

United Nations Educational, Scientific and Cultural Organization
and
International Atomic Energy Agency
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

**SUPERSYMMETRIC MUSINGS ON THE PREDICTIVITY
OF FAMILY SYMMETRIES**

Kenji Kadota¹

*Michigan Center for Theoretical Physics, University of Michigan,
Ann Arbor, MI, 48104, USA,*

Jörn Kersten²

*University of Hamburg, II. Institute for Theoretical Physics,
Luruper Chaussee 149, 22761 Hamburg, Germany*

and

Liliana Velasco-Sevilla³

The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

We discuss the predictivity of family symmetries for the soft supersymmetry breaking parameters in the framework of supergravity. We show that unknown details of the messenger sector and the supersymmetry breaking hidden sector enter into the soft parameters, making it difficult to obtain robust predictions. We find that there are specific choices of messenger fields which can improve the predictivity for the soft parameters.

MIRAMARE – TRIESTE

June 2010

¹kadota@umich.edu

²joern.kersten@desy.de

³lvelasco@ictp.it

1 Introduction

One approach towards understanding the observed fermion masses and mixings is employing family symmetries. In a wide class of such models, one assigns the representations of the corresponding group in such a way that the family symmetry forbids Yukawa couplings, while the matter fields couple to a number of flavon (or familon) and vector-like messenger fields. When these heavy messenger fields are integrated out, one obtains an effective theory with non-renormalisable couplings between matter and flavon fields. Then the flavon fields develop the vacuum expectation values (vevs) that break the family symmetry spontaneously. This generates non-renormalisable Yukawa couplings, which are suppressed by a power of the small ratio of flavon vev to messenger mass. As this power varies for different elements of the Yukawa matrices, one can naturally obtain hierarchical fermion masses [1].

In supersymmetric theories, family symmetries also restrict the soft supersymmetry (SUSY) breaking parameters [2,3], provided that the mechanism mediating SUSY breaking to the visible sector operates at a scale where the family symmetry is unbroken. This is the case for gravity-mediated SUSY breaking, for example, where the characteristic scale is the Planck mass M_P . If all matter fields transform under a three-dimensional representation of a family symmetry, only soft scalar mass matrices proportional to the unit matrix are allowed. The trilinear scalar couplings have to vanish like the Yukawa couplings.

The breaking of the family symmetry leads to off-diagonal entries in the soft mass matrices, suppressed by powers of the ratio of flavon vevs to messenger masses. Furthermore, non-zero trilinear couplings are generated, which are not guaranteed to be proportional to the Yukawa couplings. In principle the deviations of the soft parameters from the pattern of the Constrained Minimal Supersymmetric Standard Model (CMSSM) can be calculated within a particular family model, and it has been found that they can be sufficiently small to be compatible with the experimental bounds [3–9]. Therefore, one may expect that the SUSY flavour problem is absent even after family symmetry breaking. If there is a CP symmetry which is spontaneously broken together with the family symmetry, then one can also address the SUSY CP problem [2–7, 10].

Thus, in addition to explaining the fermion masses and mixings, family symmetries could give calculable corrections to the soft SUSY breaking parameters, which would offer additional experimental tests of family symmetries because those soft parameters can be probed experimentally by measuring flavour- and CP-violating observables.

In this work, we discuss to what extent family symmetries can indeed yield robust predictions for the soft parameters. In Sec. 2, after a general discussion of the formalism in the supergravity framework, we argue that the predictivity is severely limited unless the messenger sector and the SUSY-breaking hidden sector are known, as illustrated by a concrete example in Sec. 2.2. After-

wards, Sec. 3 discusses a possibility to gain predictivity by modifying the messenger sector, and Sec. 4 compares the ensuing predictions for a particular model with the experimental constraints.

2 Soft Parameters from Family Symmetries

2.1 General Formalism

Studies of how family symmetries restrict the squared masses and trilinear couplings of supersymmetric fields in the effective supergravity theory have now been brought forward for some time [3–9]. These studies made use of effective potentials restricted only by the symmetries of the models. We stress in this section that understanding the ultra-violet (UV) completion of such effective models, in particular specifying the properties of the messenger fields, is crucial for the assessment of the predictivity for the soft SUSY breaking parameters. We therefore re-enumerate a specific approach which takes into account all the ingredients for the family symmetry breaking and its connection to SUSY breaking in the supergravity context, similar in spirit to a study of Yukawa textures in [14].

We assume that the matter fields F ($F = Q, L$) and f^c ($f = u, d, e, \nu$) transform under a three-dimensional irreducible representation $\mathbf{3}$ of a non-Abelian family symmetry. The flavons $\bar{\phi}$ transform under the conjugate representation $\bar{\mathbf{3}}$. Where necessary, we indicate components by the family indices $i, j = 1, 2, 3$ (e.g. $F = (Q_1, Q_2, Q_3)$ and $Q_1 = (u, d)$ is the usual quark $SU(2)_L$ doublet of the first family). The messengers are denoted by χ and can be either singlets or triplets of the family symmetry.

Explicitly, we take the following steps in order to derive the observable quantities:

1. We start from supergravity with the superpotential

$$W = W_{\text{O}} + W_{\text{H}} . \tag{1}$$

As an example, we take

$$W_{\text{O}} = M_{\chi_0^f} \bar{\chi}_0^f \chi_0^f + M_{\chi_2^f} \bar{\chi}_2^f \chi_2^f + \lambda_H F_i H_f \chi_0^f + \lambda_1 \bar{\chi}_0^f \bar{\phi}_1^f \chi_2^f + \lambda_2 \bar{\chi}_2^f \bar{\phi}_2^f f_i^c + \mu H_u H_d , \tag{2}$$

where λ_H, λ_1 and λ_2 are $\mathcal{O}(1)$ couplings, F, f and i are understood to be summed over and $H_{e,\nu} \equiv H_{d,u}$. In this example, χ_0^f and χ_2^f transform as $\bar{\mathbf{3}}$ and $\mathbf{1}$, respectively, under the family symmetry. The superpotential (2) will lead to some elements of the Yukawa matrices. In order to generate the remaining elements, realistic models contain additional flavons and messengers with couplings and masses analogous to the ones shown. We will ignore this complication for the time being.

On the other hand, W_{H} involves only hidden-sector fields h_m , viz. the flavons and a further field¹ h responsible for the breaking of SUSY by $\langle \mathcal{F}_h \rangle \neq 0$. We assume that the couplings

¹In general, a set of fields.

λ_H , λ_1 and λ_2 are real and do not depend on h . The field h has to be a singlet under the family symmetry, since otherwise the breaking of SUSY, which involves vevs for both h and \mathcal{F}_h in general, would also break the family symmetry.

The Kähler potential has the most general form allowed by gauge and family symmetries,

$$K = \sum_{\alpha} C_{\alpha}^{\dagger} C_{\alpha} + \sum_{n\alpha} \frac{\zeta_n C_{\alpha}^{\text{T}}}{M_{\text{P}}^2} |\bar{\phi}_n C_{\alpha}^{\text{T}}|^2 + \sum_{\alpha} \xi_{C_{\alpha}}(h_m) C_{\alpha}^{\dagger} C_{\alpha} + \left[Z_H(h_m) H_u H_d + \sum_n Z_{\chi_n^f}(h_m) \bar{\chi}_n^f \chi_n^f + \text{h.c.} \right] + \dots + K_{\text{H}}(h_m), \quad (3)$$

where C_{α} is any field from the observable sector, whereas $C^{\text{T}} = (F, f^c, \chi_0^{\dagger}, \bar{\chi}_0^f)$ refers to family triplets only. The functions ξ , Z and K_{H} are family singlets and depend only on the hidden-sector fields h_m . For simplicity, we use the minimal form $K_{\text{H}} = h^{\dagger} h + \sum_n \bar{\phi}_n^{\dagger} \bar{\phi}_n$ in the following, since any different choice would not lead to qualitative changes of the discussion. Finally, ζ are numerical coefficients. Family and gauge indices are not shown explicitly. The dots represent terms that contain more than two fields C_{α} or are suppressed by higher powers of M_{P} . We choose a basis where the leading-order Kähler potential, i.e. the first term in Eq. (3), is minimal.

We assume the hierarchy of scales²

$$\langle \bar{\phi} \rangle \lesssim M_{\chi} \ll M_{\text{P}}, \quad (4)$$

because we

- (a) need sufficiently small Yukawa couplings (with a relatively weak hierarchy between one pair of flavon vevs and messenger masses due to the large y_t), and
- (b) would like to ensure that the Planck-suppressed terms with coefficients ζ in Eq. (3), which give rise to off-diagonal corrections to the soft masses after the breaking of the family symmetry,³ are negligible compared to similar terms that arise after integrating out the messengers and are suppressed by M_{χ}^2 , as we will discuss shortly.

2. We integrate out the messengers by solving the equations $\partial W / \partial \chi = \partial W / \partial \bar{\chi} = 0$ for the messenger fields [15]. In the example of Eq. (2), this yields the effective superpotential (cf. Fig. 1)

$$W_{\text{O}} = \frac{\lambda_H \lambda_1 \lambda_2}{M_{\chi_0^f} M_{\chi_2^f}} F_j \bar{\phi}_{1j} H_f \bar{\phi}_{2i} f_i^c \quad (5)$$

²Here and in the following, χ refers to some messenger. The indices identifying one particular field will be shown only if they are relevant. Likewise, $\bar{\phi}$ will denote some flavon.

