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INCOHERENT SPATIAL OPTICAL SOLITONS IN PHOTOREFRACTIVE CRYSTALS

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Abstract

In this paper we have presented a theoretical investigation on the propagation properties of incoherent solitons in photorefractive media which is characterized by noninstantenous saturating nonlinearity. Using a mutual coherence function approach, we have obtained the equation of existence curve for such solitons. We have discussed the coherence characteristics of these solitons. We have found that solitons with a particular radius can possess two different critical powers. The dynamical evolution of these solitons has been discussed in detail by both analytical and numerical simulation.

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I. Introduction

One of the recent developments in the field of optical solitons is the identification and demonstration of the existence of incoherent or partially coherent spatial optical solitons [1-3]. Unlike coherent optical solitons, phase of the partially coherent light beams fluctuates randomly, yet there are nonlinear media in which they can create a smooth light induced refractive index change that subsequently lead to the trapping of the beam and spatial soliton formation. Ideally, the formation of incoherent solitons requires that the media should have a noninstantenous nonlinearity, for example, biased photorefractive crystals, which exhibit a strong and sufficiently slow saturable Kerr like nonlinearity [1, 2]. Such nonlinear media respond to the time averaged light intensity, which means that even though the phase of the light beam fluctuates randomly, it still creates a smooth light induced refractive index change and subsequent trapping of the beam. However, incoherent optical solitons have also been experimentally observed in nonlinear media with instantaneous response and subsequently theoretical investigations have been carried out to understand their dynamics [4,5]. Recently, Piccanti et al. reported the first observation of incoherent solitons and their interactions in nematic liquid crystals [6,7]. Liquid crystals possess strong instantaneous saturable Kerr-like nonlinearity, and their nonlinear response time ranges from tens to hundreds of milliseconds, which is sufficiently slow facilitating for spatial solitons to form [8].

Incoherent solitons have attracted particularly strong attention as they open the possibility of using light sources with degraded or poor coherence in soliton based optical signal processing. A great deal of theoretical efforts has been directed towards understanding solitons of incoherent light [9-14]. Theoretical descriptions of partially coherent solitons have been carried out following three different approaches, in particular, (i) coherent density method [9,10], self consistent modal theory [11,12] and mutual coherence function [13]. Krolikowski et al. investigated the propagation of a partially coherent beam in a nonlinear medium characterized by logarithmic nonlinearity [13]. They have also investigated propagation characteristics of partially coherent beams in spatially nonlocal nonlinear media with a logarithmic type of nonlinearity [14]. The propagation properties of spatiotemporal incoherent white light solitons in nonlocal nonlinear media with logarithmic nonlinearity [15] and elliptic incoherent accessible solitons in strongly nonlocal media with noninstantenous Kerr nonlinearity [16] have been studied. Most recently, elliptic incoherent solitons in strongly nonlocal media with anisotropic Kerr nonlinearity has been also addressed [17].

In this paper, we discuss the propagation properties of incoherent solitons in photorefractive media that is possessing eaneous saturating Kerr nonlinearity. Using the mutual coherence function approach, we obtain the existence curve of these solitons. We have found that, both the total power and the coherence characteristics of these incoherent solitons determine their propagation properties. Detail numerical simulation has been carried out to identify the behavior of the spatial width and coherent radius.

II. Incoherent Propagation Model

Consider the propagation of a partially coherent optical beam in a noninstantenous nonlinear medium. The refractive index of the material is given by $n = n_0 + \delta n(I)$, where n_0 and $\delta n(I)$ denote the linear and nonlinear contribution to the refractive index, I is the intensity of the optical beam. The nonlinear part of the refractive index is due to noninstantenous saturating photorefractive nonlinearity. The refractive index change $\delta n(I)$ may be taken as [18]

$$\delta n(I) = \frac{-\alpha}{1+I}\,, \tag{1}$$

where α is a constant related to the properties of the medium. We start with the nonlinear Schrödinger equation governing the paraxial propagation of the partially coherent beam in the nonlinear medium:

$$i\frac{\partial\psi(\vec{r})}{\partial z} + \frac{1}{2k_0}\nabla_r^2\psi(\vec{r}) + k_0\delta n\big(I(r)\big)\psi(r) = 0, \tag{2}$$

where $\psi(\vec{r})$ is the slowly varying envelope of the electric field of the optical beam, k_0 is the wave number in free space, and ∇_r^2 is the transverse Laplacian representing diffraction of the beam. For optical waves at two different spatial points $\vec{r}_1$ and $\vec{r}_2$, the slowly varying envelopes $\psi(\vec{r}_1)$ and $\psi(\vec{r}_2)$ respectively obey:

