Neutrino at Collider - I

“Parity restored at TeV scale?”

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Summer School on Particle Physics

ICTP — April 2011
Are we satisfied with the SM?

Gravity?
Dark Matter?
The Higgs? Hierarchy?

*SM aesthetically incomplete*

Accidental symmetries, $B$, $L$?

Can we have new physics at collider?

Neutrino masses are new physics

Dirac or Majorana?

Low scale?

Key questions: which symmetry? at which scale?
Outline

New physics - SM already needs extension

- Neutrino mass
  Majorana - Dirac - generic
- Consequences
  $0\nu\beta\beta$
  versus Cosmology?
  New physics at TeV?

Beyond SM

- Further hints from Quantum Numbers
- Let’s restore Parity, Left-Right at TeV scale
- Constraints
- Back to $0\nu\beta\beta$
  typell example
- LNV @ Collider
  Signals
- Outlook
Neutrino have mass

From oscillations we know their mass differences

\[ m_2^2 - m_1^2 = 7.6 \times 10^{-5} \text{ eV}^2 \]

\[ |m_3^2 - m_2^2| = 2.4 \times 10^{-3} \text{ eV}^2 \]

and mixing angles, \( \theta_{12} = 35^\circ \pm 4^\circ, \theta_{23} = 45^\circ \pm 8^\circ, \theta_{13} < 13^\circ. \)

From oscillations we don’t know:

- The absolute neutrino mass scale
  (direct searches, cosmology: \( m_{1,2,3} < 1 \text{ eV} \))

- The mass hierarchy
  (normal \( m_1 < m_2 < m_3 \) or inverted \( m_3 < m_1 < m_2 \)?)

- Dirac or Majorana
  \((\nu \neq \nu^c \text{ or } \nu \equiv \nu^c \text{ [Majorana '37]})\)
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Theory?

What about theory?

In the SM:

- Lepton Number conserved.  (also family $L_e$, $L_\mu$, $L_\tau$ separately!)
- Only left neutrinos, there is no renormalizable mass term.
- Effective theory: a $D = 5$ nonrenormalizable operator?

BSM:

- Or new states.
- Question: is it low or high scale physics?
- Physical consequences.
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Neutrino masses

- **Dirac mass** ($\Delta L = 0$) – need Right-Handed neutrino $\nu_R$
  \[ M_D \overline{\nu_R} \nu_L + h.c. \equiv M_D \nu_R^c C \nu_L \rightarrow M_D \nu_R^\ast \nu_L \beta \delta^{\alpha \beta} + h.c. \]
  
  $M_D$ generic complex.
  
  Generated with familiar Yukawa term, $y_D H \bar{\ell}_L \nu_R$.

- **Majorana mass** ($\Delta L = 2$)
  \[ M_L (\overline{\nu^c_L}) \nu_L + h.c. \equiv M_L \nu_L^t C \nu_L \rightarrow M_L \nu_L \alpha \nu_L \beta \epsilon^{\alpha \beta} + h.c. \]
  
  $M_L$ symmetric!
  
  Breaks total lepton number $L$. (as family ones, $L_e$, $L_\mu$, $L_\tau$.)
  
  Generated only as effective operator, $\frac{\lambda}{M}(\bar{\ell}H)(H\ell)$.

[Mohapatra, Pal, “Massive neutrinos in physics and astrophysics”]
[Denner et al, “Compact Feynman rules for Majorana fermions”, PLB291]
[Dreiner, Haber, Martin, “Feynman Rules using two-component spinor notation”]
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Seesaw (type-I)

Once present, the singlet $\nu_R$ can have renormalizable Majorana mass. So,

$$\begin{pmatrix} \nu_L & \nu_R^c \end{pmatrix} \begin{pmatrix} 0 & M_D^t \\ M_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}.$$ 

- **Seesaw**: if $M_R \gg M_D$, the mass matrix is $\begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$,

$$M_\nu \simeq -M_D^t M_R^{-1} M_D, \quad M_\nu \simeq M_R,$$

$M_R$ large $\Rightarrow$ $M_\nu$ small.

(eigenstates: light Majorana and heavy Majorana)

[Minkowski '77, Mohapatra Senjanović '79, GRS '79, Glashow '79; Yanagida '79]
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But what can $M_D$ and $M_R$ be?
Scales $m_D, m_R$ quite free... (yukawa perturbativity, $M_D < 500$GeV)

Some scenarios using $m_\nu = m_D^2/m_R \lesssim 1$eV

- $m_D \sim 100$GeV – (like heavy quarks?)
  \[
m_D^2/m_\nu = m_R \gtrsim 10^{13-15}$GeV, \]  
  High scale physics

  Fits with GUT scenario, related to $B$?, ...