³In contrast, the terms involving $\xi(h_m)$ do not lead to flavour violation, since ξ are singlets under the family symmetry.

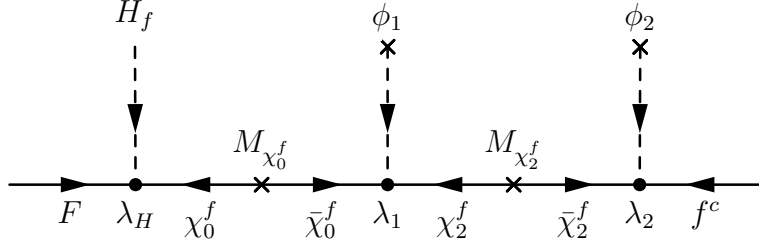


Figure 1: Feynman diagram responsible for the Yukawa couplings. Under the SU(3) family symmetry, $\chi_1^f \sim \bar{\mathbf{3}}$, and $\chi_2^f \sim \mathbf{1}$.

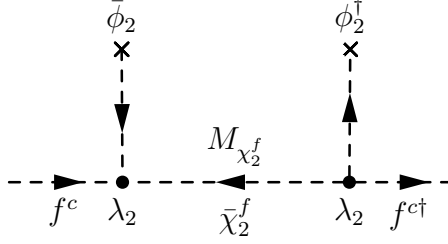


Figure 2: Diagram relevant for the Kähler potential.

and the effective Kähler potential

$$\begin{aligned}
K &= F_i^\dagger F_i \left[1 + \xi_F + \frac{\lambda_H^2 (1 + \xi_{\bar{\chi}_0^f})}{M_{\chi_0^f}^2} H_f^\dagger H_f \right] \\
&+ f_i^c f_j^{c\dagger} \left[\delta_{ij} + \xi_{f^c} \delta_{ij} + \frac{\lambda_2^2 (1 + \xi_{\chi_2^f})}{M_{\chi_2^f}^2} \bar{\phi}_{2i} \bar{\phi}_{2j}^\dagger \right] \\
&+ H_f^\dagger H_f (1 + \xi_{H_f}) + (Z_H H_u H_d + \text{h.c.}) + \dots + K_H \\
&\equiv \tilde{K}_{F_i^\dagger F_j} F_i^\dagger F_j + \tilde{K}_{f_i^c f_j^{c\dagger}} f_i^c f_j^{c\dagger} + \tilde{K}_{H_f^\dagger H_f} H_f^\dagger H_f + (Z_H H_u H_d + \text{h.c.}) + \dots + K_H,
\end{aligned} \tag{6}$$

where we have omitted M_{P} -suppressed terms proportional to ζ and terms suppressed by higher powers of messenger masses. Thus, we obtain non-minimal terms suppressed by messenger masses. This can be visualised by diagrams like the one shown in Fig. 2. Note that in this way the messengers are integrated out already around M_{P} , not only at their own mass scales. This should not be a problem as long as we do not aspire high-precision calculations including the running of parameters between M_{P} and M_χ .

3. From the effective potentials we calculate the scalar potential. It contains $\frac{\partial W_{\mathcal{O}}}{\partial \phi}$, which yields an important contribution to the trilinear scalar couplings. The minimisation of the potential yields vevs for all hidden sector fields and their \mathcal{F} terms, breaking both SUSY and the family symmetry.
4. We take the flat limit, i.e. $M_{\text{P}} \rightarrow \infty$ and $m_{3/2}^2 = \langle e^{K_{\text{H}}/M_{\text{P}}^2} |W_{\text{H}}|^2 \rangle / M_{\text{P}}^4 = \text{const.}$ [16].

This removes the dynamical degree of freedom h from the theory. In contrast, both the flavon vevs $\langle \bar{\phi} \rangle$ and the dynamical fields $\bar{\phi}$ are still present, since they have couplings to the observable sector that are suppressed by M_X rather than M_P . It is only at the scale $\langle \bar{\phi} \rangle < M_P$ that they decouple. Again, this should not be a problem as long as we do not aim to calculate the running of parameters between M_P and $\langle \bar{\phi} \rangle$.

5. We rescale the superpotential of the visible sector,

$$W'_O = W_O \left\langle \frac{W_H^*}{|W_H|} e^{\frac{1}{2M_P^2} \sum_m |h_m|^2} \right\rangle \equiv \mathcal{N} W_O. \quad (7)$$

This is necessary in order to obtain the usual globally supersymmetric contribution $\sum_\alpha |\partial W'_O / \partial C_\alpha|^2$ to the scalar potential. The rescaling is absorbed in the effective Yukawa couplings,

$$Y'_{f_i^c F_j H_f} \equiv \mathcal{N} Y_{f_i^c F_j H_f} \equiv \mathcal{N} \lambda_H \lambda_1 \lambda_2 \frac{\langle \bar{\phi}_2 \rangle_i \langle \bar{\phi}_1 \rangle_j}{M_{\chi_0^f} M_{\chi_2^f}}. \quad (8)$$

$Y_{f_i^c F_j H_f}$ denotes the ij component of the matrix $Y_{\alpha\beta\gamma}$ coupling the fields $C_\alpha = f^c$, $C_\beta = F$ and $C_\gamma = H_f$. Note that the rescaled Yukawa couplings Y' are the ones directly related to observable quantities (up to canonical normalisation) that are determined by the fit to the fermion masses.

6. The scalar potential now consists of the globally supersymmetric part and soft SUSY breaking terms. Assuming that no \mathcal{D} terms contribute to SUSY breaking, we determine the latter using Eqs. (11, 12) of [17], which in our notation become

$$m_{\bar{\alpha}\beta}^{\prime 2} = m_{3/2}^2 \langle \tilde{K}_{\bar{\alpha}\beta} \rangle - \left\langle \mathcal{F}^{*\bar{m}} \left(\partial_{\bar{m}}^* \partial_n \tilde{K}_{\bar{\alpha}\beta} - (\partial_{\bar{m}}^* \tilde{K}_{\bar{\alpha}\gamma}) \tilde{K}^{\gamma\bar{\delta}} \partial_n \tilde{K}_{\bar{\delta}\beta} \right) \mathcal{F}^n \right\rangle, \quad (9a)$$

$$a'_{\alpha\beta\gamma} = \langle \mathcal{F}^m \rangle \left[\left\langle \frac{\partial_m K_H}{M_P^2} \right\rangle Y'_{\alpha\beta\gamma} + \frac{\mathcal{N} \partial Y_{\alpha\beta\gamma}}{\partial \langle h_m \rangle} \right] - \langle \mathcal{F}^m \rangle \left[\left\langle \tilde{K}^{\delta\bar{\rho}} (\partial_m \tilde{K}_{\bar{\rho}\alpha}) \right\rangle Y'_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right], \quad (9b)$$

where $\tilde{K}_{\bar{\alpha}\beta} \equiv \frac{\partial^2 K}{\partial C_{\bar{\alpha}}^\dagger \partial C_\beta}$ with $C = (F, f^{c\dagger}, H_f)$ and where $\tilde{K}^{\gamma\bar{\delta}}$ denotes the elements of the inverse matrix. Besides, $\partial_m \equiv \partial / \partial h_m$, $\partial_m^* \equiv \partial / \partial h_m^*$, and e.g. $\langle \mathcal{F}^{\bar{\phi}_1} \rangle \partial / \partial \bar{\phi}_1 \equiv \langle \mathcal{F}^{\bar{\phi}_{1i}} \rangle \partial / \partial \bar{\phi}_{1i}$. We have expressed the formula for the trilinear couplings in terms of Y' for convenience, where it is possible without ambiguity. Primes denote parameters before canonical normalisation. There are different \mathcal{F} -term vevs associated to each flavon, $\langle \mathcal{F}^{\bar{\phi}_n} \rangle = c_n m_{3/2} \langle \bar{\phi}_n \rangle$ [2, 3], where $c_n \neq c_m$ for $n \neq m$.⁴

⁴Here we use $\langle \mathcal{F}^m \rangle = \langle e^{K/(2M_P^2)} \frac{|W_H|}{M_P^2} \rangle \langle K_H^{m\bar{n}} (K_{\bar{n}} + \frac{W_{\bar{n}}^*}{W^*}) \rangle$. For the flavons $\langle |\mathcal{F}^{\bar{\phi}_n}|^2 \rangle$ behaves as $m_{3/2}^2 c_n^2 \left| \left\langle K_{\bar{\phi}_n} + \frac{W_{\bar{\phi}_n}}{W} \right\rangle \right|^2$, then it is assumed that the term containing $|K_{\bar{\phi}_n}|^2$ is the dominant one. Formally the coefficients c_n should be determined from the process that sets completely the minimum of the scalar potential and so depends on details of how SUSY is broken. However, since the \mathcal{F} terms in general are proportional to $\bar{\phi}_n$ the coefficients c_n are expected to be $\mathcal{O}(1)$.

