Introductory Lectures on Collider Physics

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(Dated: June 14, 2011)
I. KINEMATICS

Let’s begin by reminding ourselves of what we all learned back in ordinary quantum mechanics. If we want to study a potential $V(x)$ by scattering some kind of particle $a$ off of it, we prepare a beam of particles with four-momentum $p_a$, and measure their outgoing (scattered) four momenta $p_1$ (see Figure 1a). The 4-vector $Q \equiv p_1 - p_a$ describes the momentum transfer in each reaction, and the amplitude for scattering for a given $Q$ is given by the Fourier transform of $V$:

$$A(Q) \propto \int d^3 \vec{x} e^{i \vec{q} \cdot \vec{x}} V(\vec{x}) .$$

Since large values of $|\vec{q}|$ correspond to small distance structure, the large $|\vec{q}|$ events provide information about small structure in $V$.

Most often in particle physics, we are more interested in the short distance physics of another particle. We can replace $V(x)$ with another particle $b$, and study the $2 \rightarrow 2$ reaction,

$$p_a + p_b \rightarrow p_1 + p_2 ,$$

(see Figure 1b) in which case we simultaneously study the short distance behavior of both $a$ and $b$ in the large $Q \equiv p_1 - p_a$ regime. In fact, particles 1 and 2 need not even be the same as $a$ and $b$, so for high enough energies we may succeed at producing new particles we have never seen before. In any case, energy-momentum conservation implies,

$$p_a + p_b = p_1 + p_2 .$$
It is easy to choose any coordinate system we like by applying Lorentz transformations to all four of the momenta. In practice, a convenient choice is the center of momentum (CoM) frame, in which \( \vec{p}_a = -\vec{p}_b \). Since these momenta define a line (and nothing in the initial state distinguishes anything going on in the transverse directions\(^1\), we can always choose the axis defined by \( \vec{p} \) to be the \( \hat{z} \)-axis. In this case, we can write the 4-momenta of the incoming particles,

\[
 p_a \equiv (E_a, 0, 0, p) \\
 p_b \equiv (E_b, 0, 0, -p)
\] (4)

where \( E_a \equiv \sqrt{p^2 + m_a^2} \) and \( E_b \equiv \sqrt{p^2 + m_b^2} \) so the particles are on-shell. An important thing to notice is that the quantity \( p \) completely specifies the momenta of the initial state (if our particles have spins we should also define those somehow). We can encode this information in a Lorentz-invariant way by defining Mandelstam \( s \),

\[
 s \equiv (p_a + p_b)^2 = (p_1 + p_2)^2 = (E_{a}^{CoM} + E_{b}^{CoM})^2
\] (5)

where the second equality follows from energy conservation and the last emphasizes that these are related to the initial energy in the center of mass frame. That final expression makes it obvious that \( s \geq (m_a + m_b)^2 \) and it should also be clear that \( s \geq (m_1 + m_2)^2 \).

Now let’s discuss the final state particles 1 and 2. In the CoM frame, they must have equal and opposite spatial momenta, since \( p_a + p_b \) has only a time-component by construction in that frame. We can always choose our \( \hat{x} \)-axis to lie in the plane formed by \( \vec{p}_1 \) and \( \vec{p}_a \), and we can write the four-momenta as,

\[
 p_1 \equiv (E_1, p's_\theta, 0, p'c_\theta) \\
 p_2 \equiv (E_2, -p's_\theta, 0, -p'c_\theta)
\] (6)

where \( E_1 \equiv \sqrt{p'^2 + m_1^2} \) and \( E_2 \equiv \sqrt{p'^2 + m_2^2} \) and I have introduced the short-hand notation \( s_\theta \equiv \sin \theta \) and so on. Energy conservation tells us that \( E_1 + E_2 = E_a + E_b \), or written out as functions of \( p \) and \( p' \), that the magnitude \( p' \) is determined entirely by \( p \) and the four masses. Thus, \( p' \) will always be the same in a \( 2 \rightarrow 2 \) reaction involving the same particles and fixed \( p \).

