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ENTANGLEMENT GENERATION WITH DEFORMED BARUT-GIRARDELLO COHERENT STATES AS INPUT STATES IN A UNITARY BEAM SPLITTER

K. Berrada[1]
Laboratoire de Physique Théorique, Faculté des Sciences, Université Mohammed V-Agdal, Av. Ibn Battouta, B.P. 1014, Agdal Rabat, Morocco
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

A. Benmoussa[2]
Department of Physics and Astronomy, Lehman College, The City University of New York, Bronx, New York 10468-1589, USA

and

Y. Hassouni[3]
Laboratoire de Physique Théorique, Faculté des Sciences, Université Mohammed V-Agdal, Av. Ibn Battouta, B.P. 1014, Agdal Rabat, Morocco.

Abstract

Using linear entropy as a measure of entanglement, we investigate the entanglement generated via a beam splitter using deformed Barut-Girardello coherent states. We show that the degree of entanglement depends strongly on the q-deformation parameter and amplitude Z of the states. We compute the Mandel Q parameter to examine the quantum statistical properties of these coherent states and make a comparison with the Glauber coherent states. It is shown that these states are useful to describe the states of real and ideal lasers by a proper choice of their characterizing parameters, using an alteration of the Holstein-Primakoff realization.

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[1]kberrada@ictp.it
[2]adil.benmoussa@lehman.cuny.edu
[3]y-hassou@fsr.ac.ma

1 Introduction

Quantum entanglement is one of the most striking features of quantum mechanics which plays a key role in various quantum information processing and transmission tasks [1], including quantum cryptographic key distribution [2], quantum teleportation [3], superdense coding [4], quantum computation [5], and more recently, metrology [6, 7]. Consequently, the characterization and the quantification of entanglement have attracted much attention and various entanglement measures have been proposed, such as linear entropy [8, 9, 10], concurrence [11, 12, 13, 14], entanglement of formation [15] and negativity [16, 17]. The fundamental problems for entanglement phenomenon is to find a method to determine whether a given state of a composite quantum system consisting of two or more subsystems is entangled or not and to choose the best measure quantifying the amount of entanglement.

The preparation of entangled states has been widely studied and constitutes an essential step in many quantum information processing and transmission tasks. Recently, various devices have been proposed and realized experimentally to generate quantum entanglement, such as beam splitter [18, 19], Cavity QED [20], NMR systems [21], etc.

The entanglement generated by a beam splitter has been studied in considerable detail by Kim et *al.* [22]. They showed that, in order to obtain entangled output states of a beam splitter, a necessary condition is that at least one of the input fields should be nonclassical. It was recently shown by Ivan et *al.* that if the input state to a beam splitter is a product state of the vacuum in one mode and a state with a sub-Poissonian distribution, then the output state will be entangled [23]. More recently, we have generalized this result to the states with super-Poissonian and sub-Poissonian statistics, by investigating the entanglement generated via a 50:50 beam splitter when a deformed spin coherent state is incident on one input port and a ground state is incident on the other [8]. We have distinguished two cases: in the classical $q \to 1$ limit, the deformed spin coherent states go to Glauber coherent states in the limit of high spin, $j \to \infty$, and thus become product states after beam splitter, whereas for $q \neq 1$, the states are entangled as they pass through a beam splitter and the entanglement is strongly dependent on the q-deformation parameter and the amplitude Z of the state.

Another concept widely used and applied in quantum information theory is the notion of coherent or quasiclassical states. These states make a very useful tool for the investigation of various problems in physics [24, 25, 26]. Coherent states were first introduced by Schrödinger [27] in the context of the harmonic oscillator, who was interested in finding quantum states which provide a close connection between quantum and classical formulations of a given physical system. Later, the notion of coherent has became very important in quantum optics due to Glauber [28], as eigenstates of the annihilation operator $\hat{a}$ of the harmonic oscillator, while he demonstrated that these states have the interesting property of minimizing the Heisenberg uncertainty relation. Next, the following important coherent states are $SU(2)$ and $SU(1,1)$, coherent states introduced by Peremolov [29, 30] which describe several systems and also have some applications in quantum optics, statistical mechanics, nuclear physics, and condensed matter physics [31, 32, 33].