As mentioned, we are treating the flavons as part of the hidden sector associated to the breaking of SUSY and therefore there are also non-zero vevs for their \mathcal{F} terms, although they are not the main contribution to SUSY breaking, the leading source of course being the family-blind field h . It is also important to note that if there was only one flavon in the theory and thus only one \mathcal{F} term, then we can immediately see from Eqs. (9) that when going to the canonical basis there would be no off-diagonal terms, even with a non-trivial Kähler metric. On the other hand it can be quickly computed [4] that with at least two different flavons and consequently different \mathcal{F} terms, the soft mass matrices have the same structure as the Kähler metric but with different $\mathcal{O}(1)$ coefficients in each component,

$$m_{\tilde{f}_i^c \tilde{f}_j^{c\dagger}}^2 \sim \mathcal{O}(1) m_{3/2}^2 \langle \tilde{K}_{\tilde{f}_i^c \tilde{f}_j^{c\dagger}} \rangle, \quad (10)$$

where of course the precise values of the $\mathcal{O}(1)$ coefficients depend on the details of the Kähler potential and the \mathcal{F} terms.

7. We normalise the visible-sector fields to obtain canonical kinetic terms,

$$F \rightarrow \hat{F} \equiv V_F^{-1} F \quad , \quad f^c \rightarrow \hat{f}^c \equiv f^c V_{f^c}^{-1\dagger} \quad , \quad H_f \rightarrow \hat{H}_f \equiv \tilde{K}_{H_f^\dagger H_f}^{\frac{1}{2}} H_f \quad , \quad (11)$$

where the (non-unitary) matrices V diagonalise the Kähler metric,⁵

$$V_F^\dagger \tilde{K}_{F^\dagger F} V_F = \mathbb{1} \quad , \quad V_{f^c}^\dagger \tilde{K}_{f^c f^c} V_{f^c} = \mathbb{1} \quad . \quad (12)$$

Consequently, the transformations of the soft parameters and the Yukawa couplings are given by

$$m_{\tilde{F}^\dagger \tilde{F}}'^2 \rightarrow \hat{m}_{\tilde{F}^\dagger \tilde{F}}^2 \equiv V_F^\dagger m_{\tilde{F}^\dagger \tilde{F}}'^2 V_F \quad , \quad (13a)$$

$$m_{\tilde{f}^c \tilde{f}^{c\dagger}}'^2 \rightarrow \hat{m}_{\tilde{f}^c \tilde{f}^{c\dagger}}^2 \equiv V_{f^c}^\dagger m_{\tilde{f}^c \tilde{f}^{c\dagger}}'^2 V_{f^c} \quad , \quad (13b)$$

$$a'_{\tilde{f}^c \tilde{F} H_f} \rightarrow \hat{a}_{\tilde{f}^c \tilde{F} H_f} \equiv \tilde{K}_{H_f^\dagger H_f}^{-\frac{1}{2}} V_{f^c}^\dagger a'_{\tilde{f}^c \tilde{F} H_f} V_F \quad , \quad (13c)$$

$$Y'_{f^c F H_f} \rightarrow \hat{Y}_{f^c F H_f} \equiv \tilde{K}_{H_f^\dagger H_f}^{-\frac{1}{2}} V_{f^c}^\dagger Y'_{f^c F H_f} V_F \quad . \quad (13d)$$

8. Flavour-violating parameters are computed in the super-CKM (SCKM) basis where the Yukawa couplings are diagonal,

$$\tilde{Y}_{f^c F H_f} = U_R^{f\dagger} \hat{Y}_{f^c F H_f} U_L^f = \text{diag} \quad , \quad (14)$$

⁵At the order we are considering the Kähler potential does not mix different fields F or f^c . Hence, every block $\tilde{K}_{F^\dagger F}$ and $\tilde{K}_{f^c f^c}$ in the Kähler metric can be diagonalised with a different matrix. Likewise, the block associated to the Higgs fields is diagonal. We use $\tilde{K}_{F^\dagger F}$ to denote the matrix whose ij element is $\tilde{K}_{F_i^\dagger F_j}$, and analogously for other quantities.

and we have the corresponding transformations for the soft terms,

$$\tilde{a}_{\tilde{f}^c \tilde{F} H_f} = U_R^{f\dagger} \hat{a}_{\tilde{f}^c \tilde{F} H_f} U_L^f, \quad (15a)$$

$$\tilde{m}_{\tilde{f},LL}^2 = U_L^{f\dagger} \hat{m}_{\tilde{F}^\dagger \tilde{F}}^2 U_L^f, \quad (15b)$$

$$\tilde{m}_{\tilde{f},RR}^2 = U_R^{f\dagger} \hat{m}_{\tilde{f}^c \tilde{f}^c}^2 U_R^f. \quad (15c)$$

In summary, we would like to emphasise two crucial points for the predictivity of these scenarios. A first consequence of the supergravity formalism, including a UV completion with both a sector breaking SUSY and a sector breaking the family symmetry, is the explicit form (8) of the Yukawa couplings, containing information on both sectors. In the supergravity literature the dependence on the family-blind sector is a well-known fact. However, so far this has not been considered in works studying family symmetries in the effective theory approach. Second, the relations (9) between the parameters describing the Yukawa couplings and those responsible for the soft parameters are sensitive to many details of the UV completion, as we shall illustrate in the following sections.

2.2 A Conventional Example with a Triplet Messenger

Let us consider the model presented in [11] with an $SU(3) \times U(1) \times U(1)'$ family symmetry as a conventional example. Besides the fields mentioned in the previous section, another flavon $\bar{\phi}_3$ and additional family-singlet messengers χ_1^f, χ_3^f are present. The superpotential is a straightforward generalisation of Eq. (2).⁶ The Yukawa couplings stem from diagrams of the type shown in Fig. 1 [12], involving χ_0^f and one more messenger χ_n^f with $n = 1, 2, 3$. The flavons develop vevs $\langle \bar{\phi}_3 \rangle \propto (0, 0, 1)$, $\langle \bar{\phi}_2 \rangle \propto (0, 1, -1)$, and $\langle \bar{\phi}_1 \rangle \propto (1, 1, 1)$. We assume the hierarchy

$$\frac{\langle \bar{\phi}_3 \rangle_3^2}{M_{\chi_3^f}} \gg \frac{\langle \bar{\phi}_2 \rangle_2^2}{M_{\chi_2^f}} \gg \frac{\langle \bar{\phi}_1 \rangle_2 \langle \bar{\phi}_2 \rangle_2}{M_{\chi_1^f}} \sim \frac{\langle \bar{\phi}_1 \rangle_2 \langle \bar{\phi}_2 \rangle_2}{M_{\chi_2^f}}. \quad (16)$$

Then the Yukawa matrices are approximately given by

$$Y'_{f^c F H_f} \approx \mathcal{N} \lambda_H \begin{pmatrix} 0 & \lambda_1 \lambda_2 \frac{\langle \bar{\phi}_1 \rangle_1 \langle \bar{\phi}_2 \rangle_2}{M_{\chi_0^f} M_{\chi_1^f}} & \lambda_1 \lambda_2 \frac{\langle \bar{\phi}_1 \rangle_1 \langle \bar{\phi}_2 \rangle_3}{M_{\chi_0^f} M_{\chi_1^f}} \\ \lambda_1 \lambda_2 \frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_1 \rangle_1}{M_{\chi_0^f} M_{\chi_2^f}} & \lambda_2^2 \frac{\langle \bar{\phi}_2 \rangle_2^2}{M_{\chi_0^f} M_{\chi_2^f}} & \lambda_2^2 \frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_2 \rangle_3}{M_{\chi_0^f} M_{\chi_2^f}} \\ \lambda_1 \lambda_2 \frac{\langle \bar{\phi}_2 \rangle_3 \langle \bar{\phi}_1 \rangle_1}{M_{\chi_0^f} M_{\chi_2^f}} & \lambda_2^2 \frac{\langle \bar{\phi}_2 \rangle_3 \langle \bar{\phi}_2 \rangle_2}{M_{\chi_0^f} M_{\chi_2^f}} & \lambda_3^2 \frac{\langle \bar{\phi}_3 \rangle_3^2}{M_{\chi_0^f} M_{\chi_3^f}} \end{pmatrix}, \quad (17)$$

which can fit the appropriate fermion masses and mixings [8, 13]. For brevity, we use a single value λ_n for all the $\mathcal{O}(1)$ couplings of each flavon $\bar{\phi}_n$, and likewise a single λ_H . In general, one

⁶We do not mention several other fields and many details which are important for the model but not relevant for our discussion. For example, an extra Higgs field H_{45} , whose vev is proportional to the hypercharge, plays a role in generating the 23-blocks of the Yukawa matrices.

could distinguish between many different values, introducing λ_n^{f0m} for the coupling between $\bar{\phi}_n$, $\bar{\chi}_0^f$ and χ_m^f , and λ_n^{fm} for the coupling between $\bar{\phi}_n$, $\bar{\chi}_m^f$ and f^c . However, precisely keeping track of these complications is not necessary, since all predictions will only be up to $\mathcal{O}(1)$ factors.

For example, up to factors of order unity one obtains

$$Y'_{dcQH_d} \sim \begin{pmatrix} 0 & \epsilon_d^3 & -\epsilon_d^3 \\ \epsilon_d^3 & \epsilon_d^2 & -\epsilon_d^2 \\ -\epsilon_d^3 & -\epsilon_d^2 & 1 \end{pmatrix} \quad (18)$$

for the down-type quarks, which is compatible with observations for $\epsilon_d \sim 0.13$ [8].