\(^1\) Including spins! Since we can measure at most one component of the spin (if any) of \( a \) and \( b \), we can choose the spin measurement axis to also be the \( \hat{z} \)-axis.
We can encode the dependence on the scattering angle $\theta$ in a Lorentz invariant,

$$t \equiv (p_1 - p_a)^2 = (p_b - p_2)^2 = m_1^2 + m_a^2 - 2p_1 \cdot p_a = m_1^2 + m_a^2 - 2(E_{a CoM}^C - p p' \cos \theta). \tag{7}$$

 Needless to say, if we know $s$ and $t$, we know everything about the momenta involved in a $2 \to 2$ reaction. Notice that $t = Q^2$ from our earlier discussion, and it tells us about the 4-momentum transfer.

The physical range of $t$ can be easily determined by remembering that $-1 \leq \cos \theta \leq 1$.

Thus, we have:

$$t_1 \leq t \leq t_0, \tag{8}$$

where,

$$t_{0,1} = \left(\frac{m_a^2 - m_1^2 - m_b^2 + m_2^2}{2\sqrt{s}}\right)^2 - (p \mp p'\theta)^2. \tag{9}$$

For very high energy reactions (all energies much larger than all masses):

$$p_a \to (\sqrt{s}/2, 0, 0, \sqrt{s}/2)$$
$$p_b \to (\sqrt{s}/2, 0, 0, -\sqrt{s}/2)$$
$$p_1 \to (\sqrt{s}/2, \sqrt{s}/2\cos \theta, 0, \sqrt{s}/2\sin \theta)$$
$$p_2 \to (\sqrt{s}/2, -\sqrt{s}/2\sin \theta, 0, -\sqrt{s}/2\cos \theta)$$

and the limits on $t$ become $t_0 \to 0$ and $t_1 \to -s$. This helps make it clear why we need high energies to probe short distances: it is not enough to have high energy (large $s$), but we need high energy to reach large values of $t$, for which we have wide angle scattering which probes short distances.

Before closing the section on kinematics, I should mention that some people define a third Mandelstam invariant,

$$u \equiv (p_1 - p_b)^2 = (p_a - p_2)^2. \tag{10}$$

Since it is redundant with $s$ and $t$, we have a relation among the three:

$$s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2. \tag{11}$$
II. CROSS SECTIONS

If we are looking for events of some kind, we can write the rate at which they occur in a way which is independent of how we prepare the initial particles going into the reaction in terms of the cross section $\sigma$,

$$N = \sigma L \epsilon$$

where $N$ is the number of events observed, $\sigma$ is the cross section for the reaction, typically measured in barns (1 bn = $10^{-24}$ cm$^2$) or GeV$^{-2}$. $L \equiv \int dt \mathcal{L}$ is the integrated luminosity, which represents how much collision data was collected, and can be expressed in barn$^{-1}$ or GeV$^2$. Finally, $\epsilon$ is a dimensionless number that represents the fact that particle detectors typically have a limited efficiency to record every particle produced. In practice, $\epsilon$ is something the experimentalists need to determine for themselves (and then tell to the world when they commission their detectors). $L$ is determined by how long the accelerator was running and collecting data.

To compute a cross section, we compute the matrix element squared ($|\mathcal{M}|^2$) for the quantum transition of interest, and we sum over the allowed final states,

$$d\sigma = \frac{1}{2s} \left( \prod_{i=1}^{N} \frac{d^3 \vec{p}_i}{(2\pi)^3} \frac{1}{2E_i} \right) (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_i p_i) \ |\mathcal{M}(p_a, p_b \rightarrow \{p_i\})|^2$$

where the first factor corrects for the flux of the incoming massless particles, and the 4-delta function enforces energy momentum conservation. If our particles have spins, we can specify them as part of $\mathcal{M}$, or (more often) we cannot prepare the spin of the initial state or measure the spins of the final state. In such cases, we sum over the final spins and average over the initial ones. We call the matrix element squared after this operation $|\mathcal{M}|^2$. In these lectures, we will always be interested in these summed/averaged matrix elements.

Note that we wrote $d\sigma$ in the CoM frame, but it is invariant under boosts along the beam axis, so we will rarely have to worry about that fact. Also notice that each final state particle has 3 independent momentum components, for $3N$ total. Since there are 4 energy-momentum constraints, the total number of independent quantities is $3N - 4$. In a $2 \rightarrow 2$ reaction, these would be the $\theta$ and $\phi$ angles describing the final state momenta, where we previously ignored $\phi$ because we knew the matrix element cannot possibly depend on it.
III. $e^+e^- \rightarrow \mu^+\mu^-$

Let’s think about $e^+e^- \rightarrow \mu^+\mu^-$. Since $m_\mu \sim 200 \times m_e$, if we have enough energy to make muons in the first place, it must be a good approximation to drop $m_e$ compared to either $m_\mu$ or $\sqrt{s}$. So we will treat the electron (but not, for now, the muon) as massless.

