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ANNIHILATION OR CREATION OF PARTICLES: AN AUTONOMOUS REACTION-DIFFUSION PROCESS

Dedicated to Prof. Zahra Rhanavard, Former Dean of the Alzahra University
on the occasion of her 8th years efforts.

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Abstract

Annihilation or creation of particles on a D-dimensional region with boundaries is considered. Particles can be injected or extracted at the boundary as well. Two general behaviours of this system are investigated. The stationary behaviour of this system, and the dominant way of the relaxation of the system towards its stationary state. Based on the first behaviour, static phase transitions (discontinuous changes in the stationary profiles of the system) are investigated. Based on the second behaviour, dynamical phase transitions (discontinuous changes in the relaxation-times of the system) are studied. The investigation is specialized to systems in which the evolution equation of one-point functions are closed (the autonomous system).

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1 Introduction

A reaction-diffusion system consists of a collection of particles (of one or several species) moving and interacting with each other with specific probabilities (or rates in the case of continuous time variable). The aim of studying such systems, is of course to calculate the time evolution of such systems. But to find the complete time evolution of a reaction-diffusion system, is generally a very difficult (if not impossible) task.

Various methods have been used to study the reaction-diffusion system: analytical techniques, approximation methods, and simulation. The success of the approximation methods, may be different in different dimensions, as for example the mean field techniques, working good for high dimensions, generally do not give correct results for low dimensional systems. A large fraction of analytical studies belong to low-dimensional (specially one-dimensional) systems, as solving low-dimensional systems should, in principle, be easier [1–14].

Various classes of reaction-diffusion systems are called exactly-solvable, in different senses. In [15–17], integrability means that the N-particle conditional probabilities' S-matrix is factorized into a product of 2-particle S-matrices. This is related to the fact that for systems solvable in this sense, there are a large number of conserved quantities. In [18–27], solvability means closedness of the evolution equation of the empty intervals (or their generalization).

Consider a reaction-diffusion system (on a lattice) with open boundaries. By open boundaries, it is meant that in addition to the reactions in the bulk of the lattice, particles at the boundaries do interact with some external source. An important question is to find the possible phase transitions of the system. By phase transition, we mean a discontinuity in some behavior of the system with respect to its parameters. Such discontinuities, may arise in two general categories: in the stationary (large time) profiles of the system, and in the time constants determining the evolution of the system. In the first case, static phase transitions are dealt with; in the second case, dynamical phase transitions. There are systems for which the equation of motion for the one-point function (the probability that a certain site be occupied) is closed, that is independent of the multi-point functions [28–30]. Among these systems is the so called voting model (or a generalization of the Glauber model at zero temperature). In [31] a voting system on a one-dimensional lattice was studied, for which at the boundaries of the lattice there are injection or extraction of the particles. Based on the evolution of the one-point functions, it was shown there that the system exhibits two kinds of phase transitions: a static phase transition, corresponding to a discontinuous change in the stationary profile of the one-point function; and a dynamical one, corresponding to a discontinuous change in the behaviour of the relaxation time of the system toward its stationary state. In [32–34], the phase structures of extensions of such systems on a one-dimensional lattice were investigated. All of these are restricted to the case of a one-dimensional lattice.

A multi-dimensional extension of the voting model, on a continuum was studied in [37]. Here we would like to study the presence of annihilation or creation of particles in the volume for this model. This is a simple autonomous model in higher-dimension with a term for annihilation or creation proportional to the density profile in the volume.

The scheme of the present article is as follows. In section 2, a review of the multi-dimensional voting model on continuum and evolution equation for the density of the particles are presented. In section 3, the generalization of the evolution equation of this model to have annihilation or creation of particles in the volume is given. In section 4, the time-independent of the system is studied and it is shown that the system exhibits no static phase transition. In section 5, the relaxation of the system towards its stationary state is studied and it is shown that the system for some regimes exhibits a dynamical phase transition. Section 6 is devoted to the concluding remarks.