$$i\frac{\partial\psi(\vec{r}_1)}{\partial z} + \frac{1}{2k_0}\nabla_{\vec{r}_1}^2\psi(\vec{r}_1) + k_0\delta n\big(I(\vec{r}_1)\big)\psi(\vec{r}_1) = 0\,, \tag{3}$$

$$i\frac{\partial\psi(\vec{r}_2)}{\partial z} + \frac{1}{2k_0}\nabla_{\vec{r}_2}^2\psi(\vec{r}_2) + k_0\delta n\big(I(\vec{r}_2)\big)\psi(\vec{r}_2) = 0\,. \tag{4}$$

To express coherence properties of the incoherent beam, we introduce the mutual coherence function Γ, which is defined as $\Gamma(\vec{r}_1,\vec{r}_2) = \langle\psi(\vec{r}_1)\psi(\vec{r}_2)\rangle$, where the angular brackets denote temporal averaging. The time average intensity is obtained from the coherence function as $I(\vec{r}) = \Gamma(\vec{r}_1,\vec{r}_2) = \langle\psi(\vec{r})\psi(\vec{r})\rangle$. By virtue of eqs(3) and (4), the mutual coherence function satisfies following equation:

$$i\frac{\partial\Gamma(\vec{r}_1,\vec{r}_2)}{\partial z} + \frac{1}{2k_0}\left(\nabla_{\vec{r}_1}^2 - \nabla_{\vec{r}_2}^2\right)\Gamma(\vec{r}_1,\vec{r}_2) + k_0\big\{\delta n\big(I(\vec{r}_1)\big) - \delta n\big(I(\vec{r}_2)\big)\big\}\Gamma(\vec{r}_1,\vec{r}_2) = 0. \tag{5}$$

We introduce new variables $\vec{R} = \frac{\vec{r}_1+\vec{r}_2}{2}$, $\vec{p} = \vec{r}_1 - \vec{r}_2$ and with these independent variables it is straightforward to transform equation (5) in the following form:

$$i\frac{\partial\Gamma_{12}(\mathrm{R},\overrightarrow{p},z)}{\partial z}+\frac{1}{2k_0}\nabla_{\vec{R}}\cdot\nabla_{\vec{p}}\,\Gamma_{12}(\mathrm{R},\overrightarrow{p},z)+k_0\{\delta n(\Gamma_{11})-\delta n(\Gamma_{22})\}\Gamma_{12}(\mathrm{R},\overrightarrow{p},z)=0 \quad , \tag{6}$$

where we have defined $\Gamma_{ij}=\Gamma(\vec{r}_i,\vec{r}_j)$, $i,j=(1,2)$, $\Gamma_{11}=\Gamma(\vec{r}_1,\vec{r}_1)=I(\vec{r}_1)$ and $\Gamma_{22}=\Gamma(\vec{r}_2,\vec{r}_2)=I(\vec{r}_2)$. For simplicity we assume that the incident optical field $\psi(\vec{r})$ possesses Gaussian statistics, therefore the correlation function can be written at z=0 as [13]

$$\Gamma(\vec{r}_1,\vec{r}_2,z=0)=exp\left[-\frac{r_1^2+r_2^2}{2\rho_0^2}-\frac{|\vec{r}_1-\vec{r}_2|^2}{r_c^2}\right], \tag{7}$$

where ρ_0 and r_c respectively denote the initial width of the optical field and coherence radius of beam. Using variables as $\vec{R}$ and $\vec{p}$, the coherence function can be recasted as

$$\Gamma(\vec{R}\ \ ,\vec{p}\ \ ,z=0)=exp\left[-\frac{|\vec{R}|^2}{\rho_0^2}-\frac{|\vec{p}|^2}{\sigma_0^2}\right], \tag{8}$$

where σ_0 is the effective coherence radius and defined as $\frac{1}{\sigma_0^2}=\frac{1}{r_c^2}+\frac{1}{4\rho_0^2}$. We therefore look for solution of Eq(6) in the Gaussian –Schell model form:

$$\Gamma(\vec{R}\ \ ,\vec{p}\ \ ,z)=A(z)exp\left[-\frac{|\vec{R}|^2}{\rho^2(z)}-\frac{|\vec{p}|^2}{\sigma^2(z)}+i\vec{R}\cdot\vec{p}\mu(z)\right], \tag{9}$$

