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MAGNETIZATION REVERSAL THROUGH SOLITON IN A SITE-DEPENDENT WEAK FERROMAGNET

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Abstract

Switching the magnetization of a magnetic bit through flipping of soliton offers the possibility of developing a new innovative approach for data storage technologies. The spin dynamics of a site-dependent ferromagnet with antisymmetric Dzyaloshinskii-Moriya interaction is governed by a generalized inhomogeneous higher order nonlinear Schrödinger equation. We demonstrate the magnetization reversal through flipping of soliton in the ferromagnetic medium by solving the two coupled evolution equations for the velocity and amplitude of the soliton using the fourth order Runge-Kutta method numerically. We propose a new approach to induce the flipping behaviour of soliton in the presence of inhomogeneity by tuning the parameter associated with Dzyaloshinskii-Moriya interaction which causes the soliton to move with constant velocity and amplitude along the spin lattice.

1 Introduction

The study of magnetization reversal process in magnetic systems is one of the fundamental issues in magnetic data storage. The magnetization reversal process obtained through an understanding of the underlying magnetization dynamics is an important issue mainly because the dynamic process is nonlinear in nature. The magnetization reversal process is normally based on a coherent rotation of the magnetization and a propagation of domain walls in the presence of the magnetic field [1]. In that case, the switching of magnetization is accomplished by domain wall motion which is a relatively slow process compared to a reversal by coherent rotation [2]. Conventionally, the bit state has been changed by applying an instantaneous magnetic field pulse, which causes the magnetization reversal process by the precession of magnetization [3-4]. Most of the available results on the magnetization reversal process is based on experimental studies and numerical simulations and analytical results are very limited [5-8]. One can't rule out the possibility of magnetization reversal without applying external magnetic fields.

The study of nonlinear spin excitations in the Heisenberg model of ferromagnets with different magnetic interactions have been identified as integrable models with localized spin excitations such as solitons [9-13]. In addition to this, there exists an important type of antisymmetric interaction, the Dzyaloshinskii-Moriya (D-M) interaction, which is less spoken about in the literature of nonlinear dynamics due to the mathematical complexity of their representations in the Hamiltonian and in the governing dynamical equations. The combination of low symmetry and spin-orbit coupling was shown by Dzyaloshinskii [14] and Moriya [15] to give rise to anisotropic exchange coupling. Dzyaloshinskii has shown that the spin superstructure gives rise to a non-vanishing antisymmetric spin coupling vector which is parallel to the trigonal axis in $\alpha - Fe_2O_3$. Moriya has shown how the processes involving an additional virtual transition due to spin-orbit coupling can cause an anisotropic exchange interaction as a correction to the isotropic Anderson superexchange term and introduced a new term in the spin Hamiltonian which is the Dzyaloshinskii-Moriya interaction. In particular, when the symmetry around the magnetic ions is not high enough, this unfamiliar but important antisymmetrical coupling leading to the mechanism of weak ferromagnetism results in the canting of spins. Such a coupling would open up entirely new possibilities in data storage technologies such as weak ferromagnetic memory elements that could be read out nondestructively via the accompanying magnetization. Weak ferromagnetism plays an important role in describing insulators, spin glass, low temperature phases of copper oxide superconductors, phase transition [16-20]. In a recently proposed spin based quantum computer architecture, the exchange interaction especially the weak ferromagnetic interaction between the spins plays a fundamental role in establishing two-qubit entanglement [21].

The nonlinear dynamics of inhomogeneous systems have been widely investigated [22-26] and are expected to have many applications in the construction of magnetic memory devices, logic gates and so on. In Ref. [27], it is pointed out that the site-dependent biquadratic ferromagnetic spin chain with different type of inhomogeneities exhibits soliton excitations and further exploited for the magnetization reversal process. Also it has been demonstrated that in Ref. [28], the site-dependent ferromagnetic spin chain with nonlinear inhomogeneity admits shape changing property during its evolution. This shape changing property can be exploited to reverse the magnetization without loss of energy which may have potential applications in magnetic memory and recording devices. Thus it has become increasingly important to investigate the magnetization reversal process by exploiting the localized coherent structure of solitons in a weak ferromagnetic medium. Conceived by the above, in this paper we present a controlled magnetization reversal phenomenon by the flipping of solitons in a site-dependent weak ferromagnet. The organization of the paper is as follows. In Sec. II, we formulate the model Hamiltonian and derive the equation of motion in semiclassical limit. We carry out multiple scale perturbation method to derive evolution equations for soliton parameters and construct the perturbed soliton solution in Sec. III. In Sec. IV, we establish the controlled magnetization reversal process under the influence of D-M interaction using fourth order Runge-Kutta method numerically. Finally, the paper is concluded in Sec. V.

2 Soliton dynamics of a ferromagnet with D-M interaction

In this section of the paper, we investigate the nature of nonlinear spin excitations especially in the form of soliton by analyzing the underlying dynamics of a ferromagnet with site-dependent Dzyaloshinskii-Moriya interaction, we formulate the model Hamiltonian as

$$\begin{aligned} H = -\sum_i \big[& J_1 f_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + J_2 f_i S_i^z S_{i+1}^z + J_3 \vec{D}_i . (\vec{S}_i \times \vec{S}_{i+1}) \\ & - A (S_i^z)^2 - A' (S_i^z)^4 \big], \end{aligned} \tag{1}$$

where $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$ is a three component spin vector. Here J_1 and J_2 are the exchange interaction parameters due to spin-spin coupling in the $S^x - S^y$ plane and along the S^z-direction respectively. The term proportional to J_3 represents the Dzyaloshinskii-Moriya weak interaction and the antisymmetric coupling $(\vec{S}_i \times \vec{S}_{i+1})$ acts to cant the spins. The function f_i characterize the variation in the bilinear exchange interaction. The terms proportional to A and A' correspond to the single site uniaxial anisotropic energy due to crystal field effect and the easy axis of magnetization is chosen along the z-direction. In the case of usual weak ferromagnets, the Dzyaloshinskii vector is assumed to be a constant which can be related to the symmetry of the magnetic crystals. Unlike the usual case, here the D-M interaction varies along the spin chain, thus introducing an inhomogeneity in the antisymmetric interaction.

Generally inhomogeneity in magnetic materials arise because of the following two factors (i) if the distance between neighbouring magnetic atoms varies along the chain and the degree of overlapping of electronic wave function varies from site to site. Thus the interaction between the spins depends upon the site in the crystal lattice, which is known as site-dependent interaction. E.g., charge transfer complexes TCNQ, $Ni(CN)_4$, organo-metallic insulators, TTF-bisdithiolenes and $Ni(Co)_4$. (ii) If the atomic wave function itself varies from site to site although the atoms themselves may equally be spaced. This type of inhomogeneity occurs when magnetic insulators placed in a weak, static and inhomogeneous electric field. It can also be simulated by the deliberate introduction of impurities or organic complexes in the vicinity of a bond so as to alter the electronic wave functions without causing appreciable lattice distortion. The dimensionless form of the Hamiltonian can be written as

$$\begin{aligned} H = -\frac{1}{2}\sum_i \Big[& J_1 f_i(\hat{S}_i^+\hat{S}_{i+1}^- + \hat{S}_i^-\hat{S}_{i+1}^+) + 2J_2 f_i \hat{S}_i^z\hat{S}_{i+1}^z \\ & -iJ_3 g_i\Big\{D^+\big[\hat{S}_i^z\hat{S}_{i+1}^- - \hat{S}_i^-\hat{S}_{i+1}^z\big] + D^-\big[\hat{S}_i^+\hat{S}_{i+1}^z - \hat{S}_{i+1}^+\hat{S}_i^z\big] \\ & +D^z\big[\hat{S}_i^-\hat{S}_{i+1}^+ - \hat{S}_i^+\hat{S}_{i+1}^-\big]\Big\} - 2A(\hat{S}_i^z)^2 - 2A'(\hat{S}_i^z)^4\Big], \end{aligned} \tag{2}$$

where $S_i^{\pm} = \hat{S}_i^x \pm i\hat{S}_i^y$, $\hat{S}_i = \frac{\vec{S}_i}{\hbar}$, $D^{\pm} = D^x \pm iD^y$ and $\vec{D}_i = g_i\vec{D}$, the function g_i characterize the variation in D-M interaction along the spin chain. The dimensionless spin operator $\hat{S}_i$ satisfies the commutation relations

$$[\hat{S}_n^+, \hat{S}_m^-] = 2\delta_{mn}\hat{S}_n^z \quad \text{and} \quad [\hat{S}_n^{\pm}, \hat{S}_m^z] = \mp\delta_{mn}\hat{S}_n^{\pm}, \tag{3}$$

with $\hat{S}_n.\hat{S}_n = S(S+1)$. Now, we introduce the Holstein-Primakoff transformation [29,30] for the spin operators in terms of boson operators

$$\hat{S}_n^+ = \sqrt{2S}\ \big[1 - \frac{a_n^\dagger a_n}{2S}\big]^{1/2} a_n, \tag{4}$$

$$\hat{S}_n^- = \sqrt{2S}\ a_n^\dagger\big[1 - \frac{a_n^\dagger a_n}{2S}\big]^{1/2}, \tag{5}$$

$$\hat{S}_n^z = \big[S - a_n^\dagger a_n\big], \tag{6}$$

and the bosonic operators $a_n^\dagger$ and a_n satisfy the Bose commutation relations

$$\big[a_n, a_m^\dagger\big] = \delta_{nm} \quad \text{and} \quad \big[a_n, a_m\big] = \big[a_n^\dagger, a_m^\dagger\big] = 0. \tag{7}$$