- $m_D \lesssim$ MeV – Now one can have much lower $m_R$:
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m_D^2/m_\nu = m_R \lesssim$ TeV, \]  
  Collider scale

More interesting:

$m_R$ associated to physical states: observable (see later)

Seesaw-I not the only possibility...
Seesaw (type-I) - at which scale?

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Some scenarios using $m_\nu = m_D^2/m_R \lesssim 1\text{ eV}$ ignoring mixings

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High scale physics

Fits with GUT scenario, related to $B^-$?, ... [Bajc lectures]

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Fits with GUT scenario, releted to $B\?,\ldots$ [Bajc lectures]

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Seesaw-I not the only possibility...
Seesaw (type-II)

- In a $SU(2) \times U(1)_Y$ theory, the lepton doublet $\ell$ can couple also with a triplet scalar field $\Delta_L \in (3, 1)$:

$$\mathcal{L}_{Y\Delta} = Y_\Delta \ell_L^t \tau_2 \Delta_L \ell_L$$

with symmetric $Y_\Delta$. In components

$$\Delta_L = \begin{pmatrix} \delta^+ / \sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+ / \sqrt{2} \end{pmatrix}$$

- If it has a (neutral!) VEV $\langle \delta^0 \rangle = v_L$, it generates a neutrino Majorana mass $M_L \nu^t_L \nu_L$, with

$$M_L = Y_\Delta v_L.$$

- The triplet couples to Higgs, $m_\Delta^2 \Delta^2 + m_\Delta H \Delta H$. (\(m_\Delta \gg v\))

So it has a naturally small VEV, \(v_L \sim v^2 / m_\Delta\).

$$M_\nu \sim Y_\Delta v^2 / m_\Delta$$

Again, large \(m_\Delta \rightarrow \) small \(M_L\).
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Masses, general

Seesaw type-I plus type-II lead to the general scenario:

\[
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\end{pmatrix}
\begin{pmatrix}
M_L & M^t_D \\
M_D & M_R
\end{pmatrix}
\begin{pmatrix}
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\end{pmatrix}.
\]

with \(M_L, M_D \ll M_R\).

- Eliminating the \(M_D\) mixing, one gets \(\begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}\), with

\[
M_\nu \simeq M_L - M^t_D \frac{1}{M_R} M_D, \quad M_N \simeq M_R.
\]

- Note, now that there can be cancelations to get light \(M_\nu\).

And there can be cancelations also inside \(M^t_D M_R^{-1} M_D\).

(see Casas-Ibarra parametrization of \(M_D\))
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Dirac vs Majorana
Seesaws
Diagonalization
Lepton Violation
$0\nu\beta\beta$
Experiments
New Physics

Masses, diagonalization

Now, as for quarks, mass eigenstates are not flavour ones.
Charged leptons-neutrino mismatch enters Left charged current.

\[ M_e = V_{eL} m_e V_{eR}^\dagger \]
\[ M_\nu = V_{\nu L} m_\nu V_{\nu R}^\dagger \]
\[ U_{PMNS} = V_{eL}^\dagger V_{\nu L} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} = \begin{bmatrix} c_{13} & s_{13}e^{-i\delta} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & e^{i\alpha_2} & 1 \end{bmatrix} \]

- **Dirac mass, generic complex**
  \[ V_{\nu L} \neq V_{\nu R} \]
  so 5 external phases irrelevant.
  (Kinetic, current and masses respect $U(1)_{L_x}$!)
  Only $\mathcal{CP}$ from the 'Dirac' phase, as in CKM ($U_{e3}$ suppressed).

- **Majorana mass, complex symmetric**
  \[ V_{\nu R} \equiv V_{\nu L}^* \]
  Now the two phases $\alpha_1$ and $\alpha_2$ can not be removed!
  (i.e. Majorana mass breaks lepton numbers!)
  These phases however appear only in LNV processes.
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\end{bmatrix}
\]

\[
M_\nu = V_{\nu L} m_\nu V_{\nu R}^\dagger
\]

\[
= \begin{bmatrix}
e^{i\alpha_e} & 0 & 0 \\
e^{i\alpha_\mu} & 0 & 0 \\
e^{i\alpha_\tau} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix}
\begin{bmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
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- **Dirac** mass, generic complex \( V_{\nu L} \neq V_{\nu R} \)
  - so 5 external phases irrelevant.
  - (Kinetic, current and masses respect \( U(1)_{Lx} \)!) Only \( CP \) from the 'Dirac' phase, as in CKM (\( U_{e3} \) suppressed).