2 Definition of the model on continua

Consider the model defined in [37]. Let us briefly review it in the following: Imagine a multi-dimensional lattice, each site of which is either empty ($\emptyset$) or contains a single particle (A), and let there be a reaction between two neighboring sites like

$$\begin{aligned} A\emptyset &\to AA, && \text{with the rate } u_l,\\ \emptyset A &\to \emptyset\emptyset, && \text{with the rate } u_l. \end{aligned} \tag{1}$$

A lattice was studied at the boundaries of which injection and extraction of particles can take place. It was shown that these models are autonomous, meaning that the evolution equation of the n-point functions contain only n- or less-point functions. Here, we are considering the reaction between a site i (the left site), which is the ending point of the link l, and another site (the right site) which is the starting point of the same link. n_i is the particle number operator at the site i of the lattice and from (1), it is seen that the evolution equation for the one-point function is

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle n_i\rangle=\sum_l [u_l\,(\langle n_{i-l}\rangle-\langle n_{i-l}\,n_i\rangle)-u_l\,(\langle n_i\rangle-\langle n_{i-l}\,n_i\rangle)], \tag{2}$$

where by the site index $i-l$, it is meant a site which is the starting point of the link l, the ending point of which is the site i. It can be seen that the two-point functions on the right-hand side of (2) cancel each other. So,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle n_i\rangle=\sum_l u_l\,(\langle n_{i-l}\rangle-\langle n_i\rangle). \tag{3}$$

Now, assume that the one-point function is a slowly-varying function of its argument (i). In this case, one can define a smooth particle density function of the continuous position variable $\mathbf{r}$, with

$$\rho(\mathbf{r}_i):=\frac{1}{\mathcal{V}}\,\langle n_i\rangle, \tag{4}$$

where $\mathbf{r}_i$ is the position of the lattice site i, and $\mathcal{V}$ is the *specific hypervolume* of a site. Then (3) can be rewritten as

$$\frac{\partial}{\partial t}\,\rho = \sum_l u_l\,\left[-\boldsymbol{\delta}_l\cdot\nabla + \frac{1}{2}\,(\boldsymbol{\delta}_l\cdot\nabla)^2\right]\,\rho, \tag{5}$$

where higher-derivative terms have been neglected. Using suitable coordinates for $\mathbf{r}$, one can write the second derivative as

$$\frac{1}{2}\sum_l u_l\,(\boldsymbol{\delta}_l\cdot\nabla)^2 = \sum_a\left(\frac{\partial}{\partial x^a}\right)^2, \tag{6}$$

where x^a's are the coordinates of $\mathbf{r}$. So, (5) is rewritten as

$$\frac{\partial}{\partial t}\,\rho = (-\mathbf{v}\cdot\nabla + \nabla^2)\rho, \tag{7}$$

where

$$\mathbf{v} := \sum_l u_l\,\boldsymbol{\delta}_l. \tag{8}$$

Eq. (7) is nothing but a diffusion equation combined with a drift velocity $\mathbf{v}$.

Suppose that (7) holds for the interior of the region V. Integrating (7) on V, one arrives at

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}V\;\rho = -\oint_{\partial V}\mathrm{d}S\;\mathbf{n}\cdot\mathbf{v}\,\rho + \oint_{\partial V}\mathrm{d}S\;\mathbf{n}\cdot\nabla\rho. \tag{9}$$

The first term on the right-hand side is the rate of change of the total number of the particles inside, as a consequence of the drift, while the second term is the effect of injecting or extracting particle at the boundary. The boundary condition

$$\mathbf{n}\cdot\nabla\rho = \alpha - \beta\,\rho, \qquad \text{at the boundary} \tag{10}$$

corresponds to an injection rate of α per unit hyperarea of the boundary, and an extraction rate of β per unit hyperarea per particle density at the boundary. In general, one can take α and β position-dependent.