For treating the problem semi-classically at sufficiently low temperature we use a truncated semiclassical expansion for $\hat{S}_n^+$ and $\hat{S}_n^-$ in the following form

$$\tilde{S}_n^+ = \sqrt{2}\Big[1 - \frac{\epsilon^2}{4}a_n^\dagger a_n - \frac{\epsilon^4}{32}a_n^\dagger a_n a_n^\dagger a_n - \frac{\epsilon^6}{128}a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n - O(\epsilon^8)\Big]\epsilon a_n, \tag{8}$$

$$\tilde{S}_n^- = \sqrt{2}\epsilon a_n^\dagger\Big[1 - \frac{\epsilon^2}{4}a_n^\dagger a_n - \frac{\epsilon^4}{32}a_n^\dagger a_n a_n^\dagger a_n - \frac{\epsilon^6}{128}a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n - O(\epsilon^8)\Big], \tag{9}$$

where $\tilde{S}_n^{\pm} = \frac{\hat{S}_n^{\pm}}{S}$ and while writing the above expansion it is assumed that $a_n^\dagger a_n \ll 2S$, $\epsilon = 1/\sqrt{S}$ is a small dimensionless parameter. Using the expansions presented in Eqs. (8-9), the Hamiltonian (2) can be written in terms of the bosonic operators as a power series in ϵ as given in Eq. (A1) in the appendix. Having known the Hamiltonian, the dynamics of spins can be expressed in terms of the Heisenberg equation of motion for the Bose operator,

$$i\hbar\frac{\partial a_n}{\partial t} = [a_n, H] = F(f_n, g_n, a_n^\dagger, a_n, a_{n+1}^\dagger, a_{n+1}). \tag{10}$$

After introducing Glauber's coherent-state representation [31] for the Bose operators $a_n^\dagger|u\rangle = u_n^*|u\rangle$, $a_n|u\rangle = u_n|u\rangle$, $|u\rangle = \Pi_n|u_n\rangle$ with $\langle u|u\rangle = 1$, where $|u(n)\rangle$ is the coherent-state eigenvector for the operator a_n and u_n is the coherent amplitude for the system in the state $|u\rangle$, we write down the discrete equation of motion $< u|a_j|u >$ corresponding to the new Hamiltonian as given in Eq. (A2) in the appendix. For treating the problem semi-classically at low temperature and long wave length limit, it is appropriate to make continuum approximation by replacing $u_j(t)$ by $u(x,t)$ and by expanding

$$u_{j\pm1} = u(x,t) \pm \gamma u_x + \frac{\gamma^2}{2!}u_{xx} \pm \frac{\gamma^3}{3!}u_{xxx} + \frac{\gamma^4}{4!}u_{xxxx} \pm O(\gamma^5), \tag{11}$$

where $x = j\gamma$, γ is the lattice parameter and suffix x represents the partial derivative with respect to x. Also we introduce the expansion for $f_{i\pm1}$ and $g_{i\pm1}$ in the same fashion

$$f_{j\pm1} = f(x,t) \pm \gamma f_x + \frac{\gamma^2}{2!}f_{xx} \pm \frac{\gamma^3}{3!}f_{xxx} + \frac{\gamma^4}{4!}f_{xxxx} \pm O(\gamma^5), \tag{12}$$

$$g_{j\pm1} = g(x,t) \pm \gamma g_x + \frac{\gamma^2}{2!}g_{xx} \pm \frac{\gamma^3}{3!}g_{xxx} + \frac{\gamma^4}{4!}g_{xxxx} \pm O(\gamma^5). \tag{13}$$

Using Eq. (11) and Eqs. (12-13) in Eq. (A2) as presented in the appendix, and retaining terms proportional to $\gamma^m\epsilon^n$ up to order O(m+n=5), we obtain the following generalized higher order inhomogeneous nonlinear Schrödinger equation as given in Eq. (A3) in the appendix. Eq. (A3) describes the dynamics of spins in a Heisenberg anisotropic ferromagnet with D-M interaction in the continuum limit under semiclassical approximation. A close inspection and the results of singularity structure analysis on Eq. (A3) reveals that it contains several well known and atleast four integrable models at different orders of ϵ, governed by (i) cubic NLS, (ii) MKdV, (iii) Hirota and (iv) fourth order NLS equations [23]. The elementary spin excitations can be expressed in terms of solitons for specific choices of parameters and only when the exchange inhomogeneity appears in the form of a linear function. Now a natural question arises as to what will be the nature of the nonlinear spin excitations in the more general case i.e, other than the conditions for integrability. To answer this question, we carry out a multiple scale perturbation analysis following the procedure of Kodama and Ablowitz [32]. Further, we are very much interested to exploit the coherent nature of solitons for exhibiting the lossless reversal of magnetization using the inherent inhomogeneity present in the weak ferromagnetic interaction.

3 Effect of discreteness due to D-M interaction

In order to investigate the effect of discreteness due to the presence of inhomogeneous D-M interaction, we recast Eq. (A3) in the following form

$$iu_t + h_1'u_{xx} + h_2'|u|^2u + i(h_3'D^zu_x + h_4'D^zu_{xxx} + h_5'D^z|u|^2u_x) = 0, \tag{14}$$

by redefining the coefficients $h_1' = \alpha_1 x + \beta_1$, $h_2' = \alpha_2 x + \beta_2$, $h_3' = \alpha_3 x + \beta_3$, $h_4' = \alpha_4 x + \beta_4$, $h_5' = \alpha_5 x + \beta_5$, $\alpha_1 = 2J_1 k\gamma^2$, $\beta_1 = 2J_1 k_1\gamma^2 - J_1 k\gamma^3 - iJ_3 k\gamma^3 D^z$, $\alpha_2 = -4k\epsilon^2(J_1 - J_2)$, $\beta_2 = -4\epsilon^2(k_1 - \frac{\gamma}{2}k)(J_1 - J_2) + 2iJ_3 k\epsilon^2\gamma D^z - 4\epsilon^2 A - 24\epsilon^2 A'$, $\alpha_3 = -4J_3 k\gamma$, $\beta_3 = -\frac{2ikJ_1\gamma^2}{D^z} - 4J_3 k_1\gamma + 2J_3 k\gamma^2$, $\alpha_4 = -\frac{2J_3 k\gamma^3}{3}$, $\beta_4 = -\frac{2J_3 k_1\gamma^3}{3}$, $\alpha_5 = 4J_3 k\epsilon^2\gamma$, $\beta_5 = 4J_3 k_1\epsilon^2\gamma$, $f(x) = g(x) = kx + k_1$ and $D^{\pm} = 0$, the D-M interaction is restricted to the z-direction. We try to understand the effect of inhomogeneity in the D-M interaction on the spin soliton, for that we assume $h_j'(x) = h_{j0} + \lambda h_j(x)$, where $i = 1, 2, ..., 5$, $h_{30} = h_{40} = h_{50} = 0$, λ is a small parameter and $h_j(x)$ is a linear function of x. After appropriate rescaling of t and redefining of λ, we get

$$iu_t + u_{xx} + \frac{1}{2}|u|^2 u - \lambda[h_1 u_{xx} + h_2|u|^2 u + i(h_3 D^z u_x + h_4 D^z u_{xxx} + h_5 D^z |u|^2 u_x)] = 0. \quad (15)$$