We see that each Yukawa coupling depends on a product of two different masses, for instance

$$Y'_{f_2^c F_3 H_f} = \mathcal{N} \lambda_H \lambda_2^2 \frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_2 \rangle_3}{M_{\chi_0^f} M_{\chi_2^f}} \equiv -\epsilon_f^2. \quad (19)$$

On the other hand, due to the quantum numbers under the family symmetry, the family triplet messengers χ_0^f cannot occur in the diagrams relevant for the Kähler potential. Therefore, the family-dependent terms in K depend only on one messenger mass,⁷ for example, from Eq. (6),

$$\tilde{K}_{f_2^c f_3^c f} = \lambda_2^2 (1 + \xi_{\chi_2^f}) \frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_2^\dagger \rangle_3}{M_{\chi_2^f}^2} \equiv -\tilde{\epsilon}_f^2. \quad (20)$$

More generally, the effective superpotential operators responsible for Yukawa couplings, which can be determined by a fit to the fermion masses, depend on a different combination of messenger masses than the effective operators in the Kähler potential. Consequently, the couplings of the latter operators are *not* determined. This has profound implications for the predictivity of the theory for the SUSY breaking parameters.

In order to study this issue without introducing unnecessary complications, let us consider only those contributions to the soft parameters which stem from the flavons $\bar{\phi}_1$ and $\bar{\phi}_2$. The calculation is performed as specified in the previous subsection. Using the effective Kähler potential (6) and the Yukawa couplings (17), we obtain contributions to some elements of the soft masses and trilinears. Further elements are easily obtained by exchanging $\bar{\phi}_1$ and $\bar{\phi}_2$ or by replacing $\bar{\phi}_1$ with $\bar{\phi}_2$ (and correspondingly for the associated parameters λ_n , c_n and $M_{\chi_n^f}$).

Let us first define two limiting cases (where “fixed” means fixed to yield the correct Yukawa couplings).

Case 1 $M_{\chi_0^f} M_{\chi_2^f} = \text{fixed}$, $M_{\chi_0^f} \gtrsim \langle \bar{\phi}_2 \rangle \Rightarrow M_{\chi_2^f} \gg \langle \bar{\phi}_2 \rangle$, $\tilde{\epsilon}_f \ll 1$

The messenger mass $M_{\chi_0^f}$ is not much larger than the flavon vev. Then the second messenger mass $M_{\chi_2^f}$ has to be very large, and K is family-blind to a good approximation.

⁷The second messenger mass does appear in the term involving F and H , but this term is family-blind and its contribution to the SUSY breaking parameters is suppressed by $\langle H_f \rangle^2 M_{\chi_0^f}^{-2}$. Besides, there is a term proportional to $|\bar{\phi}_1|^2 |\bar{\phi}_2|^2 M_{\chi_0^f}^{-2} M_{\chi_2^f}^{-2}$, which is also too small to play a significant role.

Case 2 $M_{\chi_0^f} M_{\chi_2^f} = \text{fixed}$, $M_{\chi_2^f} \gtrsim \langle \bar{\phi}_2 \rangle \Rightarrow M_{\chi_0^f} \gg \langle \bar{\phi}_2 \rangle$, $\tilde{\epsilon}_f \lesssim 1$

As $M_{\chi_2^f}$ is now rather small relative to the vev,⁸ we find significant deviations from a family-blind Kähler potential.

The setup under consideration allows us to choose both of the above cases, so that it does not predict which one is realised. The question is then how much the soft parameters differ between these cases.

The soft scalar masses depend only on the \mathcal{F} terms and on the Kähler potential [17], which is very different in our two limiting cases. Canonical normalisation does not lead to any qualitative change here, since the corresponding transformations (13a) are determined by \tilde{K} only and not by parameters involving the Yukawa couplings. Therefore, we have to conclude that in fact the unknown parameters in the Kähler potential prevent us from predicting the soft scalar masses.

More precisely, we cannot predict the soft masses of the superpartners of the right-handed fermions. We do find that there are no corrections to the $SU(2)_L$ doublet scalar mass matrix $m_{\tilde{F}^\dagger \tilde{F}}^2$, since there are no $SU(2)_L$ doublet messengers [18].⁹

The structure of the trilinear scalar couplings before canonical normalisation is

$$\begin{aligned} a'_{\tilde{f}_i^c \tilde{F}_j H_f} &= m_{3/2} \left[T_h \frac{|\langle h \rangle|^2}{M_{\text{P}}^2} + T_{\tilde{f}_i^c \tilde{F}_j H_f} \right] Y'_{\tilde{f}_i^c F_j H_f} \\ &\quad - m_{3/2} c_2 \lambda_2^2 \frac{\langle \bar{\phi}_2 \rangle_i}{M_{\chi_2^f}^2} \sum_k \langle \bar{\phi}_2^\dagger \rangle_k \left(1 - \mathcal{O} \left(\frac{|\langle h \rangle|^2}{M_{\text{P}}^2} \right) \right) Y'_{\tilde{f}_i^c F_j H_f}, \end{aligned} \quad (21a)$$

where

$$T_h = 1 + \mathcal{O}(\lambda_H) + \mathcal{O}(\lambda_1) + \mathcal{O}(\lambda_2), \quad (21b)$$

$$T_{\tilde{f}_i^c \tilde{F}_j H_f} = c_1 p_1^{ij} + c_2 p_2^{ij}, \quad (21c)$$

and where we have neglected terms suppressed by $\langle \bar{\phi} \rangle^2 / M_{\text{P}}^2$. Besides, p_n^{ij} stands for the power of $\langle \bar{\phi}_n \rangle$ that appears in $Y'_{\tilde{f}_i^c F_j H_f}$, e.g. $p_2^{23} = 2$. The first term in Eq. (21a), from $\partial_h \tilde{K}_H$ and the second line in Eq. (9b), gives a family-blind contribution proportional to $Y'_{\tilde{f}_i^c F_j H_f}$. We define the familiar CMSSM-like parameter

$$A_0 \equiv m_{3/2} T_h \frac{|\langle h \rangle|^2}{M_{\text{P}}^2}. \quad (22)$$

The part of the second line of Eq. (9b) involving $\tilde{K} \partial_{\bar{\phi}} \tilde{K}$ reproduces the second line of Eq. (21a). Finally, the second term of Eq. (9b) contains $\partial_{\langle \phi_m \rangle} Y$ and yields the contribution $m_{3/2} T_{\tilde{f}_i^c \tilde{F}_j H_f} Y'_{\tilde{f}_i^c F_j H_f}$

⁸In order to reach $\tilde{\epsilon}_f \sim 1$, Eq. (4) has to be relaxed a bit, allowing for $M_{\chi_2^f} < \langle \bar{\phi}_3 \rangle$. This seems reasonable, since χ_2^f does not couple to $\bar{\phi}_3$, so that no dangerous terms containing $\langle \bar{\phi}_3 \rangle / M_{\chi_2^f}$ (without any accompanying suppression factors) can arise. If instead $M_{\chi_2^f} \geq \langle \bar{\phi}_3 \rangle$, one finds that $\tilde{\epsilon}_f$ cannot become much larger than ϵ_f .

⁹A purely left-handed (i.e. $SU(2)_L$ doublet) messenger sector is excluded because in this case the $SU(2)_L$ symmetry would lead to $\epsilon_u = \epsilon_d$.

to the trilinear coupling $a'_{\tilde{f}_i^c \tilde{F}_j H_f}$. This term can give a non-trivial family dependence even with a canonical Kähler potential due to the non-trivial dependence of the Yukawa couplings on the flavon fields. Furthermore, this term does not depend on the unknown parameters $\tilde{\epsilon}_f$ in the Kähler potential and hence it could be directly linked to observable quantities. However, the predictivity for the physical trilinears is limited by two effects.

Firstly, the second line in Eq. (21a) depends on the unknown parameter $\tilde{\epsilon}_f$. This line also contains different elements of the Yukawa matrices, which can be much larger than the element appearing in the first line, $Y'_{f_k^c F_j H_f} \gg Y'_{f_i^c F_j H_f}$ for some values of k . Then Case 1 for a' differs considerably from Case 2, since some terms of the second line dominate for sufficiently large $\tilde{\epsilon}_f^2$. For example, consider the element $a'_{\tilde{f}_2^c \tilde{F}_3 H_f}$. Up to Planck-suppressed terms, the second line of Eq. (21a) reads

$$-m_{3/2} c_2 \lambda_2^2 \left[\frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_2^\dagger \rangle_2}{M_{\chi_2^f}^2} Y'_{f_2^c F_3 H_f} + \frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_2^\dagger \rangle_3}{M_{\chi_2^f}^2} Y'_{f_3^c F_3 H_f} \right] \sim -m_{3/2} c_2 [\tilde{\epsilon}_f^2 \epsilon_f^2 - \tilde{\epsilon}_f^2].$$

In Case 2 the parameter $\tilde{\epsilon}_f$ can easily be much larger than ϵ_f , so that the second line dominates over the first one, which is proportional to the Yukawa coupling $Y'_{f_2^c F_3 H_f} \sim \epsilon_f^2$. Thus, the order of magnitude of $a'_{\tilde{f}_2^c \tilde{F}_3 H_f}$ is changed compared to Case 1.