From now on, for simplicity we restrict ourselves to the case that the volume V is a D-dimensional hyperball with radius R, the boundary of which is a hypersphere.

3 A simple model for the annihilation or creation of particles

For the simplest generalization of this model, one can consider an additional term in the evolution equation to have creation or annihilation of particles proportional of the density profiles in the volume. Consider

$$\frac{\partial}{\partial t}\,\rho = (-\mathbf{v}\cdot\nabla + \nabla^2)\rho + m\rho, \tag{11}$$

and

$$\mathbf{n}\cdot\nabla\rho = \alpha - \beta\,\rho, \qquad \text{at the boundary} \tag{12}$$

where for $m < 0$, annihilation and for $m > 0$, creation of particles takes place in a volume. Here we would like to find the behaviour of the systems in the thermodynamics limit for the case, $m \neq 0$.

4 The time-independent state and the static phase transition

Let ρ_0 be the time-independent solution to (11) and (12). Using the ansatz

$$F_{\mathbf{q}}(\mathbf{r}) := \exp(\mathbf{q}\cdot\mathbf{r}) \tag{13}$$

(with $\mathbf{q}$ a constant vector) as a time-independent solution to (11), one arrives at

$$\mathbf{q}\cdot\mathbf{q} - \mathbf{v}\cdot\mathbf{q} = m, \tag{14}$$

and

$$(\mathbf{q} - \frac{\mathbf{v}}{2})^2 = \frac{\mathbf{v}\cdot\mathbf{v}}{4} - m, \tag{15}$$

which leads to

$$\mathbf{q} = \frac{1}{2}(\mathbf{v} + \mathbf{v}'), \tag{16}$$

where $\mathbf{v}'$ is an arbitrary constant vector subject to the condition

$$\mathbf{v}'\cdot\mathbf{v}' = \sqrt{\mathbf{v}\cdot\mathbf{v} - 4m}. \tag{17}$$

So, one can write the general time-independent solution to (11) as

$$\begin{aligned}\rho_0(\mathbf{r}) &= \int \mathrm{d}\Omega'\ \tilde{A}(\Omega')\, F_{\mathbf{q}}(\mathbf{r}),\\ &= \int \mathrm{d}\Omega'\ A(\Omega')\ \exp\left\{\frac{1}{2}\left[(\mathbf{v}+\mathbf{v}')\cdot\mathbf{r} - |\mathbf{v}+\mathbf{v}'|\,R\right]\right\},\\ &=: \int \mathrm{d}\Omega'\ A(\Omega')\ \exp[G(\mathbf{v}',\mathbf{r})],\end{aligned} \tag{18}$$

where Ω' denotes the angular coordinates of $\mathbf{v}'$, and A is an arbitrary function. It is easy to see that the maximum value of G is zero, and this maximum value is reached at a point on the boundary ($r = R$), where $\mathbf{r}$ is parallel to $\mathbf{v} + \mathbf{v}'$. For large values of R and r, G is a rapidly-varying function and the integral is mainly determined from that point of the integration region which maximizes G.

Here we separately consider two cases; the annihilation and creation of the particles in the volume (corresponding respectively to $m < 0$, $m > 0$).

1 -**Annihilation,** $m < 0$

Consider annihilation of particles in the volume. From Eq.(17),

$$|\mathbf{v}'| \geq |\mathbf{v}| \tag{19}$$

Then for every $\mathbf{r}$ we can find a $\mathbf{v}'$ that $\mathbf{q_0} = \frac{1}{2}(\mathbf{v}+\mathbf{v}')$ will become parallel to $\mathbf{r}$. We denote this particular value of $\mathbf{q}$ by $\mathbf{q_0}$. Therefore the equation (18) with condition (19), shows that the stationary density profile does exist; it is continuous and zero at thermodynamics limit ($R \to \infty$) at the boundary. Now for phase transition we examine the slope of the the density profile at the boundary.