We study Eq. (15) perturbatively by treating terms proportional to λ as weak perturbation. When $\lambda = 0$, Eq. (15) reduces to the completely integrable cubic NLS equation which admits N-soliton solutions [33]. The envelope one soliton solution for cubic NLS can be written as

$$u = \eta sech\eta(\theta - \theta_0)exp[i\xi(\theta - \theta_0) + i(\sigma - \sigma_0)], \quad (16)$$

where $\theta_t = -2\xi$, $\theta_x = 1$, $\sigma_t = \eta^2 + \xi^2$, $\sigma_x = 0$. The parameter η and ξ are related to the scattering parameter of the inverse scattering transform (IST) analysis. Now we write η, ξ, θ, θ_0 and σ_0 as functions of a new time scale $T = \lambda t$ and hence $u = \hat{u}(\theta, T; \lambda)\exp[i\xi(\theta - \theta_0) + i(\sigma - \sigma_0)]$. Then under the assumption of quasi-stationarity, we expand $\hat{u}$ in terms of λ using Poincare type expansion as

$$\hat{u}(\theta, T; \lambda) = \hat{u}_0(\theta, T) + \lambda\hat{u}_1(\theta, T) + ..., \quad (17)$$

where $\hat{u}_0 = \eta sech\eta(\theta - \theta_0)$ and making use of u in Eq. (15), at O(λ) we obtain

$$-\eta^2\hat{u}_1 + \hat{u}_{1\theta\theta} + 2\hat{u}_0^2\hat{u}_1^* + 4\hat{u}_0^2\hat{u}_1 = F_1(\hat{u}_0), \quad (18)$$

where

$$\begin{aligned} F_1(\hat{u}_0) = {} & (\xi_T(\theta - \theta_0) - \xi\theta_{0T} - \sigma_{0T})\hat{u}_0 + h_1(\hat{u}_{0\theta\theta} - \xi^2\hat{u}_0) + h_2|u_0|^2 u_0 - h_3 D^z\xi\hat{u}_0 \\ & -h_4 D^z(3\xi\hat{u}_{0\theta\theta} - \xi^3\hat{u}_0) - h_5\xi D^z|u_0|^2 u_0 + i\big[-u_{0T} + 2h_1\xi\hat{u}_{0\theta} + h_3 D^z\hat{u}_{0\theta} \\ & +h_4 D^z(\hat{u}_{0\theta\theta\theta} - 3\xi^2\hat{u}_{0\theta}) + h_5 D^z|\hat{u}_0|^2\hat{u}_{0\theta}\big]. \end{aligned} \quad (19)$$

After substituting $\hat{u}_1 = \hat{\phi}_1 + i\hat{\psi}_1$, where $\hat{\phi}_1$ and $\hat{\psi}_1$ are real functions, in Eq. (18), we obtain

$$L_1\hat{\phi}_1 = -\eta^2\hat{\phi}_1 + \hat{\phi}_{1\theta\theta} + 6\hat{u}_0^2\hat{\phi}_1 = \Re F_1(\hat{u}_0), \quad (20)$$

$$L_2\hat{\psi}_1 = -\eta^2\hat{\psi}_1 + \hat{\psi}_{1\theta\theta} + 2\hat{u}_0^2\hat{\psi}_1 = \Im F_1(\hat{u}_0), \quad (21)$$

where L_1, L_2 are the self-adjoint operators. The real $\Re F_1$ and imaginary $\Im F_1$ parts of F_1 is given by

$$\begin{aligned} \Re F_1(\hat{u}_0) = {} & (\xi_T(\theta - \theta_0) - \xi\theta_{0T} - \sigma_{0T})\hat{u}_0 + h_1(\hat{u}_{0\theta\theta} - \xi^2\hat{u}_0) + h_2|u_0|^2 u_0 - h_3 D^z\xi\hat{u}_0 \\ & -h_4 D^z(3\xi\hat{u}_{0\theta\theta} - \xi^3\hat{u}_0) - h_5\xi D^z|u_0|^2 u_0, \end{aligned} \quad (22)$$

$$\Im F_1(\hat{u}_0) = -u_{0T} + 2h_1\xi\hat{u}_{0\theta} + h_3 D^z\hat{u}_{0\theta} + h_4 D^z(\hat{u}_{0\theta\theta\theta} - 3\xi^2\hat{u}_{0\theta}) + h_5 D^z|\hat{u}_0|^2\hat{u}_{0\theta}. \quad (23)$$

It may be noted that $\hat{u}_{0\theta}$ and $\hat{u}_0$ are the solutions of the homogeneous parts of Eqs. (20-21) for $\hat{\phi}_1$ and $\hat{\psi}_1$ respectively and the secularity conditions yield

$$\int_{-\infty}^{\infty} \hat{u}_{0\theta} \Re F_1 d\theta = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \hat{u}_0 \Im F_1 d\theta = 0. \tag{24}$$

On substituting the values of $\hat{u}_{0\theta}$, $\hat{u}_0$, $\Re F_1(\hat{u}_0)$ and $\Im F_1(\hat{u}_0)$ in Eqs. (24) and evaluating the above integrals, for the case of linear inhomogeneity, we arrive at the following two coupled evolution equations for soliton parameters such as amplitude η and velocity ξ

$$\frac{d\xi}{dT} = (\alpha_1 - \alpha_2)\eta^2 + \alpha_1\xi^2 + \alpha_3 D^z\xi - \alpha_4 D^z\xi^3 + (\alpha_4 D^z + \frac{\alpha_5}{3}D^z)\xi\eta^2, \tag{25}$$

$$\frac{d\eta}{dT} = -2\alpha_1\xi\eta + \alpha_3 D^z\eta - 3\alpha_4 D^z\eta\xi^2 - (\alpha_4 - \frac{\alpha_5}{3})D^z\eta^3. \tag{26}$$

The above two coupled ordinary differential equations are not amenable to easy analysis and therefore need to seek appropriate numerical method like fourth order Runge-Kutta method to solve for the amplitude and velocity of the soliton.

3.1 Perturbed solitons

The perturbed soliton solutions can be constructed by solving Eq. (20) for $\hat{\phi}_1$ and Eq. (21) for $\hat{\psi}_1$ using ξ_T and η_T. The homogeneous part of Eq. (20) admits two particular solutions ϕ_{11} and ϕ_{12} which are of the form

$$\phi_{11} = sech\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0), \tag{27}$$

$$\begin{aligned}\phi_{12} = -\frac{1}{\eta}[&sech\eta(\theta-\theta_0) - \frac{3}{2}\eta(\theta-\theta_0)sech\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0)\\ &-\frac{1}{2}\tanh\eta(\theta-\theta_0)\sinh\eta(\theta-\theta_0)].\end{aligned} \tag{28}$$

The general solution for $\hat{\phi}_1$ can then be written using the formula

$$\hat{\phi}_1 = \delta_1\phi_{11} + \delta_2\phi_{12} - \phi_{11}\int_{-\infty}^{\theta}\phi_{12}\Re F_1 d\theta' + \phi_{12}\int_{-\infty}^{\theta}\phi_{11}\Re F_1 d\theta', \tag{29}$$

where δ_1 and δ_2 are the arbitrary constants. After evaluating the integrals, we obtain the general solution for $\hat{\phi}_1$ as given in Eq. (A4) in the appendix. However, the solution $\hat{\phi}_1$ contains the secular terms which makes the solution unbounded is removed by choosing the arbitrary constant δ_2 as

$$\delta_2 = 0. \tag{30}$$

Further using the boundary conditions

$$\hat{\phi}_1(0)|_{\theta_0=0} = 0\ ; \quad \hat{\phi}_{1\theta}(0)|_{\theta_0=0} = 0, \tag{31}$$

we obtain

$$\begin{aligned}\delta_1 &= \frac{5}{3}\beta_1\xi - \frac{3}{4}\alpha_4 D^z\xi + \frac{1}{8}\alpha_5 D^z\xi,\\ \xi\theta_{0T} + \sigma_{0T} &= \frac{1}{3}\alpha_1\xi - \beta_3 D^z\xi - \frac{9}{2}\beta_4 D^z\xi\eta^2 + \beta_4 D^z\xi^3 + \frac{1}{4}\beta_5 D^z\xi\eta^2.\end{aligned} \tag{32}$$

Using the above secular and boundary conditions, the explicit form of $\hat{\phi}_1$ is constructed as given in Eq. (A5) in the appendix. In the similar fashion, the solution for $\hat{\psi}_1$ can be evaluated by solving Eq. (21). The homogeneous part of Eq. (21) admits two particular solutions ψ_{11} and ψ_{12}

$$\psi_{11} = sech\eta(\theta - \theta_0), \tag{33}$$
$$\psi_{12} = \frac{1}{2\eta}[\eta(\theta - \theta_0)sech\eta(\theta - \theta_0) + \sinh\eta(\theta - \theta_0)]. \tag{34}$$