Secondly, the transformation (13c) to the canonical basis is controlled by the Kähler potential and thus again by the unknown $\tilde{\epsilon}_f$. In order to estimate the possible change, we choose $V_{f^c}^\dagger$ as a lower-diagonal matrix [19]. Then the canonically normalised trilinears are given by

$$\hat{a}_{\tilde{f}_i^c \tilde{F}_j H_f} = (V_{f^c}^\dagger)_{ii} a'_{\tilde{f}_i^c \tilde{F}_j H_f} + \sum_{k < i} (V_{f^c}^\dagger)_{ik} a'_{\tilde{f}_k^c \tilde{F}_j H_f}, \quad (23)$$

up to an overall (family-blind) factor (remember that here $\tilde{K}_{F^\dagger F} \propto \mathbb{1}$ and hence $V_F \propto \mathbb{1}$). We have $(V_{f^c})_{ii} \sim 1$, $(V_{f^c})_{i \neq k} \lesssim \tilde{\epsilon}_f^2$, and $a'_{\tilde{f}_k^c \tilde{F}_j H_f} \lesssim a'_{\tilde{f}_i^c \tilde{F}_j H_f}$ for $k < i$. Thus, the corrections to each element of a' are at most of the same order of magnitude as the element itself as long as $\tilde{\epsilon}_f < 1$. In other words, they only cause a change by an $\mathcal{O}(1)$ factor but do not change the power of ϵ_f appearing in the element. Consequently, the effect of canonical normalisation is not terribly different in the two limiting cases introduced above. In conclusion, mainly due to the unknown size of the second line of Eq. (21a) one loses predictivity for the trilinear scalar couplings as well.

As a notable exception, the 33 element of the Kähler metric of the right-handed matter fields is given by

$$\tilde{K}_{f_3^c f_3^c} = 1 + \xi_{f^c} + \lambda_3^2 (1 + \xi_{\chi_2^f}) \frac{|\langle \bar{\phi}_3 \rangle_3|^2}{M_{\chi_3^f}^2} + \lambda_3^4 \frac{|\langle \bar{\phi}_3 \rangle_3|^4}{M_{\chi_0^f}^2 M_{\chi_3^f}^2}. \quad (24)$$

As the large 33 entries of the Yukawa matrices are

$$Y'_{f_3^c F_3 H_f} = \mathcal{N} \lambda_H \lambda_3^2 \frac{(\langle \bar{\phi}_3 \rangle_3)^2}{M_{\chi_0^f} M_{\chi_3^f}} \sim 1, \quad (25)$$

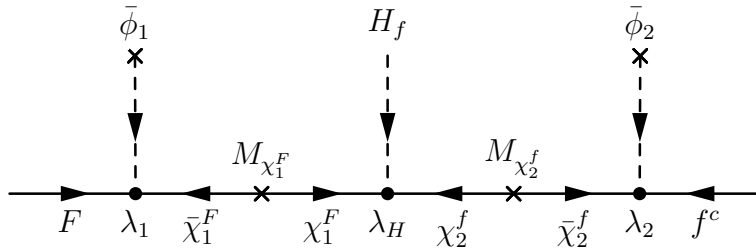


Figure 3: Another possibility for generating Yukawa couplings. All messengers are SU(3) singlets in this case.

it follows that $M_{\chi_0^f} \sim M_{\chi_3^f} \sim \langle \bar{\phi}_3 \rangle$, if we require the messengers χ_0^f and χ_3^f to be heavier than $\langle \bar{\phi}_3 \rangle$.¹⁰ Furthermore, the last term in Eq. (24), whose analogue was omitted in Eq. (6) because it is of higher order in the messenger masses and thus not important for other elements of $\tilde{K}_{f^c f^c \dagger}$, equals $|Y'_{f_3^c F_3 H_f} / \mathcal{N} \lambda_H|^2$. Thus, with the qualification mentioned in footnote 10, one can indeed gain some knowledge about the order of magnitude of the corrections to the family-universal part $(1 + \xi_{f^c})$ of $\tilde{K}_{f_3^c f_3^c \dagger}$. However, this is not the case for the remaining elements. The smallness of the Yukawa couplings other than $Y_{f_3^c F_3 H_f}$ implies a hierarchy between the relevant flavon vevs and messenger masses, which allows to vary these parameters significantly.

If the singlet messenger masses happen to be small enough to lead to measurable deviations from the CMSSM due to large $\tilde{\epsilon}_f$, there could remain some predictions in the form of correlations between different observables since the number of free parameters (including $\tilde{\epsilon}_f$) is smaller than the number of independent soft SUSY breaking parameters. However, given the large number of parameters and the complicated relations between model parameters and observables, it is questionable if such predictions could in practice be found and confirmed. Another prediction that can arise in unified models is that the parameters in the lepton sector are related to those in the quark sector [20, 21]. However, this is a consequence of the enlarged gauge symmetry rather than the family symmetry.

3 Improving Predictivity with Singlet Messengers

One possible way to realise more predictions for the SUSY breaking parameters is an extension of the theory that allows to restrict the messenger masses, as proposed in [22] for an SO(3) family symmetry, or fixes the ratios between the flavon vevs. Another way, which we will explore here, is to generate the Yukawa couplings via diagrams of the type shown in Fig. 3.¹¹ In replacement of the messengers χ_0^f , which are triplets under the family symmetry SU(3) and singlets under

¹⁰In the absence of a model for the messenger masses, even this is not predicted. Depending on the structure of higher-order corrections to the effective Kähler potential, it is possible that $M_{\chi_0^f} M_{\chi_3^f} < \langle \bar{\phi}_3 \rangle^2$ is sufficient for the consistency of the theory.

¹¹This kind of diagrams cannot be used for generating the 33 entries of the Yukawa couplings due to the quantum numbers assigned in [11]. Hence, we have to continue utilising a diagram of the type shown in Fig. 1 to generate these particular entries.

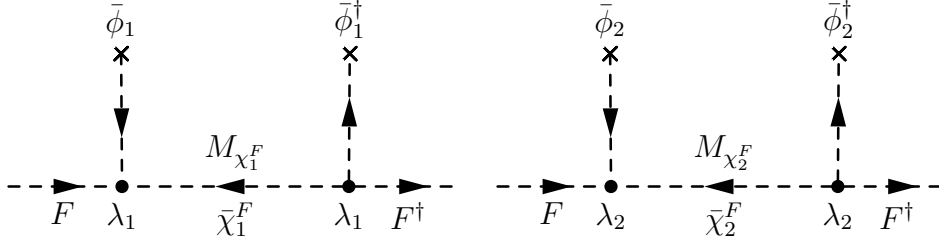


Figure 4: Diagrams relevant for the Kähler potential for the alternative messenger sector of Fig. 3.

SU(2)_L, we now employ the fields χ_1^F and χ_2^F , which are SU(3) singlets and SU(2)_L doublets.

We obtain the same effective superpotential and Yukawa couplings as before, except that χ_0^f is exchanged by χ_1^F and χ_2^F , for instance

$$Y'_{f_2^c F_3 H_f} = \mathcal{N} \lambda_H \lambda_2^2 \frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_2 \rangle_3}{M_{\chi_2^F} M_{\chi_2^f}} \equiv -\epsilon_f^2, \quad (26)$$

so that the phenomenology of the fermion sector is completely unchanged. The diagram from Fig. 2 again yields contributions to the Kähler potential of the right-handed matter fields like in Eq. (20). However, χ_n^F being family singlets, there arise the new diagrams shown in Fig. 4, which lead to non-universal corrections to the Kähler potential of the left-handed fields as well, for example

$$\tilde{K}_{F_2^\dagger F_3} = \lambda_2^2 (1 + \xi_{\chi_2^F}) \frac{\langle \bar{\phi}_2 \rangle_2 \langle \bar{\phi}_2^\dagger \rangle_3}{M_{\chi_2^F}^2} \equiv -\tilde{\epsilon}_F^2. \quad (27)$$

Thus, *all* messenger masses show up in the effective Kähler potential and the soft parameters in this case. This means that although we still have considerable freedom to adjust the expansion parameters $\tilde{\epsilon}_f$ and $\tilde{\epsilon}_F$, we predict the correlation

$$|\tilde{\epsilon}_F \tilde{\epsilon}_f| = \frac{(1 + \xi_{\chi_2^F})^{1/2} (1 + \xi_{\chi_2^f})^{1/2}}{\mathcal{N} \lambda_H} |\epsilon_f|^2. \quad (28)$$

Explicitly,

$$|\tilde{\epsilon}_Q \tilde{\epsilon}_u| \sim |\epsilon_u|^2, \quad |\tilde{\epsilon}_Q \tilde{\epsilon}_d| \sim |\epsilon_d|^2, \quad |\tilde{\epsilon}_L \tilde{\epsilon}_e| \sim |\epsilon_d|^2, \quad (29)$$

where the appearance of the quark sector parameter ϵ_d rather than an independent ϵ_e in the last relation is a particularity of the model [11]. As a consequence, no expansion parameter can be arbitrarily small, and the breaking of the family symmetry produces off-diagonal elements in all soft mass matrices. For example, we find using $\epsilon_d = 0.13$ and $\tilde{\epsilon}_d < 1$

$$\tilde{\epsilon}_Q = \frac{\epsilon_d^2}{\lambda_H \tilde{\epsilon}_d} \gtrsim 0.02. \quad (30)$$

The prediction (28) still depends on the unknown quantities λ_H , \mathcal{N} , and ξ_χ . By construction, $\lambda_H \sim 1$, like the other dimensionless couplings in the superpotential, so that this parameter does

not introduce a large uncertainty. The other unknown parameters are related to the hidden-sector field h . Although they could be significantly larger than 1 in principle, this would require $\langle h \rangle \gg M_{\text{P}}$ or a rather large number of additional hidden-sector fields with vevs close to M_{P} . Thus, it seems reasonable to expect these quantities to be of order unity as well. For instance, for the Polonyi model [23] $\langle h \rangle = (\sqrt{3} - 1) M_{\text{P}}$, so that $\mathcal{N} = \exp(\langle h \rangle^2 / 2M_{\text{P}}^2) \approx 1.3$.