where $A(z)$ and $\mu(z)$ denote the amplitude and the phase variation of the mutual coherence function, respectively. $\rho(z)$ and $\sigma(z)$ are width and effective coherence radius of the beam. This last parameter signifies the degree of coherence of the beam. Expanding photorefractive nonlinearity $\delta n(\Gamma_{11})$ and $\delta n(\Gamma_{22})$ around $r_1^2\approx 0$ and $r_2^2\approx 0$ respectively and neglecting higher order terms which are small, we obtain following equation for the mutual coherence function:

$$i\frac{\partial\Gamma_{12}(\mathrm{R},\overrightarrow{p},z)}{\partial z}+\frac{1}{k_0}\nabla_{\vec{R}}^2\cdot\nabla_{\vec{p}}^2\Gamma_{12}(\mathrm{R},\overrightarrow{p},z)+\frac{2k_0\alpha A(z)}{(1+A(z))^2}\vec{\mathrm{R}}\cdot\vec{\mathrm{p}}\ \ \Gamma_{12}(\mathrm{R},\overrightarrow{p},z)=0 \tag{10}$$

Inserting eq(9) in the equation(10), equating real and imaginary parts separately to zero, we get following two equations:

$$\frac{dA}{dz}+2A\frac{R^2}{\rho^3}\frac{d\rho}{dz}+\frac{2Ap^2}{\rho^3}\frac{d\sigma}{dz}+\frac{\mu A}{k_0}-\frac{2p^2\mu A}{k_0\sigma^2}-\frac{2R^2\mu A}{k_0\rho^2}=0\ , \tag{11}$$

and

$$-A\vec{R}\cdot\vec{p}\frac{d\mu}{dz}+\frac{4RpA}{k_0\sigma^2\rho^2}-\frac{Rp\mu^2A}{k_0}-\frac{2k_0\alpha A^2}{(1+A^2)^2\rho^2}\vec{R}\cdot\vec{p}=0\ . \tag{12}$$

Equating coefficients of R^0p^0, R^2p^0, R^0p^2 and R^1p^1 from above two equations, we get following set of ordinary differential equations:

$$\frac{dA}{dz} = -\frac{2\mu A}{k_0}, \tag{13}$$

$$\frac{d\rho}{dz} = \frac{\mu\rho}{k_0}, \tag{14}$$

$$\frac{d\sigma}{dz} = \frac{\mu\sigma}{k_0}, \tag{15}$$

and

$$\frac{d\mu}{dz} = \frac{4}{k_0\sigma^2\rho^2} - \frac{\mu^2}{k_0} - \frac{2\alpha k_0 A}{(1+A)^2\rho^2}. \tag{16}$$

Using eq(14), the last equation can be converted to the following form for the variation of the width of the soliton beam:

$$\frac{d^2\rho}{dz^2} = \frac{4}{k_0^2\sigma^2\rho} - \frac{2\alpha A}{(1+A)^2\rho}. \tag{17}$$

Above equation describes the dynamics of width $\rho(z)$ of a partially coherent beam obeying Gaussian statistics and propagating in a photorefractive medium with noninstantenous saturating nonlinearity. The dynamics of the beam is determined by the free spreading represented by the first term of the right hand side of eq(17) and nonlinearity represented by the second term in the right-hand side of this equation. From eqs(14) and (15), we immediately obtain $\frac{\rho(z)}{\sigma(z)} = \frac{\rho(0)}{\sigma(0)} = \text{constant} = N(say)$. Above relationship shows that during the propagation the beam conserves its coherence which is defined as the number of speckle within the beam diameter. During propagation the wider beam will have larger coherence radius and vice versa. From eqs(13) and (14) we immediately obtain $A(z)\rho^2(z) = A_0\rho_0^2 = \text{constant.}$, where $A_0 = A(z=0)$ and $\rho_0 = \rho(z=0)$. With the help of this relationship, eq(17) now can be re casted as:

$$\frac{d^2\rho}{dz^2} = \frac{4(\rho_0^2/\sigma_0^2)}{k_0^2\rho^3} - \frac{2\alpha A_0\rho_0^2\rho}{(A_0\rho_0^2+\rho^2)^2}. \tag{18}$$