Knowing the two particular solutions, the general solution for $\hat{\psi}_1$ can be obtained through using the formula

$$\hat{\psi}_1 = \delta_3\psi_{11} + \delta_4\psi_{12} - \psi_{11}\int_{-\infty}^{\theta}\psi_{12}\Im F_1 d\theta' + \psi_{12}\int_{-\infty}^{\theta}\psi_{11}\Im F_1 d\theta', \tag{35}$$

where δ_3 and δ_4 are the arbitrary constants. Using the two particular solutions in $\hat{\psi}_1$ and after evaluating the integrals, we obtain the general solution for $\hat{\psi}_1$ as given in Eq. (A6) in the appendix. In order to remove the secularity terms which makes the solution unbounded, we choose

$$\delta_4 = 0. \tag{36}$$

On using the boundary conditions

$$\hat{\psi}_1(0)|_{\theta_0=0} = 0\ ;\ \hat{\psi}_{1\theta}(0)|_{\theta_0=0} = 0, \tag{37}$$

we get

$$\delta_3 = -\frac{1}{6}(2\beta_1\eta - \beta_2\eta). \tag{38}$$

Using the above secular and boundary conditions, we write the final form of solution for $\hat{\psi}_1$ as presented in Eq. (A7) in the appendix. Having obtained the explicit form of $\hat{\phi}_1$ and $\hat{\psi}_1$, the first order perturbed soliton $\hat{u}_1$ can be constructed through the relation $\hat{u}_1 = \hat{\phi}_1 + i\hat{\psi}_1$. The perturbed one soliton solution $\hat{u}$ can be written down as

$$\hat{u} = \eta sech\eta\Theta + \lambda\Big[\big[-\frac{2}{3}\beta_1\xi - \frac{1}{2}\alpha_4 D^z\xi + \frac{1}{12}\alpha_5 D^z\xi - \frac{1}{4}(-2\alpha_4 D^z + \frac{1}{3}\alpha_5 D^z + \frac{4}{3}\beta_1)\xi\eta^2\Theta^2$$
$$+\frac{1}{2}(3\alpha_1\xi - \frac{3}{2}\beta_4 D^z\xi\eta^2 + \frac{1}{4}\beta_5 D^z\xi\eta^2)\Theta - \frac{1}{\eta}\alpha_1\xi\sum_{k=1}^{\infty}\frac{2^{2k}(2^{2k}-1)B_{2k}\tau^{2k+1}}{(2k+1)(2k)!}\big] sech\eta\Theta\tanh\eta\Theta$$
$$+2\alpha_1\xi\Theta sech^3\eta\Theta\tanh\eta\Theta + \big[\frac{1}{2}(-\frac{1}{2}\alpha_4 D^z + \frac{1}{12}\alpha_5 D^z + \frac{10}{3}\beta_1)\xi\eta\Theta - \frac{1}{2}\beta_1\xi$$
$$+\frac{1}{\eta}(\frac{1}{2}\alpha_1\xi - \beta_3 D^z\xi)\big] sech\eta\Theta - \big[\frac{\alpha_1\xi}{2\eta} + 4\beta_1\eta\xi\Theta - \frac{1}{2}\beta_1\xi\big] sech^3\eta\Theta + 2\beta_1\xi\eta\Theta sech^5\eta\Theta$$
$$+\big[\alpha_1\xi\Theta\tanh\eta\Theta - \frac{2}{3}\frac{\alpha_1\xi}{\eta}\big]\ln\cosh\eta\Theta sech\eta\Theta + i\Big\{\big[-\frac{1}{6}(2\beta_1 - \beta_2)\eta + \frac{1}{24}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta)$$
$$-\frac{1}{2}(\eta(\theta-\theta_0)_T - \beta_3 D^z\eta + \beta_4 D^z(3\eta\xi^2 - \eta^3))\Theta - \frac{1}{4}((\frac{4}{3}\beta_1 - \frac{2}{3}\beta_2 - 2\alpha_4 D^z + \frac{1}{3}D^z\alpha_5)\eta^3 - 2\beta_1\eta\xi^2)\Theta^2$$
$$+\big\{\frac{1}{2\eta^2}\alpha_1(\eta^2 - \xi^2) - \frac{2}{3}(2\alpha_1 - \alpha_2)\big\}\sum_{k=1}^{\infty}\frac{2^{2k}(2^{2k}-1)B_{2k}\tau^{2k+1}}{(2k+1)(2k)!}\big] sech\eta\Theta$$
$$-\big[\frac{1}{24}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta) - \frac{1}{6}(2\alpha_1 - \alpha_2)\eta\Theta + \frac{1}{6}(2\alpha_1 - \alpha_2)\big] sech^3\eta\Theta$$
$$-\big[\frac{1}{4}(6\alpha_4 D^z\eta^2 - \alpha_5 D^z\eta^2)\Theta + \frac{1}{4}(6\beta_4 D^z\eta^2 - \beta_5 D^z\eta^2)\big] sech\eta\Theta\tanh\eta\Theta$$
$$+\big[\frac{\alpha_1}{2\eta}(\eta^2 - \xi^2)\Theta - \frac{1}{3}(2\alpha_1\eta - \alpha_2\eta)\Theta + \frac{1}{3}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta)$$
$$+\frac{1}{3}(2\beta_1\eta - \beta_2\eta)\big]\ln\cosh\eta\Theta sech\eta\Theta\Big\}\Big]. \tag{39}$$

4 Controlled magnetization reversal process

In this section, we try to demonstrate that the inhomogeneity present in the D-M interaction can be a good candidate for activating magnetization or spin reversal process in a weak ferromagnet. We attempt to solve the evolution equations for soliton parameters numerically using fourth order Runge-Kutta method coupled with a time domain integration scheme and investigate the magnetization dynamics under the influence of weak ferromagnetic exchange interaction. The standard fourth order Runge-Kutta method propagates a solution over the interval by combining the information from a number of Euler-style steps. This method treats every step as sequence of identical sub-steps. We have plotted Eqs. (25-26) after numerically solving for the amplitude $\eta(T)$ and the velocity $\xi(T)$ of the soliton by choosing the initial value of $\eta(T) = \eta(0) = 0.5$ and $\xi(T) = \xi(0) = 0.3$ with a step size $h = 0.1$. In Figs. (1), we have plotted the amplitude and the velocity of the soliton when the D-M interaction is absent for the chosen parametric values $\eta_0 = 0.5$, $\xi_0 = 0.3$, $\alpha_1 = 0.05$ and $\alpha_2 = 0.01$. In the Figs. (1), it is depicted that the amplitude of the soliton increases and the velocity of the soliton decreases until it reaches the negative maximum. This implies that when the amplitude of the soliton grows infinitely the soliton moves in backward direction in the absence of weak ferromagnetic interaction. However in the presence of D-M interaction, from the Figs. (2) for $D^z = 0.34$, we observe that as time passes on the velocity and amplitude of soliton increase and when reaching a maximum value suddenly flip leading to magnetization reversal, moves in opposite direction. The soliton would have exploded had it not flipped and reversed when it moved with high speed and fast growing amplitude.

When we increase the value of D-M interaction i.e $D^z = 1.3$, the time interval between two successive reversal decreases and the soliton starts to switch more fastly but with the reduced maxima of the amplitude and velocity, as represented in Figs. (3). Surprisingly by tuning the D-M interaction parameter we can control the magnetization reversal such that the soliton is forced to move with constant amplitude and velocity. Further one can also observe that at a particular point T=85ns, more dramatically when the amplitude of the soliton is constant, it is evident that the soliton is also moving with the constant velocity from that point without exhibiting any trace of the reversal in the ferromagnetic medium, as depicted in Figs. (4). Upon further tuning of the D-M interaction parameter one can also induce switching with equal positive and negative maxima of the amplitude and velocity of the soliton. In Figs. (5), the amplitude of the soliton infact oscillates smoothly and it is also observed that when the soliton amplitude changes from positive (negative) to negative (positive), it suddenly moves either forward or backward periodically infinitely, thus establishing sustained switching.

More interestingly by changing the parameters associated with inhomogeneity and D-M interaction parameter we observe that the controlled soliton switching is established for the initial values $\eta(T) = \eta(0) = 1.3$ and $\xi(T) = \xi(0) = 0.5$ in which the amplitude of the soliton starts to oscillates and it switched to negative direction. After the equilibrium state is attained (switched state) in the negative scale, the soliton moves with constant velocity and amplitude along the spin lattice as depicted in Figs. (6). By keeping the values associated with the inhomogeneity and D-M interaction parameter, and by tuning the initial values $\eta(T) = \eta(0) = 1.3$ and $\xi(T) = \xi(0) = 0.6$, the soliton is switched to new equilibrium state in the positive direction. After switching is established, the soliton moves with constant velocity and amplitude along the spin lattice as shown in Figs. (7). Thus the initial values and D-M interaction parameter play an important role in controlling the switching of soliton.

5 Conclusion

We investigate the nonlinear dynamics of an anisotropic site-dependent ferromagnetic spin chain with Dzyaloshinskii-Moriya interaction in the semi-classical limit which is governed by a generalized inhomogeneous nonlinear Schrödinger equation through Holstein-Primakoff transformation. The effect of Dzyaloshinskii-Moriya interaction on the magnetic spin soliton of an anisotropic site-dependent ferromagnet was understood by carrying out a multiple scale perturbation analysis. We have demonstrated the magnetization reversal through flipping of soliton in the ferromagnetic medium by solving the two coupled evolution equations for the velocity and amplitude of the soliton through the fourth order Runge-Kutta method numerically. Though the spin system admits soliton when the inhomogeneity varies in a linear fashion, the flipping speed and the reversal amplitude of the soliton is controlled by the Dzyaloshinskii-Moriya interaction. This controlled lossless switching of solitons in the ferromagnetic media may find immense applications in the magnetic data-storage industry.