On the other hand, the quantum groups were introduced as a mathematical description of deformed Lie algebra that gave the possibility to construct deformed coherent states. They were introduced as a natural extension of the notion of coherent states [34]. Generalized deformation of Glauber states were constructed, see [35], as related to deformed harmonic oscillators. Deformed Peremolov and Barut-Girardello coherent states were also constructed as coherent states related to the quantum algebra $U_q\left(su(1,1)\right)$ [37, 38]. Recently, these states have attracted a lot of attention due to their possible applications in various branches of physics [36, 37, 39, 40]. Such states exhibit some nonclassical properties such as photon antibunching [41], sub-Poissonian photon statistics [42] and squeezing [43, 44], etc (for a review see Ref. [45]). It has been experimentally observed that the real laser, bunched and antibunched light possess a photon number statistics which can be super-Poissonian or sub-poissonian [46, 47].

The physical importance of the deformed coherent states lies in the fact that they offer the best description for non-ideal physical devices such as lasers (i.e. real lasers) [48]. The deformation parameter then plays the role of a tuning parameter defining how far the realized device is from the ideal one.

In this paper, we shall investigate the entanglement generated via a 50 : 50 beam splitter when a deformed Barut-Girardello coherent state is incident on one input port and a ground $|0\rangle$ is incident on the other. To achieve this, we calculate the linear entropy in these states and study its dependence on the different variables of the system. As a matter of fact, the linear entropy, and thus the entanglement of the resulting states, depends not only on the amplitude z and the $\left(U_q\left(su(1,1)\right)\right)$ representation index k, but also on the deformation parameter q which in some sense measures how far one gets from the non-deformed states. We confirm the different results obtained by evaluating the Mandel Q parameter for the same states.

The plan of the paper is as follows. In section 2, we briefly review the deformed Barut-Girardello coherent states of the $U_q\left(su(1,1)\right)$ quantum algebra. In section 3, we examine the entanglement as a result of the effect of a beam splitter on deformed Barut-Girardello coherent states using linear entropy. In section 4, we analyze the statistical properties of these states using Mandel's Q parameter. Section 5 contains a conclusion.

2 Deformed Barut-Girardello coherent states and Fock space

We begin our discussion by presenting the overall state of the art of the deformed Barut-Girardello coherent states and the representation in Fock space.

In this paper, we are interested to explain q-deformed $su(1,1)$ algebra denoted as $U_q\left(su(1,1)\right)$ algebra which has earlier been investigated extensively [49]. The $U_q\left(su(1,1)\right)$ quantum algebra is defined by the commutation relations,

$$[K_-^q, K_+^q] = [2K_z^q] \qquad , \qquad [K_z^q, K_\pm^q] = \pm K_\pm^q \ , \tag{1}$$

where its generators $(K_\pm^q, K_z^q)$ also obey Hermiticity properties, that is,

$$(K_+^q)^\dagger = K_-^q \quad , \quad (K_-^q)^\dagger = K_+^q \quad , \quad (K_z^q)^\dagger = K_z^q.$$

Here the box function $[x]_q$ is given by

$$[x]_q = \frac{q^x - q^{-x}}{q - q^{-1}} = \frac{\sinh(\gamma x)}{\sinh(\gamma)} \qquad \text{for} \qquad q = e^\gamma \qquad \gamma \in R,$$

and q is a real deformation parameter (q may take complex values, however in this paper we focus on $q \in R$).

In the classical $q \to 1$ limit, $U_q\left(su(1,1)\right)$ becomes undeformed classical $su(1,1)$ algebra. The unitary irreducible representation of the $U_q\left(su(1,1)\right)$ algebra may be obtained via the unitary representation [50] of the undeformed classical $su(1,1)$ algebra. It is described as follows:

$$\begin{aligned} K_z^q|k,m\rangle &= m|k,m\rangle \ , \\ K_\pm^q|k,m\rangle &= \left([m \pm k]_q[m \mp k \pm 1]_q\right)^{\frac{1}{2}} |k, m \pm 1\rangle \ . \end{aligned} \tag{2}$$

Here $|k,m\rangle$ is the complete orthonormal basis of the space of irreducible representation, with $m \in \{k, k+1, k+2, ...\}$ and $k \in \{\frac{1}{2}, 1, \frac{3}{2}, ...\}$ is the Bargmann index labeling the irreducible representation.

