of Standard Model particles we can imagine, with the possibilities far out-stripping our ability to cover all of them. The discovery of any resonance at the LHC will raise similar questions - what is its mass, how wide is its width, how is it produced, and what does it decay into? Having established the answers to those questions, we can then try to fit the resonance into a deeper picture of organizing principles and symmetries. And finding the answers themselves will be fun!

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