Because $\mathbf{q_0}$ is parallel to $\mathbf{r}$, then

$$\rho_0(\mathbf{r}) \approx A \exp\{\mathbf{q_0} \cdot \mathbf{r} - |\mathbf{q_0}| R\} \tag{20}$$

Therefore,

$$\nabla\rho_0(\mathbf{r}) \approx \mathbf{q_0} A \exp\{\mathbf{q_0} \cdot \mathbf{r} - |\mathbf{q_0}| R\} \tag{21}$$

In the thermodynamics limit ($R \to \infty$), the slope of the density profile in direction $\mathbf{n}$ at the boundary is zero.

Our calculation shows the density profile and derivative are zero at the boundary. This behaviour is the same in all directions and independent of $\mathbf{v}$ and changing of $\mathbf{v}$ does not cause any discontinuity in the slope of the density profile at the boundary. Therefore, we have no static phase transition in this regime.

2 -**Creation,** $m > 0$

If we consider creation of particles in the volume, from Eq. (17)

$$|\mathbf{v}'| \leq |\mathbf{v}|. \tag{22}$$

The density profile are not the same behaviour for all value of m.

a) $0 < m < \frac{\mathbf{v}\cdot\mathbf{v}}{4}$

From Eq.(16) one can define the

$$\mathbf{q} = \frac{\mathbf{v}}{\mathbf{2}} + \frac{v \sin\xi}{2}\hat{n}. \tag{23}$$

$\hat{n}$ is the unit vector in the direction of $\frac{\mathbf{v}'}{\mathbf{2}}$. Here we have a cone with height is equal $\frac{\mathbf{v}}{\mathbf{2}}$. The end of the vector $\frac{\mathbf{v}}{\mathbf{2}}$one can consider a hyperball with radius $\frac{\mathbf{v}}{\mathbf{2}}$ and ξ is the angle between the tangent lines to this ball.

$$\mathbf{q} \cdot \mathbf{R} = \frac{vR}{2}(\cos\theta + \sin\xi\cos(\theta - \phi)) \tag{24}$$

From Eq. (18) in the thermodynamic limit the stationary density profile becomes

$$\rho \approx A e^{\frac{vR}{2}(\cos\theta + \sin\xi\cos(\theta-\phi))} e^{\frac{vR}{2} f(\phi)} \tag{25}$$

$$= A e^{\frac{vR}{2} g(\theta,\phi)} \tag{26}$$

Then

$$g(\theta, \phi) = \cos\theta + \sin\xi\cos(\theta - \phi) + f(\phi). \tag{27}$$

The question here is that for which value of ϕ the function $g(\theta, \phi)$ becomes maximized

$$\frac{\partial g(\theta, \phi)}{\partial\phi} = -\sin\xi\sin(\theta - \phi) + f'(\phi) \tag{28}$$

and

$$\frac{\partial^2 g(\theta,\phi)}{\partial^2\phi} = \sin\xi\cos(\theta-\phi) + f''(\phi) \tag{29}$$

Consider ϕ^* maximizes the function $g(\theta,\phi)$, then $\frac{\partial g(\theta,\phi)}{\partial\phi}|_{\phi^*} = 0$ and $\frac{\partial^2 g(\theta,\phi)}{\partial^2\phi}|_{\phi^*} \leq 0$. Then from Eqs. (28) and (29), one can find

$$f'(\phi^*) = \sin\xi\sin(\phi^* - \theta) \tag{30}$$

and

$$\begin{aligned} f''(\phi^*) &= \frac{\partial f'}{\partial\phi^*} + \frac{d\theta}{d\phi^*}\frac{\partial f'}{\partial\theta} \\ &= -\sin\xi\cos(\theta-\phi^*) + \frac{d\theta}{d\phi^*}(-\sin\xi\cos(\theta-\phi^*)) \end{aligned} \tag{31}$$