The Barut-Girardello coherent states of $U_q\left(su(1,1)\right)$ algebra were previously investigated [51, 52, 50] in different contexts. They are defined as the eigenstates of the lowering operator K_-^q

$$K_-^q \ |Z,k\rangle_q = Z\ |Z,k\rangle_q \qquad\qquad Z \in C, \tag{3}$$

and they can be expressed as

$$|Z,k\rangle_q = \mathcal{N}\left(|Z|^2\right) \sum_{m=k}^{\infty} \frac{Z^{(m-k)}}{\sqrt{[m-k]_q![m+k-1]_q!}} |k,m\rangle \ . \tag{4}$$

where $[n]_q! = [n]_q[n-1]_q...[1]_q$ and $[0]_q! = 1$.

The normalization factor is given by

$$\mathcal{N}\left(|Z|^2\right) = \left(\frac{\mathcal{I}_l^{(q)}\left(2|Z|\right)}{|Z|^{2k-1}}\right)^{-\frac{1}{2}} \ , \tag{5}$$

where we have used the q-deformed modified Bassel function of integer order l

$$\mathcal{I}_l^{(q)}\left(2Z\right) = \sum_{m=k}^{\infty} \frac{Z^{l+2n}}{[n]_q![l+n]_q!}, \qquad\qquad \mathcal{I}_{-l}^{(q)}\left(2Z\right) = \mathcal{I}_l^{(q)}\left(2Z\right). \tag{6}$$

In the undeformed $q \to 1$ limit, the deformed Barut-Girardello coherent states reduces to the ordinary Barut-Girardello coherent states. Using these definitions, we may write the states as

$$|Z,k\rangle_q = \frac{|Z|^{k-\frac{1}{2}}}{\sqrt{\mathcal{I}^{(q)}_{2k-1}\left(2|Z|\right)}} \sum_{m=k}^{\infty} \frac{Z^{(m-k)}}{\sqrt{[m-k]_q![m+k-1]_q!}} |k,m\rangle \ . \tag{7}$$

In a manner similar to Ref. [53], and in order to apply these states in the current context one needs to express the basis vectors $|k,m\rangle$ in terms of the Fock states $|n\rangle$ ($|k,m\rangle \sim |n\rangle$). In Ref. [53], this was done using the Holstein-Primakoff realization of the $su(1,1)$ algebra.

In our case, $U_q\left(su(1,1)\right)$, one can achieve the same result using an alteration of the later realization:

$$K^q_+ = a^\dagger_q \sqrt{[2k+N]_q} \quad , \quad K^q_- = \sqrt{[2k+N]_q}\, a_q \quad , \quad K^q_z = N + k \ , \tag{8}$$

where, the a_q, $a^\dagger_q$ are deformed annihilation and creation operators acting on the Fock states as follows

$$a_q|n\rangle = \sqrt{[n]_q}\,|n-1\rangle \quad , \quad a^\dagger_q|n\rangle = \sqrt{[n+1]_q}\,|n+1\rangle \quad , \quad N|n\rangle = n|n\rangle \ . \tag{9}$$

They obey the following relation [54]

$$a_q a^\dagger_q - q a^\dagger_q a_q = q^{-N} \quad , \quad [N, a_q] = -a_q \quad , \quad [N, a^\dagger_q] = a^\dagger_q \ . \tag{10}$$

Using this realization and the one given in Eq. (8), we get the change of variables $m = n + k$ or $n = m - k$, which (when applied in Eq. (4)) yield the expression of the deformed Barut-Girardello coherent states in terms of the Fock states, as

$$|Z,k\rangle_q = \mathcal{N}\left(|Z|^2\right) \sum_{n=0}^{\infty} \frac{Z^n}{\sqrt{[n]_q![n+2k-1]_q!}} |n\rangle \tag{11}$$

with

$$\mathcal{N}\left(|Z|^2\right) = \left(\sum_{n=0}^{\infty} \frac{|Z|^{2n}}{[n]_q![n+2k-1]_q!} \right)^{-\frac{1}{2}} \ .$$

3 Effect of a beam splitter on deformed Barut-Girardello coherent states

We first discuss how a beam splitter acts on an input state comprised of a state, $|\psi\rangle$ to be studied, in one input port and a vacuum state $|0\rangle$ in the other port. We consider that the horizontal input beam always contains the state of interest while the vacuum state in the vertical input beam (see Fig. 1).