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Appendix

The Hamiltonian can be written in terms of bosonic operators as

$$
\begin{aligned}
H = -\frac{1}{2}\sum\Big[&J_1 f_n\big[2\epsilon^2(a_n a_{n+1}^\dagger + a_n^\dagger a_{n+1}) - \frac{\epsilon^4}{2}(a_n a_{n+1}^{\dagger 2} a_{n+1} + a_n^\dagger a_n^2 a_{n+1}^\dagger \\
&+ a_n^\dagger a_{n+1}^\dagger a_{n+1}^2 + a_n^{\dagger 2} a_n a_{n+1}) + \frac{\epsilon^6}{16}(2a_n^\dagger a_n^2 a_{n+1}^{\dagger 2} a_{n+1} - a_n a_{n+1}^{\dagger 2} a_{n+1} a_{n+1}^\dagger a_{n+1} \\
&- a_n^\dagger a_n a_n^\dagger a_n^2 a_{n+1}^\dagger - a_n^\dagger a_{n+1}^\dagger a_{n+1} a_{n+1}^\dagger a_{n+1}^2 + 2a_n^{\dagger 2} a_n a_{n+1}^\dagger a_{n+1}^2 \\
&- a_n^{\dagger 2} a_n a_n^\dagger a_n a_{n+1})\big] + 2J_2 f_n\big[1 - \epsilon^2 a_{n+1}^\dagger a_{n+1} - \epsilon^2 a_n^\dagger a_n + \epsilon^4 a_n^\dagger a_n a_{n+1}^\dagger a_{n+1})\big] \\
&- iJ_3 g_n\Big\{\sqrt{2}D^+\big[\epsilon(a_{n+1}^\dagger - a_n^\dagger) - \frac{\epsilon^3}{4}(a_{n+1}^{\dagger 2} a_{n+1} + 4a_{n+1}^\dagger a_n^\dagger a_n - a_n^{\dagger 2} a_n \\
&- 4a_n^\dagger a_{n+1}^\dagger a_{n+1}) - \frac{\epsilon^5}{32}(a_{n+1}^{\dagger 2} a_{n+1} a_{n+1}^\dagger a_{n+1} - 8a_{n+1}^{\dagger 2} a_{n+1} a_n^\dagger a_n \\
&- a_n^{\dagger 2} a_n a_n^\dagger a_n + 8a_n^{\dagger 2} a_n a_{n+1}^\dagger a_{n+1})\big] + \sqrt{2}D^-\big[\epsilon(a_n - a_{n+1}) \\
&- \frac{\epsilon^3}{4}(a_n^\dagger a_n^2 + 4a_n a_{n+1}^\dagger a_{n+1} - a_{n+1}^\dagger a_{n+1}^2 - 4a_{n+1} a_n^\dagger a_n) \\
&- \frac{\epsilon^5}{32}(a_n^\dagger a_n a_n^\dagger a_n^2 - 8a_n^\dagger a_n^2 a_{n+1}^\dagger a_{n+1} - a_{n+1}^\dagger a_{n+1} a_{n+1}^\dagger a_{n+1}^2 \\
&+ 8a_{n+1}^\dagger a_{n+1}^2 a_n^\dagger a_n)\big] + D^z\big[2\epsilon^2(a_n^\dagger a_{n+1} - a_n a_{n+1}^\dagger) + \frac{\epsilon^4}{2}(a_n^\dagger a_n a_n a_{n+1}^\dagger \\
&+ a_n a_{n+1}^{\dagger 2} a_{n+1} - a_n^{\dagger 2} a_n a_{n+1} - a_n^\dagger a_{n+1}^\dagger a_{n+1}^2) + \frac{\epsilon^6}{16}(2a_n^{\dagger 2} a_n a_{n+1}^\dagger a_{n+1}^2 \\
&- a_n^\dagger a_{n+1}^\dagger a_{n+1} a_{n+1}^\dagger a_{n+1}^2 - a_n^{\dagger 2} a_n a_n^\dagger a_n a_{n+1} + a_n a_{n+1}^{\dagger 2} a_{n+1} a_{n+1}^\dagger a_{n+1} \\
&- 2a_n^\dagger a_n a_n a_{n+1}^{\dagger 2} a_{n+1} + a_n^\dagger a_n a_n^\dagger a_n a_n a_{n+1}^\dagger)\big]\Big\} - 2A(1 - 2\epsilon^2 a_n^\dagger a_n \\
&+ \epsilon^4 a_n^\dagger a_n a_n^\dagger a_n) - 2A'(1 - 4\epsilon^2 a_n^\dagger a_n + 6\epsilon^4 a_n^\dagger a_n a_n^\dagger a_n \\
&- 4\epsilon^6 a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n)\Big]. \qquad (A1)
\end{aligned}
$$

The discrete equation of motion is given by

$$
\begin{aligned}
&iu_t = J_1\Big[2\epsilon^2(f_n u_{n+1} + f_{n-1}u_{n-1}) - \frac{\epsilon^4}{2}\big[2|u_n|^2(f_n u_{n+1} + f_{n-1}u_{n-1})\\
&+u_n^2(f_n u_{n+1}^* + f_{n-1}u_{n-1}^*) + f_n u_{n+1}^2 u_{n+1}^* + f_{n-1}u_{n-1}^2 u_{n-1}^*\big]\Big]\\
&+J_2\Big[-\epsilon^2(f_n + f_{n+1})u_n + \epsilon^4(f_n|u_{n+1}|^2 + f_{n-1}|u_{n-1}|^2)\Big]\\
&+iJ_3\Big\{\sqrt{2}D^+\Big[\epsilon(g_n - g_{n-1}) - \frac{\epsilon^3}{2}\big[(g_n - g_{n-1})|u_n|^2 + 2(g_n|u_{n+1}|^2\\
&-g_{n-1}|u_{n-1}|^2) - 2u_n(g_n u_{n+1}^* - g_{n-1}u_{n-1}^*)\big]\Big] + \sqrt{2}D^-\Big[\frac{\epsilon^3}{4}\big[(g_n\\
&-g_{n-1})u_n^2 - 4u_n(g_n u_{n+1} - g_{n-1}u_{n-1})\big]\Big] - D^z\Big[2\epsilon^2(g_n u_{n+1}\\
&-g_{n-1}u_{n-1}) + \frac{\epsilon^4}{2}\big[u_n^2(g_n u_{n+1}^* - g_{n-1}u_{n-1}^*) - (g_n u_{n+1}|u_{n+1}|^2\\
&-g_{n-1}u_{n-1}|u_{n-1}|^2) - 2|u_n|^2(g_n u_{n+1} - g_{n-1}u_{n-1})\big]\Big]\Big\}\\
&+4A\big[\epsilon^2 u_n - \epsilon^4|u_n|^2 u_n\big] - 8A'\big[-\epsilon^2 u_n + 3\epsilon^4|u_n|^2 u_n\\
&-3\epsilon^6|u_n|^4 u_n\big]. \qquad (A2)
\end{aligned}
$$

The generalized higher order inhomogeneous nonlinear Schrödinger equation is given by

$$
\begin{aligned}
&iu_t - \epsilon^2\Big[[2J_1(\gamma^2 f_x - \frac{\gamma^3}{2}f_{xx}) - 2iJ_3D^z(2\gamma g - \gamma^2 g_x + \frac{\gamma^3}{2}g_{xx})]u_x\\
&+[2J_1\gamma^2 f - \gamma^3 J_1 f_x - iJ_3\gamma^3 g_x D^z]u_{xx} - \frac{2}{3}iJ_3\gamma^3 g D^z u_{xxx}\\
&+iJ_3\epsilon\Big\{-\frac{\sqrt{2}D^+}{2}(\gamma g_x - \frac{\gamma^2}{2}g_{xx})|u|^2 - \frac{3\sqrt{2}D^-}{4}(\gamma g_x - \frac{\gamma^2}{2}g_{xx})u^2\\
&-2\sqrt{2}D^+(\gamma g - \frac{\gamma^2}{2}g_x)|u|_x^2 + 2\sqrt{2}(\gamma g - \frac{\gamma^2}{2}g_x)(D^+ u u_x^*\\
&-D^- u u_x)\Big\} - \epsilon^2\big[(4J_1(f - \frac{\gamma}{2}f_x) - 4J_2(f - \frac{\gamma}{2}f_x)\\
&-2iJ_3\gamma g_x D^z + 4A + 24A')|u|^2 u - 4iJ_3\gamma g D^z|u|^2 u_x\big]\Big] = 0. \qquad (A3)
\end{aligned}
$$