Applying our formula for the cross section, Eq. (13)

$$d\sigma = \frac{1}{2s} \frac{1}{2E_1} \frac{|p_1|}{2} \frac{|p_2|}{2E_2} d\Omega_1 d\Omega_2 \delta^{(4)}(p_a + p_b - p_1 - p_2)| \mathcal{M}|^2$$

where $d\Omega_i \equiv d\cos \theta_i d\phi_i$ and we’ll discuss $|\mathcal{M}|^2$ at length later on. For now let’s focus on the kinematics.

The 3 spatial pieces of the delta function require:

$$|p_2| = |p_1| = p$$
$$\theta_2 = -\theta_1$$
$$\phi_2 = \pi + \phi_1$$

and we can use those three factors to do the $d^3p_2$ integration, with the understanding that we replace $p_2 \rightarrow -p_1$ inside the matrix element as well as in $E_2$, which is now equal to $E_1$ as a result. We arrive at,

$$d\sigma = \frac{1}{2s} \frac{1}{2E_1} \frac{|p_1|}{2} \frac{p^2}{4E_1^2} \delta(E_a + E_b - E_1 - E_2) |\mathcal{M}|^2 \ d\Omega_1 dp .$$

To use the delta function, it is useful to change the integration over $dp$ into one over $dE$. That is easily accomplished by noting that $E = \sqrt{p^2 + m^2}$, so:

$$dE = \frac{p}{E} dp .$$

We will also make use of the CoM frame for which $E_a + E_b = \sqrt{s}$. Thus,

$$d\sigma = \frac{1}{2s} \frac{1}{2E_1} \frac{p(E_1)}{4E_1^2} \frac{1}{|\mathcal{M}|^2} \delta(\sqrt{s} - 2E_1) d\Omega_1 dE_1$$

$$d\sigma = \frac{1}{32\pi^2 s} \frac{p(E_1)}{\sqrt{s}} \frac{1}{|\mathcal{M}|^2} \ d\Omega .$$

Since $\phi_1$ is trivial ($|\mathcal{M}|^2$ does not depend on it), we may as well integrate over it, which physically just means that we will accept any event where $e^+e^- \rightarrow \mu^+\mu^-$ independently.
from what value $\phi_1$ happens to take. The last thing to notice is that by changing variables from $dp$ to $dE_1$, $p$ has become a function of $E_1$:

$$p(E_1) = \sqrt{E_1^2 - m^2} = \sqrt{\frac{s}{4} - m^2} = \frac{\sqrt{s}}{s} \sqrt{1 - \frac{4m^2}{s}}$$  \hspace{1cm} (20)$$

where one often sees the definition $\beta = \sqrt{1 - 4m^2/s}$ used in the literature. Altogether, this leads us to,

$$\frac{d\sigma}{d\cos \theta} = \frac{1}{32\pi s} \sqrt{1 - \frac{4m^2}{s}} |M|^2(s, \theta).$$  \hspace{1cm} (21)$$

Of course, all of the interesting stuff is actually the $s$ and $\theta$ dependence of $|M|^2$, and we will discuss this next.

In the Standard Model (SM), there are two Feynman diagrams contributing to the reaction $e^+ e^- \rightarrow \mu^+ \mu^-$ at leading order in perturbation theory (see Figure 2). They correspond to exchange of a virtual photon ($\gamma^*$) or $Z$-boson, respectively. To start out, let’s take $\sqrt{s} \ll M_Z$. In this limit, the $Z$ exchange graph is suppressed compared to the photon graph, so we can approximate the whole answer as the photon result,

$$-iM = [\bar{u}_1 i e \gamma^\mu v_2] \frac{-i g_{\mu\nu}}{s} [\bar{v}_b i e \gamma^\nu u_a],$$  \hspace{1cm} (22)$$

where $e$ is the QED gauge coupling and the ... include terms which drop out in the limit of zero electron mass. Squared and averaged/summed over initial/final spins, this is:

$$|M|^2 = \frac{e^4}{s} \left(1 + \cos^2 \theta\right).$$  \hspace{1cm} (23)$$

Thus,

$$\frac{d\sigma}{d\cos \theta} = \frac{\pi \alpha^2}{2s} \sqrt{1 - \frac{4m^2}{s}} \left(1 + \cos^2 \theta\right),$$  \hspace{1cm} (24)$$
FIG. 3: Distribution of $d\sigma/d\cos \theta$ in $e^+e^- \rightarrow \mu^+\mu^-$ as well as the dependence of the inclusive cross section on the incoming energy, $\sigma(s)$.