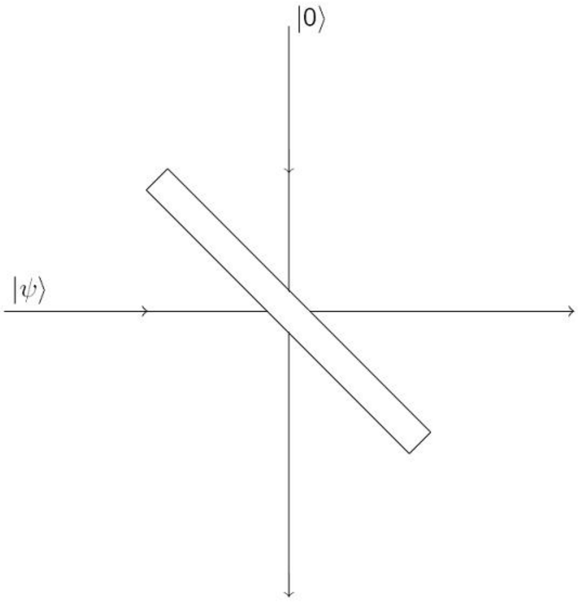

FIG. 1. A 50:50 beam splitter with a state $|\psi\rangle$ on the horizontal port and a vacuum state $|0\rangle$ on the vertical port.

Quantum mechanically, the action of a beam splitter can be described by a unitary operator $\widehat{U}_{bs}$ that relates the input state to the output state by

$$|out\rangle = \widehat{U}_{bs}|int\rangle \tag{12}$$

where the beam splitter operator is [22]

$$\widehat{U}_{bs} = \exp\left[\frac{\theta}{2}\left(a^{\dagger}b\, e^{i\phi} - ab^{\dagger}e^{-i\phi}\right)\right], \tag{13}$$

here a and b are the annihilation operators of the two input fields and $a^{\dagger}$ and $b^{\dagger}$ are their Hermitian conjugates, and ϕ denotes the phase difference between the reflected and transmitted fields.

In order to obtain the beam splitter transformation when deformed Barut-Girardello coherent states are incident on one input port, we first introduce the effect of a beam splitter on an input state comprised of a Fock state in the horizontal input beam, that is $|\psi\rangle = |n\rangle$, and a ground Fock state in the other

$$\widehat{U}_{bs}|n\rangle|0\rangle = \sum_{p=0}^{n}\binom{n}{p}^{\frac{1}{2}} T^{p}R^{(n-p)}|p\rangle|n-p\rangle. \tag{14}$$

The quantities T and R are the transmissivity and the reflectivity of the beam splitter, respectively, obeying the normalization condition $|T|^2 + |R|^2 = 1$. In this paper, we assume that our beam splitter is 50:50 and the reflected beam suffers a phase shift of $\frac{\pi}{2}$, in this case we have $T = \frac{1}{\sqrt{2}}$ and $R = \frac{i}{\sqrt{2}}$, and Eq. (14) becomes

$$\widehat{U}_{bs}|n\rangle|0\rangle = \sum_{p=0}^{n}\binom{n}{p}^{\frac{1}{2}}\left(\frac{1}{\sqrt{2}}\right)^{p}\left(\frac{i}{\sqrt{2}}\right)^{(n-p)}|p\rangle|n-p\rangle. \tag{15}$$

It is well known that if a Glauber state defined as $|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum\limits_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$ is incident on one input port and a vacuum state on the other, then the output state is separable (or factorizable) with zero entanglement

$$\begin{aligned}|out\rangle &= \widehat{U}_{bs}|\alpha\rangle|0\rangle \\ &= \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{p=0}^{\infty}\frac{\alpha^p T^p}{\sqrt{p!}}\Big|\, p\Big\rangle \otimes \sum_{n-p=0}^{\infty}\frac{\alpha^{(n-p)}R^{(n-p)}}{\sqrt{(n-p)!}}\Big|n-p\Big\rangle \\ &= \Big|\frac{\alpha}{\sqrt{2}}\Big\rangle \otimes \Big|\frac{i\alpha}{\sqrt{2}}\Big\rangle.\end{aligned} \tag{16}$$

The Glauber coherent states are the only states that when passed through one input port of a beam splitter result in product states at the output. Any other states at the input result in entangled states at the output [22].