Some additional sources of uncertainty have not been included in the above relations. As mentioned earlier, we have used only three different couplings λ_1, λ_2 and λ_H , and moreover neglected their possible dependence on h . Besides, we have defined ϵ_f before canonical normalisation. This is not entirely correct, since these parameters are determined by a fit to the *physical*, canonically normalised Yukawa couplings. However, canonical normalisation may not cause a significant change of the Yukawa matrices, if the model is to predict the fermion masses. Thus, as long as this condition is satisfied, canonical normalisation does not produce more than another $\mathcal{O}(1)$ factor.

The soft SUSY-breaking parameters are computed as in the previous section. The soft masses depend on the parameters $\tilde{\epsilon}_f$ and $\tilde{\epsilon}_F$ but not on ϵ_f , although of course now they are related. The trilinear couplings now include two terms that are not proportional to the corresponding Yukawa couplings, the second line of Eq. (21a) and in addition

$$a'_{\tilde{f}_i^c \tilde{F}_j H_f} \supset -m_{3/2} c_1 \lambda_1^2 \frac{\langle \bar{\phi}_1 \rangle_j}{M_{\chi_1^F}^2} \sum_k \langle \bar{\phi}_1 \rangle_k \left(1 - \mathcal{O} \left(\frac{|\langle h \rangle|^2}{M_{\text{P}}^2} \right) \right) Y'_{f_i^c F_k H_f}, \quad (31)$$

because of the off-diagonal term in the Kähler metric of the left-handed matter fields.

Leaving the particular model under consideration for a moment, an obvious question is whether our results can be generalised to a simple criterion for the predictivity of family models for the soft SUSY breaking parameters. We have seen that one can expect predictions, if the masses of all messengers appear in the effective Kähler potential at the order M_{χ}^{-2} . This is the case if a coupling $F\bar{\phi}\chi$ or $f^c\bar{\phi}\chi$ exists for all messengers. For $F, f^c \sim \mathbf{3}$ and $\bar{\phi} \sim \bar{\mathbf{3}}$, this requires χ to be singlets (leaving aside representations with dimension larger than 3), as in the example of this section. One could also have $\bar{\phi} \sim \mathbf{3}$, though, which would allow a coupling with triplet messengers. Hence, one cannot conclude that predictivity requires singlet messengers in general. The converse statement evidently holds provided that $F\bar{\phi}, f^c\bar{\phi} \sim \mathbf{1}$: if all messengers are singlets, then their masses appear in the effective Kähler potential. However, non-Abelian family symmetries are often extended by extra symmetries like $U(1) \times U(1)'$ in [11] under which all messengers are charged. In other words, total singlets rarely exist, limiting the use of this criterion. Consequently, one usually cannot sidestep checking the viability of each matter-flavon-messenger vertex.

4 Comparison with Experimental Constraints

In order to get an idea about the observable signatures that can be expected, let us make a very rough estimate of the parameters relevant for FCNC processes. We assume a framework where the Yukawa couplings are generated from diagrams of the type shown in Fig. 3, with the exception of the 33-entries. For the latter we employ the diagram of Fig. 1. Thus, the 33-entries of the right-handed soft mass matrices and trilinear couplings receive $\mathcal{O}(1)$ corrections.

From (13a) it is straightforward to see that the same parameters $\tilde{\epsilon}_Q$ (in case of more than one flavon) parameterise both V_F and $m_{\tilde{F}\tilde{F}}^{\prime 2}$ and analogously the flavons of the type $\tilde{\epsilon}_f$ for V_{fc} and $m_{\tilde{f}\tilde{f}}^{\prime 2}$ so the final expressions in Eq. (10) are functions only of the original parameters and consequently flavour violation bounds constrain them. Although only indirectly, since before comparing to the appropriate bounds we need to go the so called SCKM (super CKM) basis where the Yukawa couplings are diagonal. So let us describe then the changes in the structure of these matrices. One finds at energies of the order $\langle \bar{\phi} \rangle$:

$$\hat{m}_{\tilde{f}\tilde{f}}^2 \sim m_0^2 \begin{pmatrix} 1 & \tilde{\epsilon}_f^2 \epsilon_d^2 & \tilde{\epsilon}_f^2 \epsilon_d^2 \\ \cdot & 1 + \tilde{\epsilon}_f^2 & \tilde{\epsilon}_f^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad f = u, d, Q, e, L, \quad (32)$$

where we have omitted the $\mathcal{O}(1)$ coefficients and where dots stand for elements given by hermiticity. For simplicity, we have assumed a common prefactor m_0^2 in front of the matrix for all types of sfermions. Likewise, we will assume a unified gaugino mass $m_{1/2}$, so that in the limit $\langle \bar{\phi} \rangle \rightarrow 0$ we have a special case of the CMSSM.

In elements 12 and 13 of the matrix above there appears the term $\lambda_1^2 \langle |\bar{\phi}_1|^2 \rangle / M_{\chi_1^f}^2$ (from our Kähler metric given in Eq. 39). Then for the correct structure of the Yukawa couplings (used in [11] and also needed for our Yukawa matrices) the relations $\langle \bar{\phi}_1 \rangle \sim \epsilon_d \langle \bar{\phi}_2 \rangle$ and $M_{\chi_1^f} \sim M_{\chi_2^f}$ are needed.

This form of scalar masses was studied previously in [5] for the case $\tilde{\epsilon}_f = \epsilon_f$ for $f = u, d, e$ and $\tilde{\epsilon}_Q = \tilde{\epsilon}_L = 0$, i.e. diagonal mass matrices for the SU(2) doublet sfermions.

We use the one-loop renormalisation group evolution estimate [24] for the diagonal entries at low energy and neglect the running of the off-diagonal entries. Up to now we have not mentioned the neutrino sector. The inclusion of it with the use of the see-saw mechanism has been widely used in $SU(3)$ models to reproduce the right spectra and mixing for oscillating neutrinos. In most part of such models, the neutrino Yukawa couplings of $\mathcal{O}(1)$ do not influence the running because the corresponding right-handed singlet neutrino has a mass above M_{GUT} . Later, we will briefly comment on scenarios where Y_ν plays a significant role.

Due to the hierarchical Yukawa couplings, the transformation to the SCKM basis is given to

a very good approximation at low energy as follows:

$$\tilde{m}_{\tilde{d},RR}^2 \sim m_0^2 \begin{pmatrix} \mathcal{R}_{\tilde{d},RR} & \tilde{\epsilon}_d^2 \epsilon_d & \tilde{\epsilon}_d^2 \epsilon_d + \epsilon_d^3 \\ \cdot & \mathcal{R}_{\tilde{d},RR} & \tilde{\epsilon}_d^2 + \epsilon_d^2 \\ \cdot & \cdot & \mathcal{R}_{\tilde{d},RR} \end{pmatrix}, \quad (33)$$

where the factor $\mathcal{R}_{\tilde{d},RR}$ corresponds to the RGE evolution increase at low energy. The matrices $\tilde{m}_{\tilde{Q},LL}^2$, $\tilde{m}_{\tilde{L},LL}^2$ and $\tilde{m}_{\tilde{e},RR}^2$ are analogous to Eq. (33) with the replacements $\tilde{\epsilon}_d \rightarrow \tilde{\epsilon}_Q$, $\tilde{\epsilon}_d \rightarrow \tilde{\epsilon}_L$ and $\tilde{\epsilon}_d \rightarrow \tilde{\epsilon}_e$ respectively and for the leptonic cases also the RGE factors $\mathcal{R}_{\tilde{L}}$ and $\mathcal{R}_{\tilde{e}}$ are different.

We present an example using the benchmark point SPS 1a, with values $m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$ GeV and $\tan \beta = 10$, for which the estimate of [24] yields

$$(\tilde{m}_{\tilde{q},LL}^2)_{ii} \sim 30 m_0^2 \quad , \quad (\tilde{m}_{\tilde{e},LL}^2)_{ii} \sim 4 m_0^2 \quad , \quad (\tilde{m}_{\tilde{e},RR}^2)_{ii} \sim 2 m_0^2 \quad , \quad (34)$$

i.e. $\mathcal{R}_{\tilde{d},RR} \sim 30$, $\mathcal{R}_{\tilde{e},LL} \sim 4$ and $\mathcal{R}_{\tilde{e},RR} \sim 2$ for the quarks, lepton doublet and charged lepton singlets respectively.

Here we have ignored signs and assumed no severe cancellations, which can occur in fine-tuned cases. We also neglect all complex phases, so that there are no contributions to electric dipole moments and CP-violating parameters in meson mixing.

The quantities $\tilde{m}_{\tilde{u},LL}^2$ are less interesting due to the weaker experimental constraints (coming from D rather than K mixing).