Then Eq. (29),

$$\begin{aligned} \frac{\partial^2 g(\theta,\phi)}{\partial^2\phi}|_{\phi^*} =& \sin\xi\cos(\theta-\phi) - \sin\xi\cos(\theta-\phi^*) + \frac{d\theta}{d\phi^*}(-\cos\xi\sin(-\theta-\phi^*)) \leq 0 \\ =& \frac{d\theta}{d\phi^*}(-\cos\xi\sin(-\theta-\phi^*)) \leq 0 \end{aligned} \tag{32}$$

By multiplying two sides of the Eq.(32) with this term $(\frac{d\phi^*}{d\theta})^2$ we can find

$$\frac{d\phi^*}{d\theta}\cos(\theta-\phi^*) \geq 0 \tag{33}$$

Now we define $h(\theta) := g(\theta,\phi^*)$ then we have

$$h(\theta) = \cos\theta + \sin\xi\cos(\theta-\phi^*) + f(\phi^*). \tag{34}$$

Now if we find

$$\begin{aligned} \frac{dh(\theta,\phi^*)}{d\theta} =& \frac{\partial h}{\partial\theta} + \frac{d\phi^*}{d\theta}\frac{\partial h}{\partial\phi^*} \\ =& -sin\theta - \sin\xi\sin(\theta-\phi^*) \end{aligned} \tag{35}$$

and

$$\begin{aligned} \frac{d^2 h(\theta,\phi^*)}{d\theta^2} =& \frac{\partial^2 h}{\partial\theta^2} + \frac{d\phi^*}{d\theta}\frac{\partial h^2}{\partial\phi^*\partial\theta} \\ =& -cos\theta - \sin\xi\cos(\theta-\phi^*) + \frac{d\phi^*}{d\theta}(\sin\xi\cos(\theta-\phi^*)) \end{aligned} \tag{36}$$

The third term on the right-hand side of Eq. (36) is always equal or greater than zero. One should try to find the signs of the first and second terms in that equation to know about the behaviour of the function h of θ.

Then let us consider three regions, the front cone, $\theta < \xi$, between cones, $\xi < \theta < \pi - \xi$ and the back cone, $\pi - \xi < \theta < \pi$, respectively.

If $\theta < \xi$ notice that $\xi \le \pi/2$, one can find , $-1 \le \cos\theta \le 0$. Therefore, if the value of θ_0 is such that $h'(\theta_0) = 0$ then it can be seen $|\cos\theta_0| > \sin\xi|\cos(\theta - \phi^*|$ $(h''(\theta_0) > 0)$. Then the easily function h becomes minimize at θ_o. The stationary density profile in the front cone goes to infinity in the thermodynamics limit.
In the same way one can find in this region $\xi < \theta < \pi - \xi$.T he behaviour of the stationary profile of the density in the thermodynamics limit is infinity too.

Then consider that $h(\theta)$ becomes constant in the region.

In this regime the system goes to stationary state at long times, but the density profile in the thermodynamics limit ($R \to \infty$) is infinity everywhere at the boundary except for one point. It is independent of the rate of injection or extraction of particles at the boundary. The qualitative way for understanding this fact is that the value of creation in the bulk is proportional to the volume of the ball, and the value of the creation in the surface is proportional to the area of the ball. In the thermodynamic limit, the volume effect is more important than the surface effect and one can ignore the surface effect compared to the volume effect. Then the extraction of particles at the boundary can't frustrate the creation of particles in the volume.

b) $m > \frac{\mathbf{v}\cdot\mathbf{v}}{4}$

One can write Eq. (17) in the following

$$\nabla^2\tilde{\rho} + \tilde{m}\,\tilde{\rho} = 0 \tag{37}$$

where

$$\tilde{m} = m - \frac{\mathbf{v}\cdot\mathbf{v}}{4} \tag{38}$$

Then Eq. (37) is the Helmholtz equation with $\tilde{m}$. In the general case there is no solution for this equation or at least a solution with oscillatory behaviour exists. Because $\tilde{\rho}$ is a non negative function, even this solution is not possible here. Long time behaviour of the systems are not stationary in this regime.