Now let us examine the entanglement generated via 50:50 beam splitter when a deformed Barut-Girardello coherent state is injected into one input port and a vacuum state is injected into the other. In this case, the output state is obtained by using both Eqs. (12) and (14)

$$\begin{aligned}|out\rangle &= \widehat{U}_{bs}|Z,k\rangle_q|0\rangle \\ &= \mathcal{N}\left(|Z|^2\right)\sum_{n=0}^{\infty}\sum_{p=0}^{n}\left(\frac{n!}{[n]_q![2k+n-1]_q!}\right)^{\frac{1}{2}}\frac{Z^p}{\sqrt{p!}}T^p\frac{Z^{(n-p)}}{\sqrt{(n-p)!}}R^{(n-p)}|p\rangle|n-p\rangle.\end{aligned} \tag{17}$$

As measure of entanglement, we use the linear entropy (upper bound of the Von Neumann entropy) which is a good indication of the entanglement and it is reasonable in several senses as an entanglement monotone [9]. For bipartite pure state ρ_{AB} it is defined as

$$S = 1 - Tr\left(\rho_A^2\right) \tag{18}$$

where $\rho_A = Tr_B\left(\rho_{AB}\right)$ is the reduced density operator of system A. In our case, from Eq. (17) we obtain

$$\begin{aligned}\rho_A = \left(\mathcal{N}\left(|Z|^2\right)\right)^2 \sum_{p,p'=0}^{\infty}\sum_{m=0}^{\infty}\left(\frac{(m+p)!(m+p')!}{[m+p]_q![m+p']_q![2k+m+p-1]_q![2k+m+p'-1]_q!}\right)^{\frac{1}{2}} \\ \times\frac{|Z|^{2m}}{m!}|T|^{2m}\frac{Z^p\overline{Z^{p'}}}{\sqrt{p!p'!}}R^p\overline{R^{p'}}\;|p\rangle\langle p'|\,.\end{aligned} \tag{19}$$

After calculation we find for the linear entropy

$$S = 1 - \left(\mathcal{N}\left(|Z|^2\right)\right)^4 \sum_{p,p'=0}^{\infty}\sum_{m,m'=0}^{\infty}\left(\frac{X_{m,m'}^{p,p'}}{Y_{m,m'}^{p,p'}\;Z_{m,m'}^{p,p'}}\right)^{\frac{1}{2}}\frac{|Z|^{2(m+m'+p+p')}}{m!m'!p!p'!}|T|^{2(m+m')}|R|^{2(p+p')} \tag{20}$$

where,

$$
\begin{aligned}
X^{p,p'}_{m,m'} &= (m+p)!(m+p')!(m'+p)!(m'+p')! \ , \\
Y^{p,p'}_{m,m'} &= [m+p]_q![m+p']_q![m'+p]_q![m'+p']_q! \ , \\
Z^{p,p'}_{m,m'} &= [2k+m+p-1]_q![2k+m+p'-1]_q![2k+m'+p-1]_q![2k+m'+p'-1]_q! \ .
\end{aligned}
$$

We notice that in the classical $q \to 1$ limit, the expression of the linear entropy is equal to the one obtained in [53]. In fact, when $q \to 1$ the deformed Barut-Girardello coherent states tend to the ordinary Barut-Girardello coherent states which were considered in [53].

In general, the degree of entanglement of the output state is highly dependent on the values of the amplitude Z and the q-deformation parameter. In Fig.2, the linear entropy is plotted as a function of the Bargman index k for different values of the amplitude $|Z|$ while keeping the deformation parameter fixed at the value $q = 0.95$. For other values of q the graph keeps the same shape but with different numerical values of course. We see that after an initial quick change, the linear entropy tends slowly to zero with increasing k. In fact, it decreases quickly from the maximum attained at the origin to a value after which its decrease slows down tending to zero for significantly high values of the Bargman index k, which means that the entanglement tends to disappear for these values and thus, the deformed Barut-Girardello coherent states are equivalent to the high-amplitude of the optical coherent states (Glauber states). We also notice that the degree of entanglement grows with increasing amplitude $|Z|$.