We use the experimental constraints from Δm_K , $b \rightarrow s\gamma$, $\mu \rightarrow e\gamma$ etc. given in [25,26]. In the mass insertion approximation, they can be translated into constraints on the δ flavour violating parameters:

$$\begin{aligned} (\delta_{RR}^f)_{ij} &:= \frac{(\tilde{m}_{\tilde{f},RR}^2)_{ij}}{(\tilde{m}_{\tilde{f},RR}^2)_{ii}}, \\ (\delta_{LR,RL}^f)_{ij} &:= \frac{(\tilde{m}_{\tilde{f},LR,RL}^2)_{ij}}{\sqrt{(\tilde{m}_{\tilde{f},LL}^2)_{ii}(\tilde{m}_{\tilde{f},RR}^2)_{jj}}}. \end{aligned} \quad (35)$$

Then the (δ_{XY}^f) parameters are given by

$$\begin{aligned} (\delta_{LR,RL}^d)_{ij} &= \frac{v}{\sqrt{1 + \tan^2 \beta}} \left[-\frac{\tilde{a}_{\tilde{d}_i^c \tilde{Q}_j H_f}}{30m_0^2} + \frac{\mu^* \tan \beta \tilde{Y}_{\tilde{d}_i^c \tilde{Q}_j}^{\text{diag}}}{30m_0^2 \sqrt{2}} \right], \\ (\delta_{LR,RL}^u)_{ij} &= \frac{v \tan \beta}{\sqrt{1 + \tan^2 \beta}} \left[-\frac{\tilde{a}_{\tilde{u}_i^c \tilde{Q}_j H_f}}{30m_0^2} + \frac{\mu^* \tilde{Y}_{\tilde{u}_i^c \tilde{Q}_j}^{\text{diag}}}{30m_0^2 \tan \beta \sqrt{2}} \right], \end{aligned} \quad (36)$$

where $\tilde{\cdot}$ denotes the quantities in the SCKM basis. Following our discussion on the transformation of the trilinear couplings $\hat{a}_{\tilde{f}_i^c \tilde{F}_j H_f}$ due to canonical normalisation, in our example the leading terms of the trilinear couplings are always proportional to the corresponding Yukawa coupling $\hat{Y}_{\tilde{f}_i^c \tilde{F}_j H_f}$,

	Our example	Bound
$(\delta_{RR}^d)_{12}$	$\frac{\tilde{\epsilon}_d^2 \epsilon_d}{30} \sim 7 \cdot 10^{-5}$	$9 \cdot 10^{-3}$
$(\delta_{LL}^d)_{12}$	$\frac{\tilde{\epsilon}_Q^2 \epsilon_d}{30} \sim 7 \cdot 10^{-5}$	$1 \cdot 10^{-2}$
$(\delta_{LR,RL}^d)_{12}$	$\frac{v}{\sqrt{1+\tan^2\beta}} \left[-\frac{nA_0\epsilon_d^3}{30m_0^2} \right] \sim 4n \cdot 10^{-6}$	$1 \cdot 10^{-5}$
$(\delta_{LL}^d)_{23}$	$\frac{\tilde{\epsilon}_Q^2}{30} \sim 6 \cdot 10^{-4}$	$2 \cdot 10^{-1}$
$(\delta_{LL}^e)_{12}$	$\frac{\tilde{\epsilon}_L^2 \epsilon_d}{4} \sim 6 \cdot 10^{-4}$	$6 \cdot 10^{-4}$
$(\delta_{LR,RL}^d)_{23}$	$\frac{v}{\sqrt{1+\tan^2\beta}} \left[-\frac{nA_0\epsilon_d^2}{30m_0^2} \right] \sim 4n \cdot 10^{-4}$	$1 \cdot 10^{-3}$

Table 1: An example for the flavour violating parameters δ for the SPS 1a point, together with the corresponding experimental limit. For a detailed description of the formulas see the text in this section.

i.e. the first line of Eq. (21a) dominates. Then the SCKM transformation will not change the corresponding order of magnitude in $\hat{a}_{\tilde{f}_i^c \tilde{F}_j H_f}$:

$$\begin{aligned}
\tilde{a}_{\tilde{f}_i^c \tilde{F}_j H_f} &= \left[U_R^{f\dagger} \hat{a}_{\tilde{f}_i^c \tilde{F}_j H_f} U_L^f \right]_{ij} = \left[U_R^{f\dagger} \tilde{K}_{H_f^\dagger H_f}^{-\frac{1}{2}} V_{f^c}^\dagger a'_{\tilde{f}_i^c \tilde{F}_j H_f} V_F U_L^f \right]_{ij} \\
&= \mathcal{O} \left(\left[T_{\tilde{f}_i^c \tilde{F}_j H_f} + T_h \frac{|\langle h \rangle|^2}{M_{\text{P}}^2} \right] \hat{Y}_{\tilde{f}_i^c \tilde{F}_j H_f} \right) m_{3/2}.
\end{aligned} \tag{37}$$

Note that since we do not have the relations of the CMSSM case, we need to redefine an A_0 depending on each element of $a'_{\tilde{f}_i^c \tilde{F}_j H_f}$ but, since $T_{\tilde{f}_i^c \tilde{F}_j H_f}$ are expected to be $\mathcal{O}(1)$, its order of magnitude can be estimated. What we have assumed in the numerical estimates in Table 1 is that we can express $(T_h \frac{|\langle h \rangle|^2}{M_{\text{P}}^2} + T_{\tilde{f}_i^c \tilde{F}_j H_f}) = nA_0/m_{3/2}$ for a factor n of $\mathcal{O}(1)$ that depends on each element (i, j) . In our example under consideration, n can be a factor of a few (i.e. $1 + p_n^{ij}$ in Eq. (21c)) which can be well within the range to be probed by the forthcoming experiments.

In order to estimate the size of FCNCs in our setup, let us consider a very simple example ¹²:

$$\tilde{\epsilon}_Q = \tilde{\epsilon}_L = \epsilon_d \approx 0.13 \quad \Rightarrow \quad \tilde{\epsilon}_d = \tilde{\epsilon}_e = \epsilon_d \approx 0.13 \quad , \quad \tilde{\epsilon}_u = \frac{\epsilon_u^2}{\epsilon_d} \approx 0.012. \tag{38}$$

An example for the flavour violating parameters δ using the relations above is listed in Tab. 1 for the SPS 1a point, together with the corresponding experimental limits. We see that the constraints in the squark sector are easily satisfied for flavour violating parameters of the form (δ_{XX}^d) but for (δ_{XY}^d) we have an important dependence on what values are chosen for A_0 , m_0 and $\tan\beta$. If A_0 is comparatively larger than m_0 then (δ_{XY}^d) could be easily above the limit for it. Also for a large $\tan\beta$ this could be a problem. Taking $\epsilon_d \sim 0.15$, the values of $(\delta_{LR}^d)_{12,23}$ are comfortably within the limits for the point SPS1a, while, for $A_0 = -1100$ GeV, $m_0 = 200$ GeV and $\tan\beta = 10$, (δ_{12}^d) is at the limit but (δ_{23}^d) satisfies all bounds.

¹²We have taken the numerical values of ϵ_d and ϵ_u from the latest fit [8] of the kind of Yukawa matrices we are using.

The flavour violating parameters δ^u can be calculated analogously to those of δ^d . However, its corresponding experimental are not so stringent [27–29] compared to the δ^d constraints, and they are satisfied as long as the bounds on δ^d are satisfied with a choice of $\epsilon_u < \epsilon_d$. We do not show δ_{RR}^e , since they are only weakly constrained, too. Some tension can be seen in the lepton sector with $\mu \rightarrow e\gamma$, consistently with what was found in [5].

If there was a right-handed neutrino whose Yukawa coupling influenced the running of $(\hat{m}_{\tilde{L}\tilde{L}}^2)_{ij}$ we could estimate its effect as [30]:

$$(\delta\hat{m}_{\tilde{L}\tilde{L}}^2)_{ij} = \frac{-1}{8\pi^2}(3m_0^2 + A_0^2) \left[Y^{\nu\dagger} \log\left(\frac{M_{\text{GUT}}}{M_N}\right) Y^\nu \right]_{ij}.$$

The form of Y^ν is unfortunately very model-dependent since we do not know the experimental value of the absolute scale and the nature of the oscillating neutrinos. Nevertheless, let us use a form that has been widely used in $SU(3)$ models, mainly that all elements of Y^ν , except Y_{11}^ν and Y_{33}^ν are of $\mathcal{O}(\epsilon_\nu^3)$, where $\epsilon_\nu \in (\epsilon_u, 0.4)$ [8]. $Y_{11}^\nu < \mathcal{O}(\epsilon_\nu^3)$ and $Y_{33}^\nu \leq \mathcal{O}(1)$. Thus, by comparing the form of $(\delta\hat{m}_{\tilde{L}\tilde{L}}^2)_{ij}$ to that of $(\hat{m}_{\tilde{L}\tilde{L}}^2)_{ij}$, we see that for the (1, 2) and (2, 3) elements, respectively, we would compare

$$\begin{aligned} \tilde{\epsilon}_Q^2 \epsilon_d \text{ vs. } & \frac{-3}{8\pi^2} \epsilon_\nu^6 (l_2 + l_3) \\ \tilde{\epsilon}_Q^2 \text{ vs. } & \frac{-3}{8\pi^2} \epsilon_\nu^3 (l_3), \end{aligned}$$

where $l_i = \log\left(\frac{M_{\text{GUT}}}{M_{N_i}}\right) \sim \mathcal{O}(1)$. Hence only for $(\hat{m}_{\tilde{L}\tilde{L}}^2)_{23}$ we could have a sizable contribution from the right-handed neutrinos when considering values of $\epsilon_\nu \approx 0.4$.¹³