where $\alpha \equiv e^2/(4\pi)$ is the usual fine structure constant. This expression indicates that muons tend to be produced more in both the forward and backward directions, as shown in Figure 3a. If we integrate this expression over $\cos \theta$, we arrive at the dependence of the inclusive cross section on the initial energy $s$,

$$\sigma(s) = \int_{-1}^{+1} d\cos \theta \frac{d\sigma}{d\cos \theta} = \frac{4\pi\alpha^2}{3s} \sqrt{1 - \frac{4m^2}{s}}. \quad (25)$$

This function is plotted in Figure 3b, which shows the sharp turn-on at $s \simeq 4m_{\mu}^2$ and subsequent fall as $1/s$ at large energies.

**Homework:** Derive the $\theta$ and energy dependence for production of scalar muons, $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$. In reality, scalar muons must be heavy enough that it is not a good approximation to neglect the $Z$ boson, but neglect it anyway. You should find a differential cross section proportional to $\sin^2 \theta$.

The results of the exercise illustrate an important point. Though we cannot directly measure the spin of the final state particles, we can sometimes infer them through the kinematic distributions. In this case, the key is that the intermediate particle (photon) is spin-1. As a result, we can track the flow of angular momentum through the two processes:

$$e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^- \quad \text{versus} \quad e^+e^- \rightarrow \gamma^* \rightarrow \tilde{\mu}^+\tilde{\mu}^-$$

$S = 1 \quad S = 1 \quad S = 1 \quad \text{versus} \quad S = 1 \quad S = 1 \quad L = 1$
The intermediate photon requires angular momentum $J = 1$. For ordinary muons, as fermions, this is most easily realized in the $s$-wave ($L = 0$) for an $S = 1$ spin state. For scalar muons, there is no spin to make up $J = 1$, and we are forced to go to an $L = 1$ ($p$-wave) configuration.

It is worth mentioning in passing that the process $e^+e^- \rightarrow$ hadrons begins at lowest order at energies far below the $Z$ boson mass with exactly the same photon exchange, with quarks in the final state (since gluons carry no electric charge they cannot be produced at lowest order in perturbation theory) instead of muons, and the replacement of the muon electric charge by the quark charge, effectively multiplying the cross section by $\sum Q^2_q$, where $Q = +2/3$ for the up-type quarks and $-1/3$ for the down-type quarks. We’ll say more about this below in Section IV, but for now let’s note an important point: just being able to produce new particles already can lead us to discover them! In fact, both the charm and bottom quarks were discovered by looking at the process $e^+e^- \rightarrow$ hadrons as a function of energy. The quantity is defined normalized to the muon rate, 

$$R \equiv \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-},$$

and effectively just counts the number of quarks we have enough energy to produce, weighted by their electric charge squared. In Figure 4 we see a plot of experimental data for this ratio, including the jumps it experiences when the collider has enough energy to produce pairs of charm or bottom quarks.

Now let’s go back to the $Z$-exchange diagram. It looks a lot like the photon graph, with the difference that the $Z$ itself has a non-zero mass and the couplings to electrons and muons are chiral (meaning: they couple differently to left- and right-handed fermions). We will assume that the $Z$ couplings to electrons and muons are equal, as is predicted by the Standard Model and verified to exquisite accuracy [1]. The matrix element is,

$$-i\mathcal{M} = [\bar{u}_1 \gamma^\mu (g_R P_R + g_L P_L) v_2] \frac{-ig_{\mu\nu} + \ldots}{s - M_Z^2} [\bar{v}_b \gamma^\nu (g_R P_R + g_L P_L) u_a],$$

where $P_{L/R}$ are the left-handed/right-handed projectors and once again the ... refer to terms that vanish for the massless electrons. We can see from the propagator denominator that this graph will get very large when $s \approx M_Z^2$, which will allow us to neglect the photon contribution for such energies.