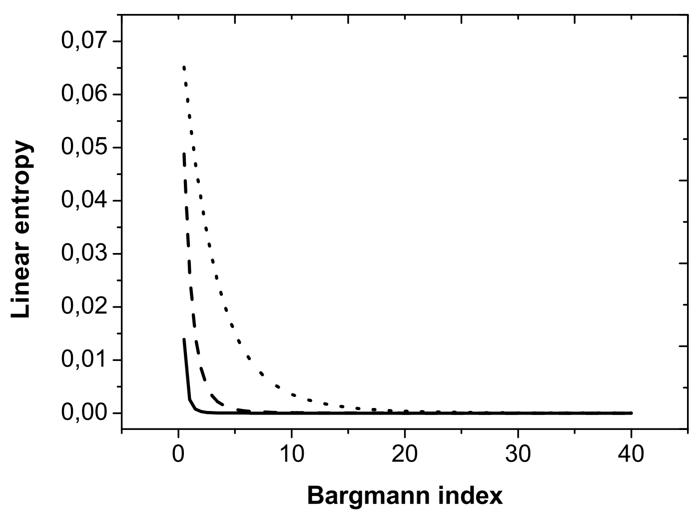

FIG. 2. Entanglement (Linear entropy) as a function of k with $q = 0.95$ for $|Z| = 9$ (dotted line), $|Z| = 3$ (dashed line) and $|Z| = 1$ (solid line).

From another side, the dependance on the deformation parameter q is shown in Fig.3, where the linear entropy is plotted as a function of the index k for $|Z| = 9$ and different values of q. It looks that as q gets farther from the undeformed $q \to 1$ limit (from left or right), the linear entropy decreases from higher values to approach more rapidly zero with smaller index k, and the deformed coherent states tend to factorize after beam splitting.

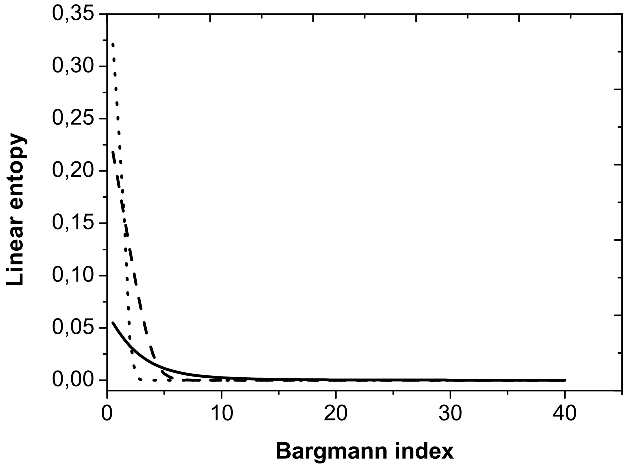

FIG. 3. Entanglement (Linear entropy) as a function of k with $|Z| = 9$ for $q = 0.2$ (dotted line), $q = 0.6$ (dashed line) and $q = 1$ (solid line).

Now, we want to examine and explain our results by using the Mandel parameter in order to investigate the statistical properties of the deformed Barut-Girardello coherent states and to compare them with Glauber coherent states. In this sense, we trace the deformed Barut-Giradello coherent state from what we think of as the least classical, $k = \frac{1}{2}$, to the most classical, $k \to \infty$. This parameter is defined as [55]

$$Q = \frac{\langle N^2 \rangle + \langle N \rangle^2 - \langle N \rangle}{\langle N \rangle} \tag{21}$$

where $\langle N \rangle$ is the average number of the state.

This parameter is a good measure to determine whether a state has a Poissonian (if $Q = 0$), Super-Poissonian (if $Q > 0$) or sub-Poissonian distribution (if $-1 \leq Q < 0$). Therefore, we believe this to be a better test than calculating the overlap of the deformed Barut-Girardello coherent states with the Glauber coherent states because for the latter it would require $Q = 0$. We use this and we proceed to prove that these coherent states tend to Glauber states in the limit of high index k, $k \to \infty$.