- **Majorana** mass, complex symmetric \( V_{\nu R} \equiv V_{\nu L}^* \)
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  - (i.e. Majorana mass breaks lepton numbers!)
  - These phases however appear only in LNV processes.
Neutrino - up to now

What we saw:

- Neutrino have masses (Dirac or Majorana)
- Need extension of the SM.
- Add heavy $\nu_R \rightarrow$ seesaw-I.
- Add heavy $\Delta_L \rightarrow$ seesaw-II.

- Majorana violates Lepton number by two units
- Two extra ‘Majorana’ CP phases in the mixing matrix $U_{PMNS}$.

let’s look at consequences...
Lepton number violation, consequences

\[ W^- \rightarrow \nu \nu W^+ \]

Lepton number violation, consequences

Outline

Neutrino

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Lepton Violation

$0\nu\beta\beta$

Experiments

New Physics
Lepton number violation, consequences

- Nuclear neutrinoless double beta decay:
  \[ ^A_ZX \rightarrow ^{A+2}_ZX + 2e^- \]
  \[ \cdots \tau_{0\nu\beta\beta} \gtrsim 10^{24} y, \text{ but testable!} \]
  (and double electron nuclear capture, \[^A_ZX + 2e^- \rightarrow ^{A-2}_ZX, \text{ etc.} \])

[Racah, Nuovo Cim. '37]
Lepton number violation, consequences

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  \[ A_X \rightarrow A_X + 2e^- \]

  \[ \tau_{0\nu\beta\beta} \gtrsim 10^{24} \text{y}, \text{ but testable!} \]

  (and double electron nuclear capture, \( A_X + 2e^- \rightarrow A_{X-2}, \) etc.)

- Collider: same sign dileptons:

  Very small for standard \( W \ldots \)
Lepton number violation, consequences

- Nuclear neutrinoless double beta decay:
  \[ ^A_Z X \rightarrow ^{A+2}_Z X + 2e^- \]
  ... \( \tau_{0\nu\beta\beta} \gtrsim 10^{24} \text{y} \), but testable!

  (and double electron nuclear capture,
  \[ ^A_Z X + 2e^- \rightarrow ^{A}_{Z-2} X \], etc.)

- Collider: same sign dileptons:
  
  Very small for standard \( W \)...

- Meson neutrinoless double beta decay, e.g. \( K^+ \rightarrow \pi^- \ell^+\ell^+ \)
  \( BR < 10^{-20} \), much less than current limits, \( BR \lesssim 10^{-10} \)

[Racah, Nuovo Cim. ’37]

[Keung Senjanović ’83]

[Littenberg Schrok, ’92]
Neutrino at Collider - I
F. Nesti

Outline

Neutrino
Dirac vs Majorana
Seesaws
Diagonalization

Lepton Violation
$0\nu\beta\beta$
Experiments
New Physics
Neutrinoless double beta decay $0\nu\beta\beta$

- Actually a loop process:
  Neutrino $p \sim 100$ MeV
  Released $Q \sim 3$ MeV.

Decay width:
$\Gamma_{0\nu} = G(Q) |M|^2$

[phase space] [amplitude]

- The amplitude is $M = 8G_F^2 \int d^4x d^4y J^\mu_{had}(x) J^\nu_{had}(y) L_{\mu\nu}(x, y)$
  where the leptonic tensor is (in momentum space)

$$L_{\mu\nu} = \bar{e} \gamma_\mu L \left[ \frac{p + M_\nu}{p^2 - M_\nu^2} \right]_{ee} \gamma_\nu R e^c$$

- LNV explicitly related to Majorana neutrino masses.
  Light neutrinos ($M_\nu \ll p \sim 100$ MeV) give

$$L_{\mu\nu} \propto M_{ee}^\mu \frac{1}{p^2}$$
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Strenght of LNV in $0\nu\beta\beta$, from standard light neutrinos:

$$M^{ee}_\nu = \sum U^2_{ei} m_i = m_1 |U^2_{e1}| + m_2 |U^2_{e2}| e^{i\alpha_1} + m_3 |U^2_{e3}| e^{i\alpha_2}$$

So, from oscillations, $|U^2_{e1}| \sim 0.6$, $|U^2_{e2}| \sim 0.25$, $|U^2_{e3}| < 0.04$, ... Majorana phases important and there can be a cancelation!
$0
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0νββ cont’d

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Possible $0νββ$, as a function of lightest neutrino mass:

Can distinguish the hierarchy. And the absolute mass.

$[\text{Vissani '02}]$
0νββ, matrix elements

**Neutrino propagator**, i.e. $1/r$ for light $e^{-mr}/r$ for heavy neutrino.

- Well approximated by its typical momentum $p \sim 100 \div 200$ MeV.
- Both for light or heavy neutrino exchange (no core suppression)

$$\left\langle \frac{m_\nu}{p^2} \right\rangle_{nu} \simeq \frac{m_\nu}{p^2}, \quad \left\langle \frac{1}{m_N} \right\rangle_{nu} \sim \frac{1}{m_N}$$
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- Real calculation, w/ nuclear models, uncertain by a factor of 20–100–200%
Neutrinoless double beta decay, cont’d

Need to avoid the much more favored single beta decay.