In fact, things seem problematic for $s \approx M_Z^2$ – the amplitude not only becomes large, but seems to be infinite right at $M_Z^2$. Such behavior is obviously unphysical. In fact, it is
an artifact of our working to leading order in perturbation theory. At the next-to-leading order, the denominator of the propagator picks up an imaginary part,

\[ G^{-1}(p^2) = p^2 - M_Z^2 + iM_Z\Gamma_Z \]  

(28)

from diagrams such as those shown in Figure 4. (That Feynman graph also corrects the real part of the propagator, and those corrections turn out to be UV divergent, and require both mass and wave function renormalization – but the imaginary part is finite). The optical theorem relates the imaginary part of the loop amplitude to the intermediate particles going onto their mass shells, and thus is guaranteed to produce the actual decay width\(^2\). Having included the imaginary part of the propagator, \(|M|^2\) is now proportional to,

\[ |M|^2 \propto \frac{1}{(s - M_Z^2)^2 + M_Z^4\Gamma_Z^2} \]  

(29)

\(^2\) You can find a much more detailed discussion along with many models of resonances discussed in my lectures at TASI-08 [3]. A copy of these lectures should be posted as supplementary information on the school website, and can also be obtained from my UCI home page. Note that they are aimed at a slightly higher level than these lectures!
FIG. 5: The rate for $e^+e^- \rightarrow$ hadrons as a function of the CoM energy, including measurements from several experiments [4].

the famous Breit-Wigner function (see figure 5).

The chiral couplings of the $Z$ lead to a more interesting angular dependence:

$$|\mathcal{M}|^2 = \frac{s}{(s-M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \left\{ (g_L^2 + g_R^2)^2 \left( 1 + \cos^2 \theta \right) + 2 (g_L^2 - g_R^2)^2 \cos \theta \right\}$$  \hspace{1cm} (30)

which we can use to separately extract $|g_L|^2$ and $|g_R|^2$ by studying the angular distributions of the outgoing muons from $Z$ decays. Let’s see how this works.

LEP produced millions of $Z$ bosons through $e^+e^-$ annihilation. From here, one easy quantity to derive is the number of $Z \rightarrow \mu^+\mu^-$ decays divided by the number of $Z \rightarrow$ hadron decays,

$$R_\mu \equiv \frac{e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-}{e^+e^- \rightarrow Z \rightarrow \text{hadrons}}$$  \hspace{1cm} (31)

It is somewhat amusing that $R_\mu = 1/R$ in terms of the quantity we looked at before at lower energies to discover the $c$ and $b$ quarks. Since this quantity accepts muons no matter
at which θ they are produced, we integrate \(d\sigma/d\cos \theta\) over \(\cos \theta\). The term proportional to \(\cos \theta\) integrates away, and we are left with a prediction for \(R_\mu\):

\[
R_\mu = \frac{(g_{L_\mu}^2 + g_{R_\mu}^2)}{\sum_q(g_{L_q}^2 + g_{R_q}^2)}.
\]

(32)

The second quantity is the “forward-backward asymmetry”, which measures the number of muons which go forward \((\cos \theta > 0)\) compared to the number which go backward \((\cos \theta < 0)\),

\[
A_{FB}^\mu = \frac{N_F - N_B}{N_F + N_B}.
\]

(33)

Taking our differential cross section and performing the integrals results in,

\[
A_{FB}^\mu = \frac{3}{4} \frac{(g_{L_\mu}^2 - g_{R_\mu}^2)}{(g_{L_\mu}^2 + g_{R_\mu}^2)} \frac{(g_{e_\mu}^2 - g_{R_e}^2)}{(g_{e_L}^2 + g_{e_R}^2)}.
\]

(34)

where just to make a point, we have separated out the electron from the muon couplings, despite their being equal in the SM. We sometimes define the asymmetry

\[
A_\mu = \frac{(g_{L_\mu}^2 - g_{R_\mu}^2)}{(g_{L_\mu}^2 + g_{R_\mu}^2)}
\]

(35)

for which \(A_{FB}^\mu = 3/4A_\mu A_e\). We can define a similar \(A_f\) for any fermion \(f\) for which we can measure \(\cos \theta\). In practice, this is all three charged leptons, \(e, \mu, \) and \(\tau\), and the heavy quarks \(b\) and \(c\) (whose decays into leptons tell us whether we have a heavy quark or a heavy anti-quark\(^3\) experiencing the decay. Obviously the top quark does not arise from \(Z\) decays, but we will see below how to measure a forward-backward asymmetry for it at Fermilab.