5 Conclusions

The potential of family symmetries to predict the soft supersymmetry breaking parameters in addition to the fermion masses has been studied actively in recent years [3–9]. In this work, instead of using effective potentials restricted only by the symmetries of a model as has been conventionally done, we have explored an ultra-violet (UV) completion to study all the ingredients for the family symmetry breaking and its connection to supersymmetry breaking in supergravity. The specification of the UV completion helped us in clarifying how the flavon and the messenger fields contribute to the soft supersymmetry breaking parameters and Yukawa couplings. We argued that, in a conventional model with triplet messenger fields, predictions for the soft parameters are hindered because they depend on unknown parameters that are not fixed by fitting the Yukawa couplings to experimental data. Those parameters, for example the messenger

¹³The situation changes when A_0 is large. First of all, the simple one loop approximation to $(\delta\hat{m}_{\tilde{L}\tilde{L}}^2)_{ij}$ given above does not describe properly the running effects any more and indeed they could play a special role in enlarging or reducing this contribution [31], therefore they may play a more important role. However this needs to be analysed using the exact running whose detailed numerical study is left for our forthcoming work.

masses, are associated with the non-canonical Kähler potential and the hidden sector breaking supersymmetry.

As one possibility to improve the situation, we have proposed a model where all the messengers are family singlets. This allowed us to derive predictions in the form of correlations between different soft parameters. Such models with predictive power robust enough to test the underlying family symmetry would deserve further examination in view of the wealth of forthcoming experimental data probing flavour and CP violation.

Acknowledgments

We would like to thank Steve King, Oleg Lebedev, Jan Louis, and Oscar Vives for helpful discussions. This work was supported by the German Science Foundation (DFG) via the Junior Research Group ‘‘SUSY Phenomenology’’ within the Collaborative Research Centre 676 ‘‘Particles, Strings and the Early Universe’’, by the Michigan Center for Theoretical Physics, and by the INFN. We thank the MCTP, the Galileo Galilei Institute for Theoretical Physics, the University of Hamburg, and the Abdus Salam International Centre for Theoretical Physics (ICTP) for their hospitality.

A Kähler Metric

With all the previous assumptions, to a good approximation the Kähler metric of the right-handed matter fields for the model of Sec. 2.2 is given by

$$\tilde{K}_{fcfc^\dagger} = \mathbb{1} + \begin{bmatrix} \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} & \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} & \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} \\ \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} & \frac{\lambda_2^{f2} \langle |\bar{\phi}_2|^2 \rangle}{M^2 \chi_2^f} + \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} & \frac{\lambda_2^{f2} \langle |\bar{\phi}_2|^2 \rangle}{M^2 \chi_2^f} + \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} \\ \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} & \frac{\lambda_2^{f2} \langle |\bar{\phi}_2|^2 \rangle}{M^2 \chi_2^f} + \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} & \frac{\lambda_3^{f2} \langle |\bar{\phi}_3|^2 \rangle}{M^2 \chi_3^f} + \frac{\lambda_2^{f2} \langle |\bar{\phi}_2|^2 \rangle}{M^2 \chi_2^f} + \frac{\lambda_1^{f2} \langle |\bar{\phi}_1|^2 \rangle}{M^2 \chi_1^f} \end{bmatrix}. \quad (39)$$

$|\bar{\phi}_a|_{i,j}^2 \equiv (\bar{\phi}_a)_i (\bar{\phi}_a)_j^\dagger$, are the i and j components of $\bar{\phi}_a$ and $\bar{\phi}_a^\dagger$ respectively. We have assumed that

$$\frac{(\langle \bar{\phi}_3 \rangle_3)^2}{M^2 \chi_3^f} \gg \frac{(\langle \bar{\phi}_2 \rangle_3)^2}{M^2 \chi_2^f} \gg \frac{(\langle \bar{\phi}_1 \rangle_3)^2}{M^2 \chi_1^f}, \quad (40)$$

in addition to Eq. (16).

References

- [1] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147** (1979) 277.
- [2] S. Abel, S. Khalil and O. Lebedev, Phys. Rev. Lett. **89** (2002) 121601 [arXiv:hep-ph/0112260].

- [3] G. G. Ross and O. Vives, Phys. Rev. **D67** (2003) 095013 [arXiv:hep-ph/0211279].
- [4] G. G. Ross, L. Velasco-Sevilla and O. Vives, Nucl. Phys. **B692** (2004) 50 [arXiv:hep-ph/0401064].
- [5] S. Antusch, S. F. King and M. Malinský, JHEP **0806** (2008) 068 [arXiv:0708.1282 [hep-ph]].
- [6] S. Antusch, S. F. King, M. Malinský and G. G. Ross, Phys. Lett. B **670** (2009) 383 [arXiv:0807.5047 [hep-ph]].
- [7] L. Calibbi, J. Jones-Perez, A. Masiero, J. h. Park, W. Porod and O. Vives, Nucl. Phys. B **831** (2010) 26 [arXiv:0907.4069 [hep-ph]].
- [8] L. Calibbi, E. J. Chun and L. Velasco-Sevilla, arXiv:1005.5563 [hep-ph].
- [9] K. A. Olive and L. Velasco-Sevilla, JHEP **0805** (2008) 052 [arXiv:0801.0428 [hep-ph]].
- [10] S. Abel, D. Bailin, S. Khalil and O. Lebedev, Phys. Lett. B **504** (2001) 241 [arXiv:hep-ph/0012145].
- [11] I. de Medeiros Varzielas and G. G. Ross, Nucl. Phys. B **733** (2006) 31 [arXiv:hep-ph/0507176].
- [12] I. de Medeiros Varzielas, arXiv:0801.2775 [hep-ph].
- [13] R. G. Roberts, A. Romanino, G. G. Ross and L. Velasco-Sevilla, Nucl. Phys. B **615**, 358 (2001) [arXiv:hep-ph/0104088].
- [14] P. H. Chankowski, O. Lebedev and S. Pokorski, Nucl. Phys. B **717** (2005) 190 [arXiv:hep-ph/0502076].
- [15] L. Brizi, M. Gomez-Reino, and C. A. Scrucca, Nucl. Phys. B **820** (2009) 193 [arXiv:0904.0370].
- [16] For a short and clear review about this, we have used: D. G. Cerdeño and C. Muñoz, “An introduction to supergravity”, *prepared for 6th Hellenic School and Workshop on Elementary Particle Physics, Corfu, Greece, 6–26 Sep 1998*.
- [17] A. Brignole, L. E. Ibáñez and C. Muñoz, in *Perspectives on Supersymmetry*, Ed. G. L. Kane (World Scientific, Singapore, 1998) pp. 125-148 [arXiv:hep-ph/9707209].
- [18] S. Antusch, S. F. King and M. Malinský, JHEP **0805** (2008) 066 [arXiv:0712.3759 [hep-ph]].
- [19] S. F. King, I. N. R. Peddie, G. G. Ross, L. Velasco-Sevilla and O. Vives, JHEP **0507** (2005) 049 [arXiv:hep-ph/0407012].

- [20] S. Baek, T. Goto, Y. Okada, and K.-I. Okumura, *Phys. Rev. D* **63** (2001) 051701 [arXiv:hep-ph/0002141].
- [21] M. Ciuchini, A. Masiero, L. Silvestrini, S. K. Vempati, and O. Vives, *Phys. Rev. Lett.* **92** (2004) 071801 [arXiv:hep-ph/0307191].
- [22] S. F. King and M. Malinský, *JHEP* **0611** (2006) 071 [arXiv:hep-ph/0608021].
- [23] J. Polonyi, Budapest preprint KFK-1977-93 (1977).
- [24] S. P. Martin, arXiv:hep-ph/9709356.
- [25] M. Ciuchini, A. Masiero, P. Paradisi, L. Silvestrini, S. K. Vempati and O. Vives, *Nucl. Phys. B* **783** (2007) 112 [arXiv:hep-ph/0702144].
- [26] See for example: W. Altmannshofer, A. J. Buras, S. Gori, P. Paradisi and D. M. Straub, *Nucl. Phys. B* **830** (2010) 17 [arXiv:0909.1333 [hep-ph]].
- [27] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *Nucl. Phys. B* **477** (1996) 321 [arXiv:hep-ph/9604387].
- [28] M. Misiak, S. Pokorski and J. Rosiek, *Adv. Ser. Direct. High Energy Phys.* **15** (1998) 795 [arXiv:hep-ph/9703442].
- [29] T. Besmer, C. Greub and T. Hurth, *Nucl. Phys. B* **609** (2001) 359 [arXiv:hep-ph/0105292].
- [30] F. Borzumati and A. Masiero, *Phys. Rev. Lett.* **57** (1986) 961;
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Phys. Rev. D* **53** (1996) 2442 [arXiv:hep-ph/9510309];
J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, *Phys. Lett. B* **357** (1995) 579 [arXiv:hep-ph/9501407].
- [31] S. T. Petcov, S. Profumo, Y. Takanishi and C. E. Yaguna, *Nucl. Phys. B* **676** (2004) 453 [arXiv:hep-ph/0306195].