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ÜBER-NATURALNESS: UNEXPECTEDLY LIGHT SCALARS FROM SUPERSYMMETRIC EXTRA DIMENSIONS

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Abstract

Standard lore asserts that quantum effects generically forbid the occurrence of light (non-pseudo-Goldstone) scalars having masses smaller than the Kaluza Klein scale, M_{KK}, in extra-dimensional models, or the gravitino mass, $M_{3/2}$, in supersymmetric situations. We argue that a hidden assumption underlies this lore: that the scale of gravitational physics, M_g, (e.g the string scale, M_s, in string theory) is of order the Planck mass, $M_p = \sqrt{8\pi G} \simeq 10^{18}$ GeV. We explore sensitivity to this assumption using the spectrum of masses arising within the specific framework of large-volume string compactifications, for which the ultraviolet completion at the gravity scale is explicitly known to be a Type IIB string theory. In such models the separation between M_g and M_p is parameterized by the (large) size of the extra dimensional volume, $\mathcal{V}$ (in string units), according to $M_p : M_g : M_{KK} : M_{3/2} \propto 1 : \mathcal{V}^{-1/2} : \mathcal{V}^{-2/3} : \mathcal{V}^{-1}$. We find that the generic size of quantum corrections to masses is of the order of $M_{KK} M_{3/2}/M_p \simeq M_p/\mathcal{V}^{5/3}$. The mass of the lighest modulus (corresponding to the extra-dimensional volume) which at the classical level is $M_\mathcal{V} \simeq M_p/\mathcal{V}^{3/2} \ll M_{3/2} \ll M_{KK}$ is thus stable against quantum corrections. This is possible because the couplings of this modulus to other forms of matter in the low-energy theory are generically *weaker* than gravitational strength (something that is also usually thought not to occur according to standard lore). We discuss some phenomenological and cosmological implications of this observation.

1 Introduction

Light scalar fields play a disproportionate role in our search for what lies beyond the Standard Model. On one hand, there are a variety of reasons why scalar fields are very useful: their expectation values provide the Lorentz-invariant order parameters for spontaneously breaking symmetries, at least within the weakly coupled limit that is under the best theoretical control. They are ubiquitous in supersymmetric and extra-dimensional theories, which are among the best motivated we have, where they arise as symmetry partners of particles having other (4D) spins. And if they are sufficiently light, scalars can do interesting things: they play important roles in many of the various cosmological scenarios that have been proposed to explain the mysteries of Cosmic Inflation (and its alternatives), Dark Matter and/or Dark Energy.

However, scalars are notoriously difficult to keep light enough to be relevant to present-day phenomenology. Because their masses are difficult to protect from receiving large quantum corrections, they are usually very sensitive to the ultra-violet (UV) sector. For instance, a light scalar ϕ coupled to a heavy field ψ through a coupling $g^2 \phi^2 \psi^2$, generically generates (see §2 below for more details) a loop correction to its mass of order

$$\delta m_\phi^2 \simeq \left(\frac{gM}{4\pi} \right)^2 , \tag{1.1}$$

from the graph shown in Fig. 1. Here M is the mass of the heavy ψ particle, and the factors of 4π are those appropriate to one loop (in four dimensions). Because of such contributions, it is often only possible to obtain $m_\phi \ll gM/4\pi$ if there is a conspiracy to cancel very precisely – often to a great many decimal places – these kinds of large loop contributions against other parameters in the underlying microscopic theory describing the ultra-violet physics.
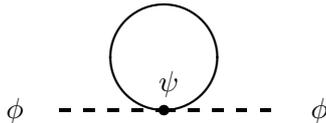

Figure 1: A large mass correction to a light scalar from a quartic coupling.

Because of this, light scalars are rarely found in a theory's low-energy limit, with the rare exceptions corresponding to when a (possibly approximate) symmetry protects the scalar from receiving large quantum corrections. On one hand, the comparative rarity of such naturally light scalars can be regarded as a feature and not a bug: it could explain why no fundamental scalars have yet been found experimentally. But on the other hand, this makes it difficult to keep scalars light enough to be useful for understanding the origin of electroweak symmetry breaking, or to be relevant for cosmology. It is this observation that lies at the core of the electroweak hierarchy problem, among others.

A great deal of attention has therefore gone towards exploring those cases where symmetries are able to protect light scalar masses without conspiracy. The known symmetries of this kind are:

(*i*) approximate shift symmetries, such as $\phi \to \phi + c$ and its nonlinear extensions, as appropriate for Goldstone and pseudo-Goldstone bosons; (*ii*) supersymmetry, for which cancellations between superpartners of opposite statistics suppress contributions from heavy particles whose mass is higher than the relevant supersymmetry-breaking scale (such as the gravitino mass, $M_{3/2}$); and (*iii*) extra dimensions, for which higher-dimensional symmetries (like gauge invariance or general covariance) can protect masses from receiving quantum corrections larger than the Kaluza-Klein (KK) scale, M_{KK}.