The general solution for $\hat{\phi}_1$ is given by

$$
\begin{aligned}
\hat{\phi}_1 = & \big[\delta_1 + \frac{3}{2}\delta_2\Theta - \frac{3}{4\eta^2}(\xi_T - \alpha_3 D^z\xi - \alpha_4 D^z(3\xi\eta^2 - \xi^3)) \\
& -\frac{5}{24}D^z(6\alpha_4 - \alpha_5)\xi - \frac{1}{4}(\xi_T - \alpha_3 D^z\xi - \alpha_4 D^z(3\xi\eta^2 - \xi^3))\Theta^2 \\
& +\frac{1}{2}(\xi\theta_{0T} + \sigma_{0T} + \beta_3 D^z\xi + \beta_4 D^z(3\xi\eta^2 - \xi^3))\Theta + \frac{4}{3}\alpha_1\xi\Theta \\
& -\frac{4}{3}\beta_1\xi - \frac{1}{\eta}\alpha_1\xi\sum_{k=1}^{\infty}\frac{2^{2k}(2^{2k}-1)B_{2k}\tau^{2k+1}}{(2k+1)(2k)!}\big] sech\eta\Theta\tanh\eta\Theta \\
& +2\alpha_1\xi\Theta sech^3\eta\Theta\tanh\eta\Theta + \big[\frac{1}{6\eta}\alpha_1\xi - \frac{\delta_2}{\eta} - \frac{1}{2}\beta_1\xi + \frac{1}{2\eta}(\xi_T \\
& -\alpha_3 D^z\xi - \alpha_4 D^z(3\xi\eta^2 - \xi^3))\Theta + \frac{1}{\eta}(\xi\theta_{0T} + \sigma_{0T} + \beta_4 D^z(3\xi\eta^2 - \xi^3)) \\
& +\frac{1}{4}(6\alpha_4 D^z\eta\xi - \alpha_5 D^z\eta\xi)\Theta + \frac{1}{4}(6\beta_4 D^z\xi\eta - \beta_5 D^z\xi\eta) \\
& +2\beta_1\xi\eta\Theta\big] sech\eta\Theta - \big[\frac{1}{2\eta}\alpha_1\xi + 4\beta_1\xi\eta\Theta - \frac{1}{2}\beta_1\xi\big] sech^3\eta\Theta \\
& +2\beta_1\eta\xi\Theta sech^5\eta\Theta + \big[\frac{1}{4\eta^2}(\xi_T - \alpha_3 D^z\xi - \alpha_4 D^z(3\xi\eta^2 - \xi^3)) \\
& +\frac{1}{12}(6\alpha_4 D^z\xi - \alpha_5 D^z\xi) - \frac{1}{3}\alpha_1\xi\Theta + \frac{1}{3}\beta_1\xi\big]\sinh\eta\Theta \\
& +\frac{\delta_2}{2\eta}\tanh\eta\Theta\sinh\eta\Theta + \big[(-\frac{2}{3\eta}\alpha_1\xi + \alpha_1\xi\Theta\tanh\eta\Theta)sech\eta\Theta \\
& +\frac{1}{3\eta}\alpha_1\xi\sinh\eta\Theta\tanh\eta\Theta\big]\ln\cosh\eta\Theta. \qquad (A4)
\end{aligned}
$$

The explicit form of $\hat{\phi}_1$ is

$$
\begin{aligned}
\hat{\phi}_1 = & [-\frac{2}{3}\beta_1\xi - \frac{1}{2}\alpha_4 D^z\xi + \frac{1}{12}\alpha_5 D^z\xi - \frac{1}{4}(-2\alpha_4 D^z + \frac{1}{3}\alpha_5 D^z + \frac{4}{3}\beta_1)\xi\eta^2\Theta^2 \\
& +\frac{1}{2}(3\alpha_1\xi - \frac{3}{2}\beta_4 D^z\xi\eta^2 + \frac{1}{4}\beta_5 D^z\xi\eta^2)\Theta - \frac{1}{\eta}\alpha_1\xi\sum_{k=1}^{\infty}\frac{2^{2k}(2^{2k}-1)B_{2k}\tau^{2k+1}}{(2k+1)(2k)!}] sech\eta\Theta\tanh\eta\Theta \\
& +2\alpha_1\xi\Theta sech^3\eta\Theta\tanh\eta\Theta + [\frac{1}{2}(-\frac{1}{2}\alpha_4 D^z + \frac{1}{12}\alpha_5 D^z + \frac{10}{3}\beta_1)\xi\eta\Theta - \frac{1}{2}\beta_1\xi + \frac{1}{\eta}(\frac{1}{2}\alpha_1\xi - \beta_3 D^z\xi)] sech\eta\Theta \\
& -[\frac{\alpha_1\xi}{2\eta} + 4\beta_1\eta\xi\Theta - \frac{1}{2}\beta_1\xi] sech^3\eta\Theta + 2\beta_1\xi\eta\Theta sech^5\eta\Theta \\
& +[\alpha_1\xi\Theta\tanh\eta\Theta - \frac{2}{3}\frac{\alpha_1\xi}{\eta}]\ln\cosh\eta\Theta sech\eta\Theta, \qquad (A5)
\end{aligned}
$$

where $\Theta = \theta - \theta_0$ and B_{2k} represents the Bernoulli number. The general solution for $\hat{\psi}_1$ is given by

$$\hat{\psi}_1 = -[\frac{1}{4}(6\alpha_4 D^z\eta^2 - \alpha_5 D^z\eta^2)\Theta + \frac{1}{4}(6\beta_4 D^z\eta^2 - \beta_5 D^z\eta^2)] sech\eta\Theta \tanh \eta\Theta$$
$$+[\frac{1}{6}(2\alpha_1 - \alpha_2) + \frac{1}{6}(2\beta_1\eta - \beta_2\eta) + \frac{1}{24}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta)$$
$$-\frac{1}{4}(\eta_T - \alpha_3 D^z\eta + \alpha_4 D^z(3\xi^2\eta - \eta^3))\Theta^2 - \frac{1}{2}(\eta(\theta - \theta_0)_T - \beta_3 D^z\eta$$
$$+\beta_4 D^z(3\eta\xi^2 - \eta^3))\Theta + \delta_3 + \frac{\delta_4}{2}\Theta] sech\eta\Theta$$
$$-[\frac{1}{24}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta) - \frac{1}{6}(2\alpha_1\eta - \alpha_2\eta)\Theta + \frac{1}{6}(2\alpha_1 - \alpha_2)] sech^3\eta\Theta$$
$$+[\frac{1}{2\eta}\alpha_1(\eta^2 - \xi^2)\Theta - \frac{1}{3}(2\alpha_1\eta - \alpha_2\eta)\Theta + \frac{1}{3}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta)$$
$$+\frac{1}{3}(2\beta_1\eta - \beta_2\eta)] \ln\cosh\eta\Theta sech\eta\Theta$$
$$+[\frac{1}{2\eta^2}\alpha_1(\eta^2 - \xi^2) - \frac{2}{3}(2\alpha_1 - \alpha_2)] sech\eta\Theta \sum_{k=1}^{\infty} \frac{2^{2k}(2^{2k}-1)B_{2k}\tau^{2k+1}}{(2k+1)(2k)!}$$
$$+\Big[\frac{\alpha_4}{2\eta} + [\frac{1}{4\eta^2}(\eta_T - \alpha_3 D^z\eta + \alpha_4 D^z(3\xi^2\eta - \eta^3)) - \frac{1}{2\eta^2}(\eta_T + \beta_1(\eta^3 - \eta\xi^2))$$
$$-\frac{1}{2\eta}\alpha_1(\eta^2 - \xi^2)\Theta + \frac{1}{3}(2\alpha_1\eta - \alpha_2\eta)\Theta + \frac{1}{3}(2\beta_1\eta - \beta_2\eta)$$
$$+\frac{1}{12}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta)] \tanh\eta\Theta\Big] \sinh\eta\Theta$$
$$+[\frac{1}{2\eta^2}\alpha_1(\eta^2 - \xi^2) - \frac{1}{3}(2\alpha_1 - \alpha_2)] \ln\cosh\eta\Theta \sinh\eta\Theta. \qquad (A6)$$

The explicit form of $\hat{\psi}_1$ is

$$\hat{\psi}_1 = [-\frac{1}{6}(2\beta_1 - \beta_2)\eta + \frac{1}{24}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta) - \frac{1}{2}(\eta(\theta - \theta_0)_T - \beta_3 D^z\eta$$
$$+\beta_4 D^z(3\eta\xi^2 - \eta^3))\Theta - \frac{1}{4}((\frac{4}{3}\beta_1 - \frac{2}{3}\beta_2 - 2\alpha_4 D^z + \frac{1}{3}D^z\alpha_5)\eta^3 - 2\beta_1\eta\xi^2)\Theta^2$$
$$+\{\frac{1}{2\eta^2}\alpha_1(\eta^2 - \xi^2) - \frac{2}{3}(2\alpha_1 - \alpha_2)\} \sum_{k=1}^{\infty} \frac{2^{2k}(2^{2k}-1)B_{2k}\tau^{2k+1}}{(2k+1)(2k)!}] sech\eta\Theta$$
$$-[\frac{1}{24}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta) - \frac{1}{6}(2\alpha_1 - \alpha_2)\eta\Theta$$
$$+\frac{1}{6}(2\alpha_1 - \alpha_2)] sech^3\eta\Theta - [\frac{1}{4}(6\alpha_4 D^z\eta^2 - \alpha_5 D^z\eta^2)\Theta$$
$$+\frac{1}{4}(6\beta_4 D^z\eta^2 - \beta_5 D^z\eta^2)] \times sech\eta\Theta \tanh\eta\Theta + [\frac{\alpha_1}{2\eta}(\eta^2 - \xi^2)\Theta$$
$$-\frac{1}{3}(2\alpha_1\eta - \alpha_2\eta)\Theta + \frac{1}{3}(6\alpha_4 D^z\eta - \alpha_5 D^z\eta)$$
$$+\frac{1}{3}(2\beta_1\eta - \beta_2\eta)] \ln\cosh\eta\Theta sech\eta\Theta. \qquad (A7)$$