We assume that the beam has the property $\frac{d\rho}{dz} = 0$ at $z = 0$ i.e., initially the beam is parallel. Integrating equation (18) once, we can obtain

$$\frac{1}{2}\left(\frac{d\rho}{dz}\right)^2 + \frac{2}{k_0^2\sigma_0^2}\left(\frac{\rho_0^2}{\rho^2} - 1\right) - \alpha A_0\left[\frac{1}{A_0+(\rho/\rho_0)^2} - \frac{1}{1+A_0}\right] = 0. \tag{19}$$

The above equation is analogous to the classical Newtonian equation describing the motion of a particle of unity mass which is moving in a potential well $V(\rho)$ that is given by

$$V(\rho) = \frac{2}{k_0^2\sigma_0^2}\left(\frac{\rho_0^2}{\rho^2} - 1\right) - \alpha A_0\left[\frac{1}{A_0+(\rho/\rho_0)^2} - \frac{1}{1+A_0}\right]. \tag{20}$$

Thus, the nature of the potential well defines the behavior of the solitons. The potential of the incoherent soliton has been illustrated for different normalized power A_0 and normalized effective coherent radius $k_0\sigma_0$ in figures 1(a) and (b), respectively. The potential well is asymmetric. The soliton is trapped in that region of the well where $V(\rho) \leq 0$. With the increase in the value of $k_0\sigma_0$, the width of the well increases. We now consider the issue of stationery beam propagation with a constant beam radius which is equivalent to the effective particle located at the bottom of the potential well. The condition for stationary propagation can be obtained from the following equation of threshold value of the normalized power $A_0(= \mathrm{A}_{0c})$:

$$\frac{A_{0c}}{(1+A_{0c})^2} = \frac{2}{\alpha k_0^2}\left(\frac{1}{\sigma_0^2}\right), \tag{21}$$

where A_{0c} is the value of A when $\frac{d^2\rho}{dz^2} = 0$.

The equation can be rewritten as

$$\frac{A_{0c}}{(1+A_{0c})^2} = \frac{1}{\alpha k_0^2}\left[\frac{2}{r_c^2} + \frac{1}{2\rho_0^2}\right] = \Sigma\ \text{(say)}, \tag{22}$$

This relation signifies that the beam spreading occurs due to diffraction and beam incoherence and the resultant beam spreading is counter balanced by the nonlinearity of the medium. From eq(22) the critical input power for the incoherent solitons is obtained as

$$A_{0c} = \frac{(1-2\Sigma)\pm\sqrt{(1-2\Sigma)^2-4\Sigma^2}}{2\Sigma}, \tag{23}$$

which implies that the effective coherent radius σ_0 should obey following inequality $\sigma_0 > \frac{2}{k_0\sqrt{\alpha}}$ i.e., incoherent soliton whose effective coherent radius is less than above value cannot propagate as a self trapped mode. It is therefore obvious that the incoherent solitons will be stable when the input power obeys a restricted value defined by the beam width and coherence characteristics. In order to get a stationary incoherent solitons the beam width ρ_0 should obey

$$\rho_0^2 = \left[\frac{2\alpha k_0^2 A_0}{(1+A_0)^2} - \frac{4}{r_c^2}\right]^{-1}. \tag{24}$$

Equation (18) can be integrated once to get

$$\int \frac{d\rho}{\sqrt{\frac{4}{k_0^2\sigma_0^2}\left(\frac{\rho_0^2}{\rho^2}-1\right)-2\alpha A_0\left[\frac{1}{A_0+(\rho/\rho_0)^2}-\frac{1}{1+A_0}\right]}} = dz \tag{25}$$

which can be easily integrated numerically to obtain the variation of beam width with distance of propagation.

When the incident power A_{0c} does not satisfy the expression (22), the incoherent solitons will undergo oscillation. The nature of such oscillations at different power and different $k_0\sigma_0$ have been demonstrated in figure 2(a) and (b) respectively. It is evident from the figure that the period of oscillation decreases with the increase in $k_0\sigma_0$, it also decreases with the increase in A_0. The oscillation within a minimum and maximum value of soliton width signifies oscillatory bound states of solitons.

III. Conclusion

In conclusion, using mutual coherence function we have studied incoherent optical solitons in photorefractive media. We have shown that the total power and the coherent characteristics together decide the propagation characteristics of solitons. We have shown that for a particular solitons radius, there exist two critical powers for stable soliton propagation. Width of a stationary soliton depends not only on power but also on coherent radius. We have also shown that, in order to form spatial solitons, the effective initial coherent radius of the soliton forming optical beam should be greater than certain value which is determined by the properties of the medium.