- In some nuclei $\beta$-decay is forbidden! [Bethe-Weizsäcker formula]

- Now, $\beta\beta$ can proceed through both $2\nu\beta\beta$, or $0\nu\beta\beta$.

How to distinguish them? – We don’t detect neutrinos.
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How to distinguish them? – We don’t detect neutrinos.
Recognized by the spectrum of electrons

In real life, the line is not so definite...
Neutrino double beta decay, evidence

- Heidelberg-Moscow experiment...

...claim of observation (!)[Klapdor-Kleingrothaus+ PLB '04, MPL '06, '10]

$$\tau_{0\nu} \sim 2.2 \times 10^{25} \text{y} \quad (6\sigma\ldots)$$

- This is conservatively translated into

$$m_{\nu}^{ee} \sim (0.4 \pm 0.2) \text{eV}$$
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If true, evidence of Majorana...
Experiments ongoing!

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Isotope</th>
<th>Mass of Isotope [kg]</th>
<th>Sensitivity $\tau_{1/2}^{0\nu}$ [yrs]</th>
<th>Sensitivity $\langle m_{\nu}\rangle$, meV</th>
<th>Status</th>
<th>Start</th>
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<tr>
<td>GERDA</td>
<td>$^{76}\text{Ge}$</td>
<td>18</td>
<td>$3 \times 10^{25}$</td>
<td>$\sim 200$</td>
<td>running!</td>
<td>2011</td>
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<td>$\sim$ 2012</td>
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<tr>
<td>MAJORANA</td>
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<td>70-200</td>
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<td>1000</td>
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<td>10-40</td>
<td>R&amp;D</td>
<td>$\sim$ 2015</td>
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<td>EXO</td>
<td>$^{136}\text{Xe}$</td>
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<td>$6.4 \times 10^{25}$</td>
<td>100-200</td>
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<tr>
<td>SuperNEMO</td>
<td>$^{82}\text{Se}$</td>
<td>100-200</td>
<td>$(1 - 2) \times 10^{26}$</td>
<td>40-100</td>
<td>R&amp;D</td>
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<td>40-120</td>
<td>R&amp;D</td>
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</tr>
</tbody>
</table>

For a recent review [Rodejohann, arXiv:1106.1334]

Stay tuned
Cosmology, limits on absolute scale (WMAP-7, SDSS, HST)

\[ \sum m_\nu \lesssim 0.4 \div 1 \text{ eV} \]  
[WMAP 95\% C.L.]

\[ \sum m_\nu \lesssim 0.17 \text{ eV} \]  
[Seljak, Slosar, Mcdonald 06]

\[ \sum m_\nu \lesssim 0.44 \div 1 \text{ eV} \]  
[Hannestad+ '08, Hamann+ '10]

…shrink…
Future clash with cosmology?

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  \ldots shrinking toward incompatibility with evidences of $0\nu\beta\beta$\ldots
  \ldots in this case, need new physics beyond light neutrinos!
New Physics - where? when?

If $m_{ee}^\nu$ excluded by cosmology, can new Physics do the job?

Try to guess at the level of effective operators...

- The 'New Physics' operator is dimension 9
  \[ O_{NP} = \lambda \frac{nnppee}{\Lambda^5} \]

- Require new physics amplitude to saturate $m_{ee}^\nu \sim eV$
  \[ A_{0\nu}^{NP} = \frac{\lambda}{\Lambda^5} \quad \leftrightarrow \quad A_{0\nu}^{m\nu} = G_F^2 \frac{m_\nu}{p^2} \]

Result, the amplitudes are comparable for (say $\lambda \sim G_F^2 M_{W}^4$)

\[ \Lambda \sim TeV. \]

...something would be expected at collider.
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Neutrino, recap

- Neutrino have mass
- Majorana? (κ, and possible $0\nu\beta\beta$).
- Possibly an effective operator: (not telling us the origin)
  \[
  \frac{\lambda}{M} (\ell H)^t (H \ell),
  \]
  [Weinberg '79]
- Realizations, e.g. type-I seesaw: (y and M quite free)
  \[
  y \bar{\ell} H \nu_R + M \nu_R^t \nu_R
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- $0\nu\beta\beta$ probes, may require new physics beyond neutrino, at TeV.
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- So... maybe TeV M hints to something? New interactions?
  ... e.g.: M breaks lepton number, B − L, ...
- Maybe we can test a low M and new forces at LHC?
  (Yes, because of ν at collider.)

Hints from quantum numbers...

Tomorrow.