3 -Results for $m = 0$

From Eq. [37], one obtains,

$$\rho_0(\mathbf{r}) \sim \begin{cases} C_1(\Omega), & r \sim R, \mathbf{r}\cdot\mathbf{v} < 0, \\ C_1(\Omega) + C_2(\Omega)\, \exp[\frac{(r-R)\,\mathbf{v}\cdot\mathbf{r}}{R}], & r \sim R, \mathbf{r}\cdot\mathbf{v} > 0. \end{cases} \tag{39}$$

From this,

$$\nabla\rho_0(r = R) \propto \mathbf{n}\,(\mathbf{n}\cdot\mathbf{v})\,\theta(\mathbf{n}\cdot\mathbf{v}), \qquad R \to \infty, \tag{40}$$

where θ is the step function. It is seen that in the thermodynamic limit ($R \to \infty$), the density profile at the boundary is stationary, unless $\mathbf{v} \cdot \mathbf{r} > 0$. So, changing $\mathbf{v}$ one can induce a discontinuous change in the slope of the density profile at the boundary. This is the static phase transition, which is seen to be independent of the injection and extraction terms, but dependent on the drift velocity.

5 The relaxation of the system towards the stationary state, and the dynamic phase transition

Starting from Eqs. (11) and (12), one arrives at

$$\frac{\partial}{\partial t}(\rho - \rho_0) = (m - \mathbf{v} \cdot \nabla + \nabla^2)(\rho - \rho_0),$$
$$=: h\,(\rho - \rho_0), \tag{41}$$

and

$$\mathbf{n} \cdot \nabla(\rho - \rho_0) = -\beta\,(\rho - \rho_0), \qquad \text{at the boundary} \tag{42}$$

where ρ_0 is the time-independent solution to Eqs. (11) and (12). Let ψ be an eigenfunction of h corresponding to the eigenvalue E. Using the ansatz (13) in the eigenvalue equation corresponding to h, one arrives at

$$\mathbf{q} \cdot \mathbf{q} - \mathbf{v} \cdot \mathbf{q} + m = E, \tag{43}$$

which leads to

$$\mathbf{q} = \frac{1}{2}(\mathbf{v} + \mathbf{v}'), \tag{44}$$

where $\mathbf{v}'$ is an arbitrary constant vector subject to the condition

$$\mathbf{v}' \cdot \mathbf{v}' = \mathbf{v} \cdot \mathbf{v} + 4\,(E - m). \tag{45}$$

We separate the two cases again, the annihilation and creation of the particles in the volume ($m < 0$, $m > 0$).

1 -**Annihilation,** $m < 0$

One has

$$\psi(\mathbf{r}) = \exp(\mathbf{v} \cdot \mathbf{r}/2) \int \mathrm{d}\Omega'\; A(\Omega')\; \exp(\mathbf{v}' \cdot \mathbf{r}/2), \tag{46}$$

where A is to be found such that the boundary condition (42) is satisfied with ψ.

If the right-hand side of Eq. (45) is positive, then $\mathbf{v}'$ is real and for large r, one can approximate ψ as

$$\psi(\mathbf{r}) \sim \exp(\mathbf{v} \cdot \mathbf{r}/2)\, A(\Omega)\; \exp(v'\, r/2), \tag{47}$$

where Ω is the angular coordinate corresponding to $\mathbf{r}$. The boundary condition (16), then becomes

$$\left[\frac{v'}{2} + \beta(\Omega) + \frac{\mathbf{n} \cdot \mathbf{v}}{2}\right] A(\Omega) = 0. \tag{48}$$

This has a nonzero solution for A, provided the expression in the parenthesis vanishes for some Ω. As $\beta \geq 0$, this happens for some (real) positive v', if and only if

$$\min\left[\beta(\Omega) + \frac{v\,\cos\phi}{2}\right] < 0, \tag{49}$$

where ϕ is the angle between $\mathbf{r}$ and $\mathbf{v}$. If Eq. (49) holds, then the range of v' for which a nonzero solution to (48) for A exists is

$$0 \leq \frac{v'}{2} \leq -\min\left[\beta(\Omega) + \frac{v\,\cos\phi}{2}\right]. \tag{50}$$

(This is true for space dimensions higher than 1D. If the space is one-dimensional, v' has only one acceptable value, as the expression in the parenthesis in Eq.(48) has only two values; of which, at most one can be zero.)