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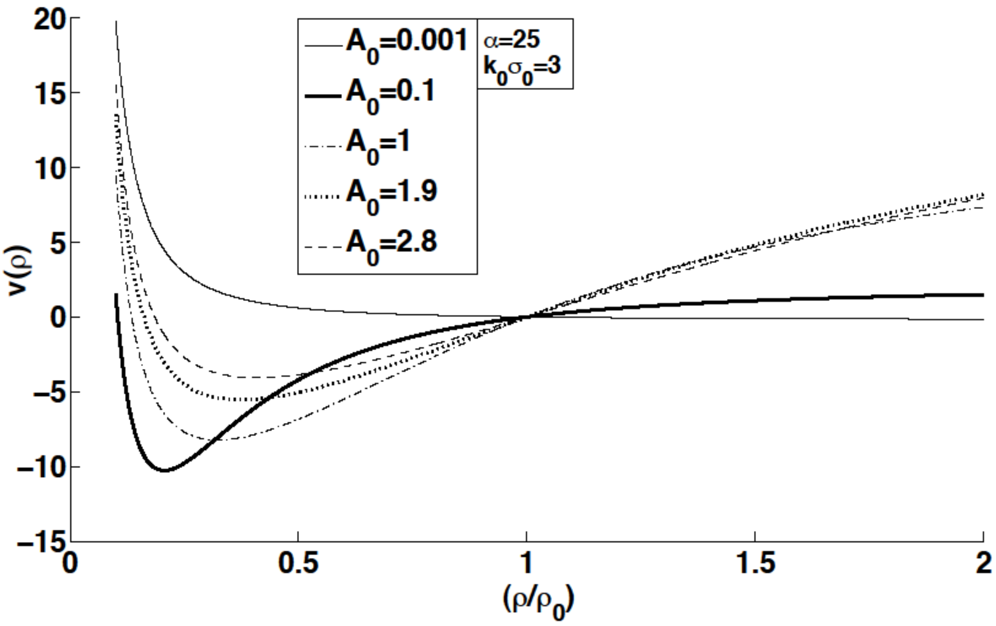
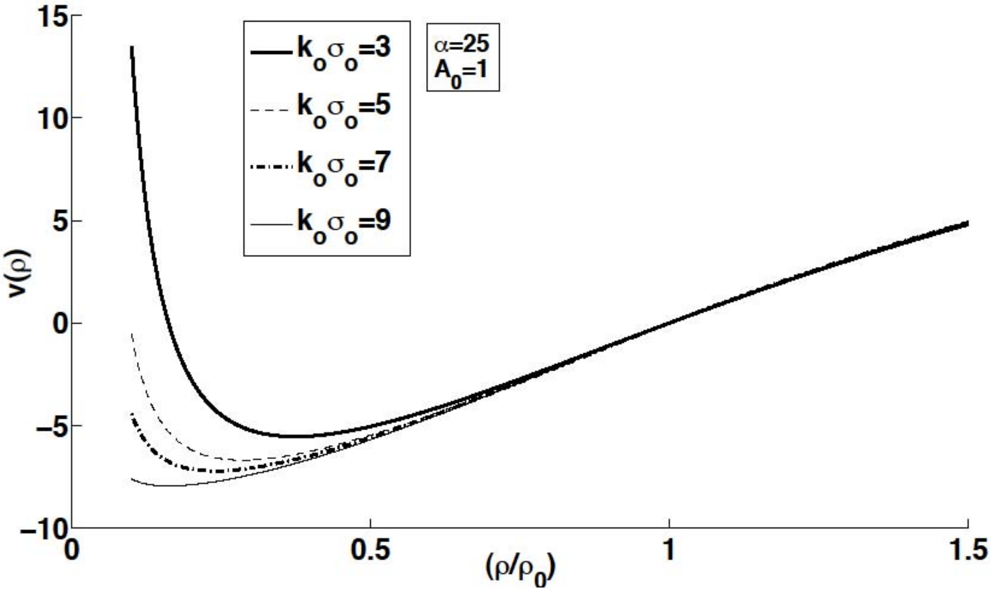
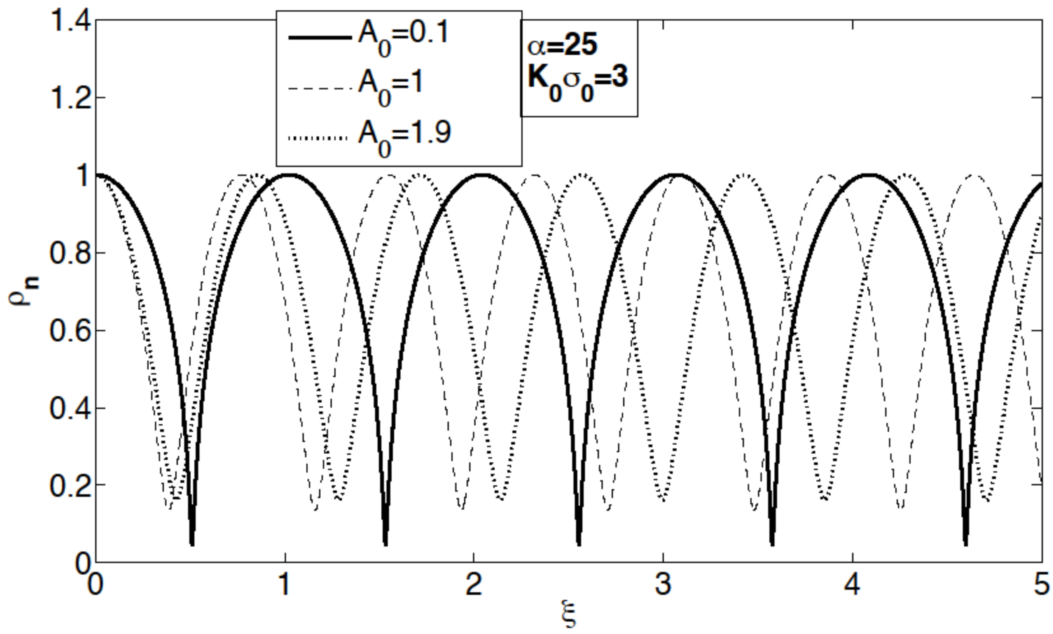