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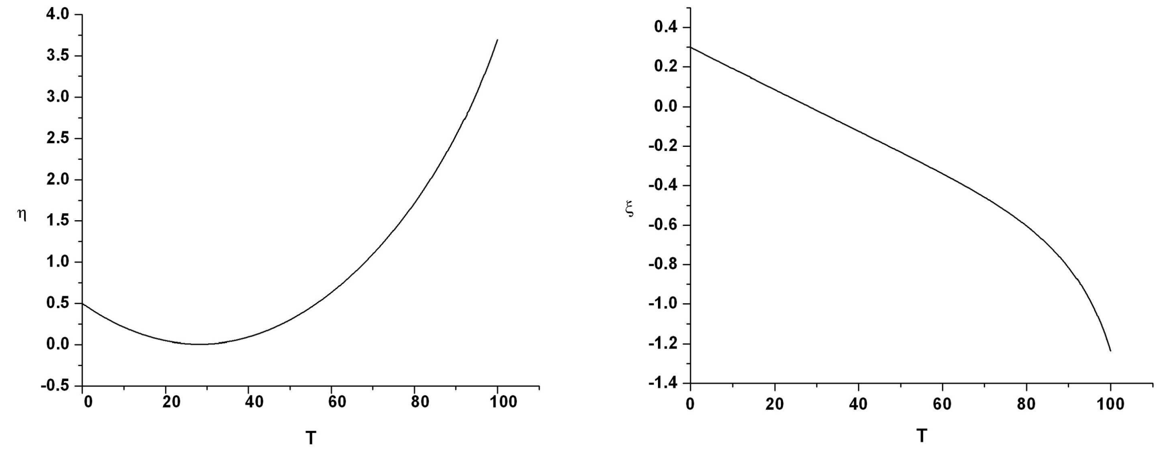

Figure 1: Evolution of soliton parameters with $\eta(0) = 0.5$, $\xi(0) = 0.3$, $\alpha_1 = 0.05$, $\alpha_2 = 0.01$ and $D^z = 0$.

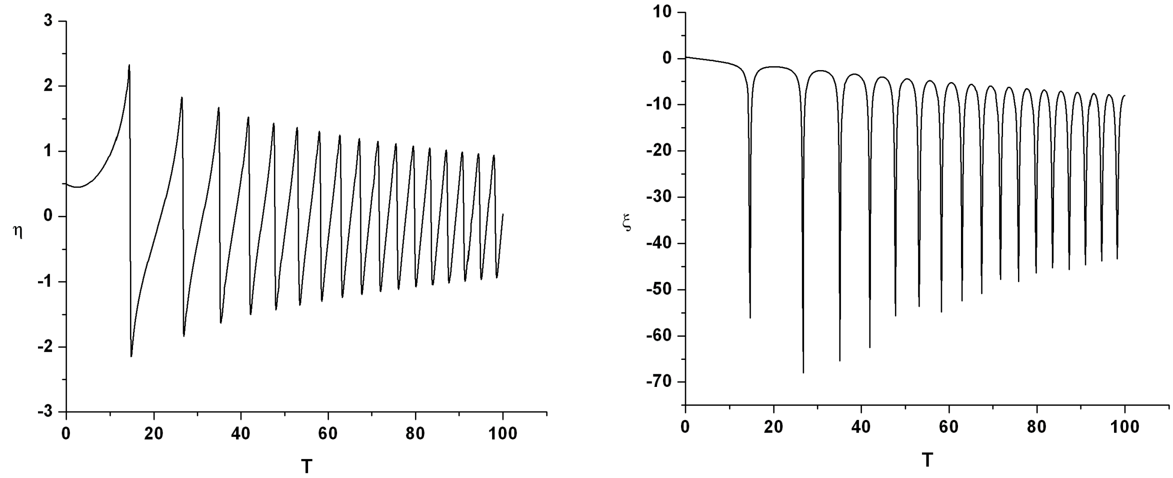

Figure 2: Evolution of soliton parameters with $\eta(0) = 0.5$, $\xi(0) = 0.3$, $\alpha_1 = 0.005$, $\alpha_2 = 0.01$, $\alpha_3 = 0.1$, $\alpha_4 = 0.05$, $\alpha_5 = 0.07$ and $D^z = 0.34$.

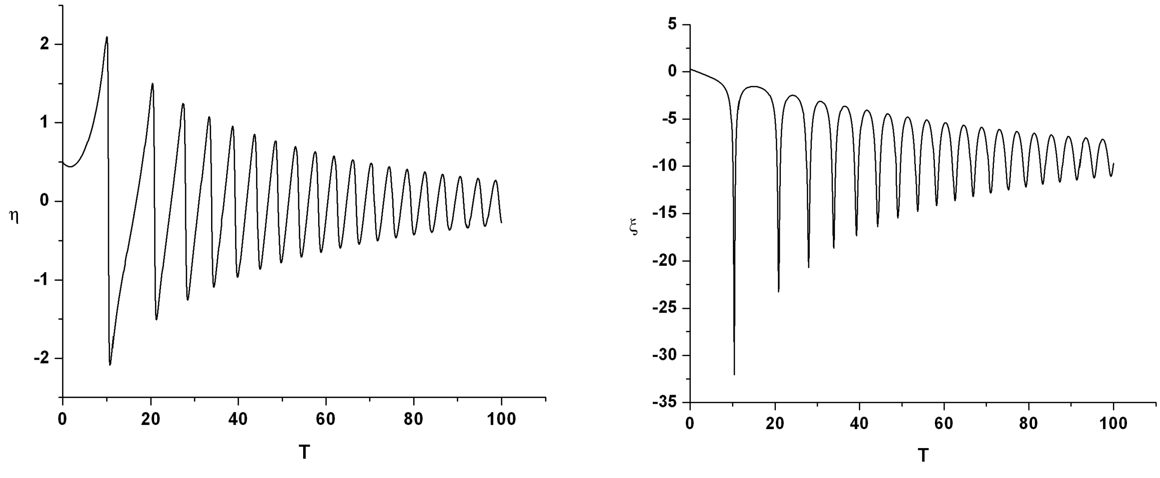

Figure 3: Evolution of soliton parameters with $\eta(0) = 0.5$, $\xi(0) = 0.3$, $\alpha_1 = 0.005$, $\alpha_2 = 0.01$, $\alpha_3 = 0.1$, $\alpha_4 = 0.05$, $\alpha_5 = 0.07$ and $D^z = 1.3$.

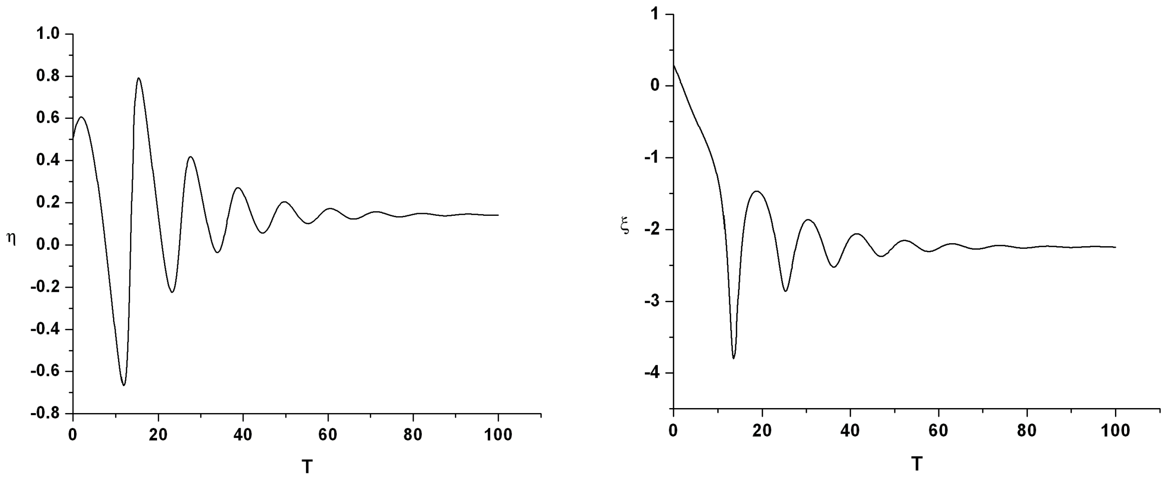

Figure 4: Evolution of soliton parameters with $\eta(0) = 0.5$, $\xi(0) = 0.3$, $\alpha_1 = 0.005$, $\alpha_2 = 0.02$, $\alpha_3 = 0.1$, $\alpha_4 = 0.005$, $\alpha_5 = 0.1$ and $D^z = -3.69$.