In Fig. 4, the parameter Q of the deformed Barut-Girardello coherent states is plotted as a function of the index k for different values of the q parameter. We show that the coherent states exhibit Poissonian and sub-Poissonian distribution depending on the values of the q parameter. This Q parameter satisfies the inequality $-1 \leq Q < 0$, approaches zero with increasing k, indicating that the deformed coherent states are close to Glauber states and it vanishes for k sufficiently large.

We notice that the plots of the Mandel Q parameter have similar behavior as the linear entropy. Indeed, the Q parameter approaches zero more rapidly with small index k as q is farther from the undeformed $q \to 1$ limit (from left or right) and it tends to zero as k becomes infinitely large.

As an important remark, when the values of q are close to one, the Mandel Q parameter moves away from zero. This means that the deformed Barut-Girardello coherent states become less quantum mechanical than the ordinary coherent states (for $q = 1$) as k becomes large, which justify our results.

It is well known that the study of the statistical properties is an important topic in quantum optics. In this way, our results show that the deformed Barut-Girardello coherent states provide a much richer structure than the undeformed ones. These deformed states may be helpful to describe the states of real and ideal laser by the proper choice of the q-deformation parameter and Bargman index k. However, these states are useful to generate and measure the entanglement and their use is not only of the theoretical purpose but also of some practical importance having in mind their experimental accessibility [48].

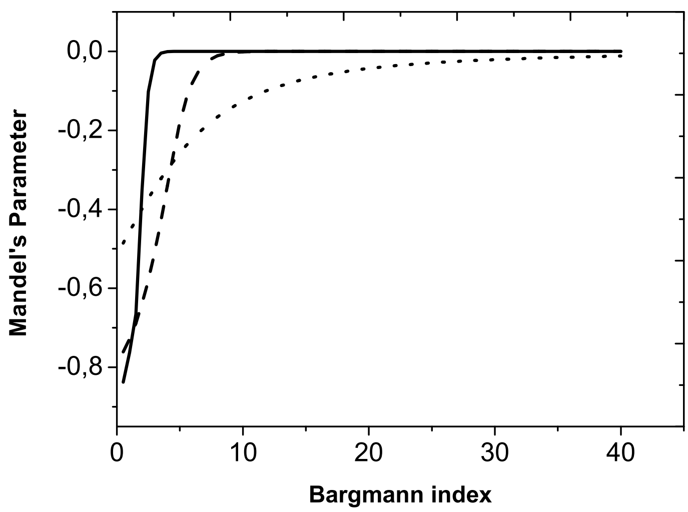

FIG. 4. Mandel's Q parameter for the deformed Barut-Girardello coherent state versus k for $q = 0.2$ (solid line), $q = 0.6$ (dashed line) and $q = 1$ (dotted line).

4 Conclusion

In the present work, we have studied the Barut-Girardello coherent states of the $U_q\left(su(1,1)\right)$ algebra and their representation in Fock space. Considering linear entropy as a measure of entanglement, we have examined and discussed the entanglement quantity for the output state of a beam splitter, given a deformed Barut-Girardello state and a vacuum state as the input states in each mode.

It is found that the generated entanglement depends heavily on the q-deformation parameter and the amplitude Z of the states. For each value of $|Z|$, after an initial quick change, the entanglement tends to zero with increasing index k. The value of the entanglement which approaches zero depends significantly on the q parameter and tends to zero as k becomes sufficiently large, in the case the deformed Barut-Girardello coherent states tend to Glauber coherent states. We also note that the entanglement of the output state gets smaller as the q-deformation parameter approaches 1 for lower values of k, whereas for higher values this behavior reverses.

We have investigated the statistical properties of the deformed Barut-Giradello coherent states by using the Mandel parameter. In this way, our aim is to make a comparison with Glauber states which allows to confirm our results.

The present results show that the deformed Barut-Girardello coherent states can be used to generate and measure the entanglement of bipartite composite systems. Also, we may use our results to discuss the entanglement of entangled deformed Barut-Girardello coherent states in vacuum environment which may open new perspectives to exploit these entangled states in the context of quantum teleportation [3], dense coding [4] and entanglement swapping [56].

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