(a)

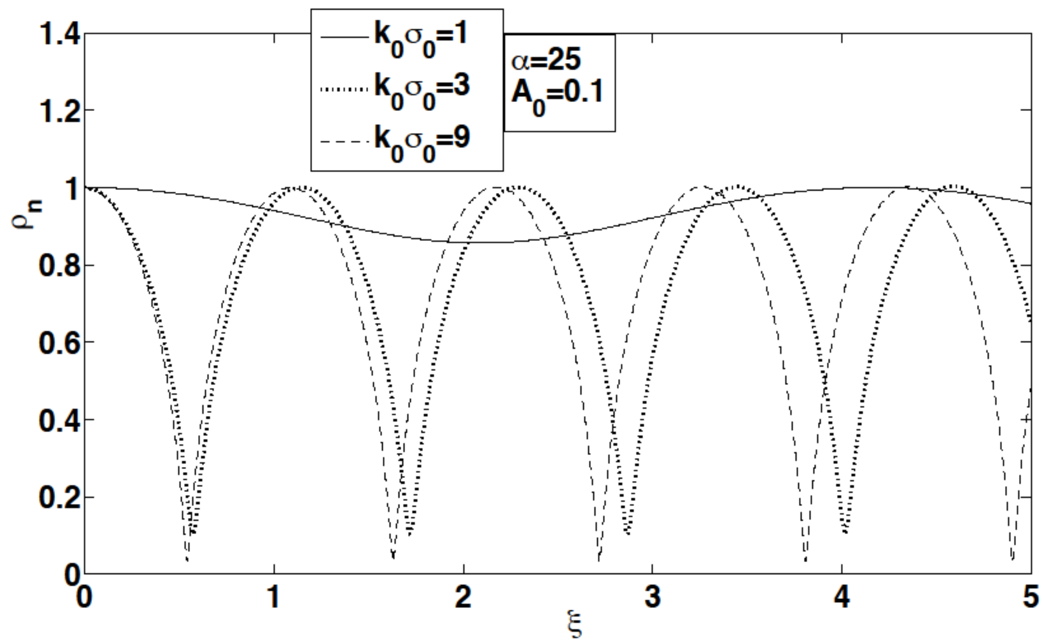


(b)

Figure 1: Variation of potential $V(\rho)$ with ρ/ρ_0. (a) For different normalized power A_0 , (b) for different $k_0\sigma_0$. $\alpha = 25$.



(a)



(b)

Figure 2: Oscillations of normalized spatial width ρ_n with normalized distance of propagation ξ. $\xi = {}^{z}/_{\sigma_0}$ and $\rho_n = \rho/\rho_0$. (a) For different normalized power A_0 , (b) for different $k_0\sigma_0$. $\alpha = 25$.