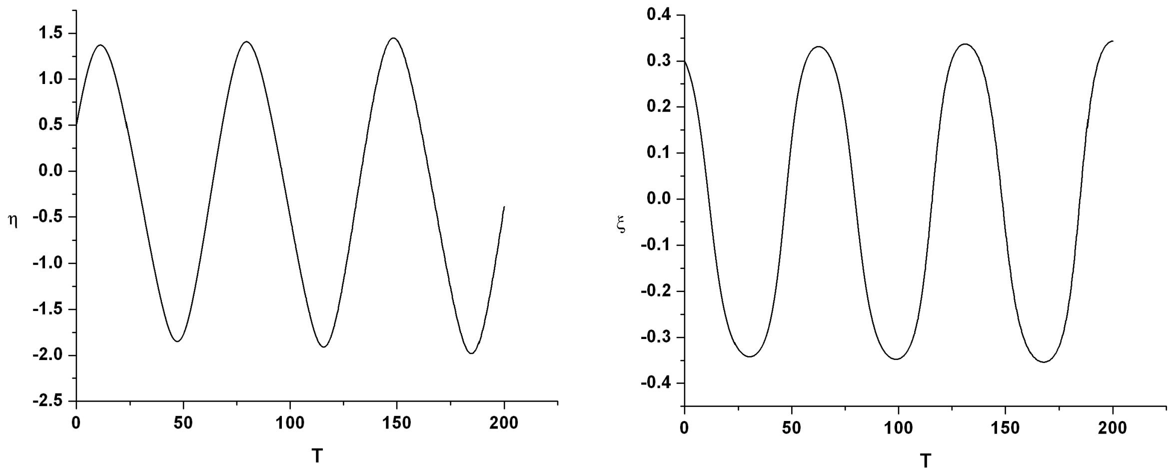

Figure 5: Evolution of soliton parameters with $\eta(0) = 0.5$, $\xi(0) = 0.3$, $\alpha_1 = 0.005$, $\alpha_2 = 0.01$, $\alpha_3 = 0.1$, $\alpha_4 = 0.005$, $\alpha_5 = 0.1$ and $D^z = -5.5$.

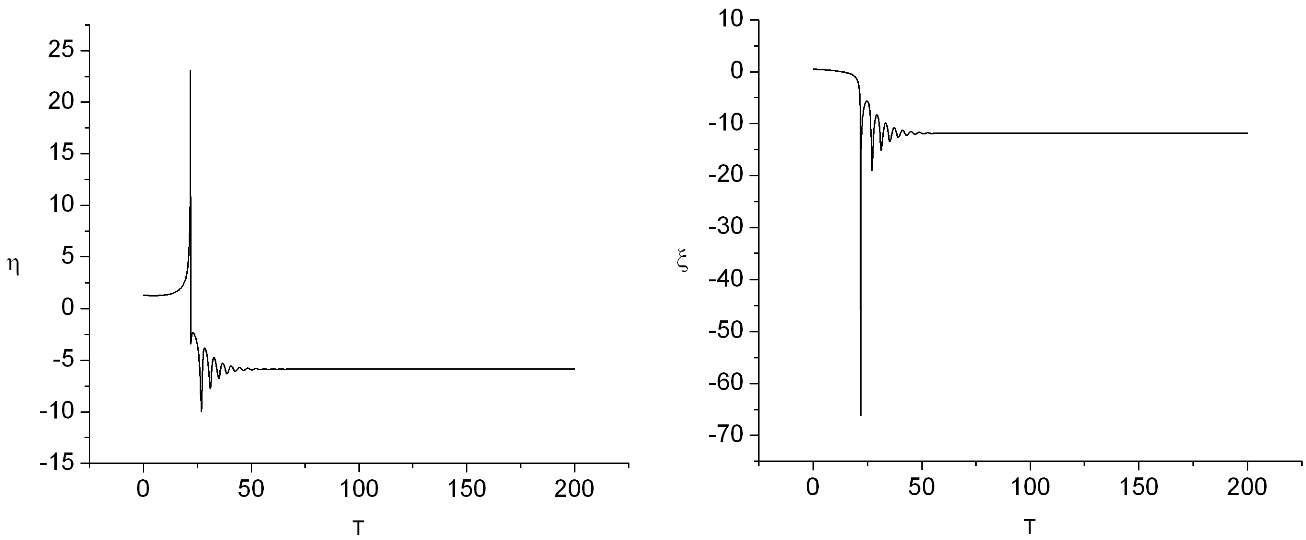

Figure 6: Evolution of soliton parameters with $\eta(0) = 1.3$, $\xi(0) = 0.5$, $\alpha_1 = 0.042$, $\alpha_2 = 0.075$, $\alpha_3 = 0.05$, $\alpha_4 = 0.1$, $\alpha_5 = 1.8$ and $D^z = 0.04$.

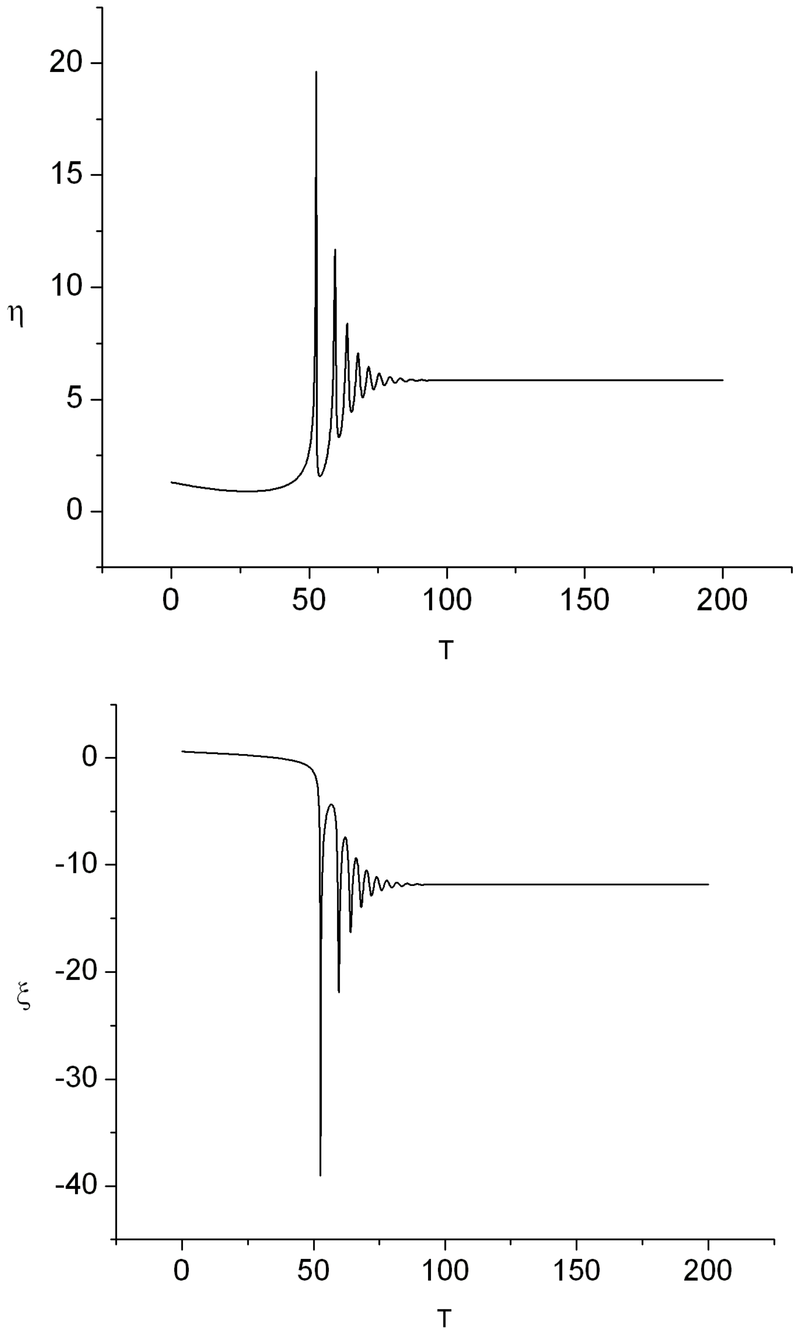

Figure 7: Evolution of soliton parameters with $\eta(0) = 1.3$, $\xi(0) = 0.6$, $\alpha_1 = 0.042$, $\alpha_2 = 0.075$, $\alpha_3 = 0.05$, $\alpha_4 = 0.1$, $\alpha_5 = 1.8$ and $D^z = 0.04$.