If relation (49) holds, then there exist eigenvalues $E' := E - m$ for h, with $E' > -\mathbf{v}\cdot\mathbf{v}/4$. Otherwise, all of the eigenvalues of h are less than or equal to $-\mathbf{v}\cdot\mathbf{v}/4$. The relaxation time of the system is

$$\tau = -\frac{1}{E'_{\max}}, \tag{51}$$

where $E'_{\max}$ is the largest eigenvalue of h. The largest value of E' is either $-\mathbf{v}\cdot\mathbf{v}/4$, or the value obtained from Eq. (45) for the largest value of v'. So,

$$\tau = \begin{cases} \frac{4}{\mathbf{v}\cdot\mathbf{v}}, & \min\left[\beta(\Omega) + \frac{v\,\cos\phi}{2}\right] > 0, \\ \frac{4}{\mathbf{v}\cdot\mathbf{v} - \{\min[2\beta(\Omega) + v\,\cos\phi]\}^2}, & \min\left[\beta(\Omega) + \frac{v\,\cos\phi}{2}\right] < 0. \end{cases} \tag{52}$$

In the first case, the system is in the fast dynamical phase, in which the relaxation time does not depend on the boundary condition. In the second case, the system is in the slow dynamical phase, in which the relaxation time is larger and does depend on the boundary conditions. This is the dynamical phase transition.

2 -**Creation,** $m > 0$

For the value of $0 < m < \mathbf{v}.\mathbf{v}/4$, energy of the system is negative and we have relaxation like (regime $m < 0$) but in the $m > \mathbf{v}.\mathbf{v}/4$ the energy is not negative and the system is not going to a relaxation state.

3 -**Results for** $m = 0$

From Ref. [37], the results are the same as ($m < 0$)

6 Concluding remarks

A simple model on a multi-dimensional continuum with creation or annihilation of particles on the volume is considered. The creation or annihilation of particles in the volume is controlled by the parameter m. There are three regimes for m and the long- times behaviour of the system is completely different in theses three regimes.

For ($m < 0$), when annihilation of particles occur in the volume, the long time behaviour of the system is stationary state and the profile of the density is continuous, and the value of this density and slope are zero at the boundary for the thermodynamics limit ($R \longrightarrow \infty$). Then the static phase transition does not take place. When there is the possibility of creation of the particle in the volume ($m > 0$), for the limit $0 < m < \mathbf{v} \cdot \mathbf{v}/\mathbf{4}$, the system has a stationary state; but the profile of the density is infinity almost everywhere at the boundary in the thermodynamics limit. Then there is no static phase transition. If the creation of particles satisfies $m > \mathbf{v} \cdot \mathbf{v}/\mathbf{4}$, there is no stationary phase. Finally, if there is no annihilation or creation of particles in the volume ($m = 0$) [37], there is a static phase transition when the direction of the drift velocity changes.

The dynamical phase transition corresponding to the relaxation of the system towards its stationary state is also different in these regimes. For these three cases, annihilation $m < 0$, creation in the limit $0 < m < \mathbf{v}.\mathbf{v}/4$, and no creation or annihilation $m = 0$, we have relaxation and the system is between two values of relaxation times (fast and slow). This is the dynamical phase transition.

But if $m > \mathbf{v}.\mathbf{v}/4$ the energy is not negative and the system is not going to a relaxation state. Therefore, there would be no dynamical phase transition.

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