**Homework:** Derive \(\overline{M}^2(e^+e^- \rightarrow Z \rightarrow f\bar{f})\) for an arbitrary fermion \(f\). Derive \(R_f\) and \(A_{FB}^f\), and check your predictions for \(R_b\) and \(A_{FB}^b\) against their measured values [1].

IV. \(e^+e^- \rightarrow \text{HADRONS}\)

We have already seen the basics of \(e^+e^- \rightarrow \text{hadrons}\). At lowest order in perturbation theory, we can compute the rate into \(qq\) pairs, and sum over the masses of all of the quarks accessible at the energy of interest. In practice, we should still worry about a few details:

\(^3\) Meson-anti-meson mixing also confuses things a little bit!
1. Because the QCD coupling strength $g_S$ is not very small, we should worry about the possibility that there could be a reasonably large chance to radiate additional quarks or gluons which will appear in our description of the final state.

2. Quarks and gluons are not asymptotic states. Because of confinement, they are confined into colorless hadrons which interact with particle detectors.

A. Hadronization

Let’s discuss the second issue first, even though it is somewhat later in our picture of how a given event evolves from the initial annihilation to being detected. We have perfect evidence that quarks and gluons are always confined at large distances into hadrons. However, because this involves QCD at very low scales, the coupling is large and we can’t use perturbation theory to understand it quantitatively. We can get some information from nonperturbative numerical simulations (lattice QCD), but even the state of the art is far away from being able to describe very complicated configurations of partons, such as occur in high energy collider reactions.

As a result, we have no first principles description of hadronization. To turn a set of models into a set of hadrons, we have to rely on models (popular computer codes such as PYTHIA [5] or HERWIG [6] contain different models, and it is worthwhile to remember that while all of them are reasonable, none of them are really absolutely correct). To discuss a definite picture, I will consider a “string”-like model, similar to (but not really the same as) the one used by PYTHIA.

First consider a $q\bar{q}$ pair, as shown in Figure 6a. Because of confinement, as they are produced and move away from one another (assuming they have some kinetic energy when created), a flux tube of gluon field stretches between them and tries to confine them. While we don’t know much about this process, we can guess that the characteristics of the flux tube are determined by the scale of nonperturbative QCD, $\Lambda \sim 300$ MeV. In particular, up to order one numbers we can expect that the transverse sides of the tube are of order $\sim 1/\Lambda$ and the energy density inside the tube is $\Lambda^4$. Thus, the total energy contained in the string is:

$$E \sim \Lambda^2 L$$ (36)
where $L$ is the string length, or in other words, the distance between the $q$ and the $\bar{q}$.

As the quarks move apart, the string stretches, converting their kinetic energy into the energy of the flux tube. This can continue either until the quarks run out of kinetic energy, or there is enough energy stored in the flux of glue to create a $q'\bar{q}'$ pair. I write these as $q'$ to emphasize that these could be differently flavored quarks than the ones I started with. In practice $m_d \sim m_u$ and $m_s$ (and even in most cases $m_c$ and $m_b$) is very small compared with the typical energies we encounter at modern colliders. After the creating the new pair of quarks, the string “snaps” into two strings, neither of which see any large color charge from the other, and so continue to evolve independently from one another (Figure 6b). Provided the quarks at its endpoint still have enough kinetic energy, each string will continue to grow, and continue to snap into pairs of light quarks when it can. Ultimately, this process will end when every quark has kinetic energy of order $\sim \Lambda$. At this point, the strings stop growing and we can identify the resulting hadrons by identifying each string with a meson. An (overly simplified) example starting from the initial production of a pair of energetic strange

\footnote{Realistic models will also produce baryons, but this is beyond the scope of our cartoon discussion.}
quarks is shown in Figure 6c.

**Moral:** We don’t *quantitatively* understand hadronization. If you are designing a measurement which depends very sensitively on the details of how it happens, you should treat whatever model you are using with deep suspicion. The differences between competing hadronization models may be large, and the spread in results they give may not capture

B. Extra Radiation : The Parton Shower

V. HADRON COLLIDERS

VI. CONCLUSIONS AND OUTLOOK

Acknowledgements

I am glad to acknowledge the excellent organization by Goran and the local organizers, and partial support from the NSF through grant PHY-0970171.