Since both the supersymmetry breaking scale and the KK scale cannot be too low without running into phenomenological difficulties, it is usually expected that the only scalars likely to be light enough to be relevant at very low energies are Goldstone (or pseudo-Goldstone) bosons, despite the fact that many candidates for fundamental theories count an abundance of scalars among their degrees of freedom. In the absence of a protective symmetry none of these scalars would survive to be light enough to observe experimentally. This is true in particular for string theory, which has both supersymmetry and extra dimensions as sources for its many scalars.

Until recently this expectation has proven hard to test, because of the technical difficulties associated with exploring the spectrum of excitations near 'realistic' vacua, far from the supersymmetric configurations for which calculations are best under control. What is new in recent times is the ability to explore non-supersymmetric vacua to see how massive the various would-be light scalars become once supersymmetry breaks.

Particularly interesting from this point of view are large volume (LV) flux compactifications [1] of Type IIB string vacua. These have the property that the same flux that fixes the scalar moduli also breaks supersymmetry, producing a very predictive spectrum of scalars whose masses depend in a predictable way on the (large) extra-dimensional volume. Since LV models in particular predict (at the classical level) the existence of non-Goldstone scalars that are parametrically much lighter than both the KK and the gravitino mass, common lore would lead one to expect that there are large loop corrections to these classical mass predictions, which lift their masses up to either M_{KK} or $M_{3/2}$.

Our purpose in this paper is estimate the generic size of radiative corrections to scalar masses in LV models, in order to estimate how these compare with the predicted classical values. We find that since loop effects are smaller than the classical predictions, the classical masses really do provide a good approximation to the full result. LV models therefore provide one of the few examples of light (non-Goldstone) scalars whose masses are naturally smaller than both the SUSY breaking and KK scales. It turns our that their masses are nonetheless (*über*) natural because of an interesting interplay between the scale of supersymmetry breaking and the size of the extra dimensions.

We organize our observations as follows. First, §2 reviews the generic size of one-loop corrections to scalar masses in four- and higher-dimensional theories, in order to set conventions and establish that our estimates reproduce standard results in standard situations. Next, §3 reviews

some of the properties about the masses and couplings of scalars that arise in the LV string vacua. This is followed, in §4, by a discussion of how large the radiative corrections are to these masses in LV models, with the estimates compared with (and shown to agree with) the results of explicit string loop calculations, when these can be done. Finally we summarize our conclusions in §5.

2 Generic loop estimates

We start with a discussion of the generic size of loop effects in four- and extra-dimensional models, contrasting how the supersymmetric case differs from the non-supersymmetric one. This section is meant to provide tools for later application, and because it does not contain new material (although perhaps presented with a slightly different point of view) it can be skipped by the reader in a hurry.

2.1 Technical naturalness in 4 dimensions

To set the benchmark for what should be regarded to be a generic quantum mass correction, consider a model involving two real scalar fields, one of which (ϕ) is light while the other (ψ) is heavy:

$$-\mathcal{L}(\phi, \psi) := \frac{1}{2}\Big[(\partial\phi)^2 + (\partial\psi)^2\Big] + \frac{1}{2}\Big[m^2\phi^2 + M^2\psi^2\Big] + \frac{1}{24}\Big[\lambda_\phi\phi^4 + 6\,g^2\phi^2\psi^2 + \lambda_\psi\psi^4\Big], \quad (2.1)$$

where $M^2 \gg m^2 > 0$.

Since ψ is heavy, we may integrate it out to obtain an effective theory describing the self-interactions of ϕ at energies below M. One of the contributing graphs is given in Fig. 1, which when evaluated at zero momentum in dimensional regularization[1] leads to

$$\begin{aligned}
\delta m^2 &\simeq g^2\mu^{2\epsilon}\int\frac{\mathrm{d}^n k}{(2\pi)^n}\left(\frac{1}{k^2 + M^2}\right) + \cdots \\
&\simeq c\frac{\mu^{2\epsilon}}{2\epsilon}\left(\frac{gM}{4\pi}\right)^2 + \cdots,
\end{aligned} \quad (2.2)$$

where $2\epsilon = n - 4 \to 0$ is the dimensional regulator, whose $1/\epsilon$ divergence is explicitly displayed by factoring it out of the dimensionless coefficient c. This divergent contribution is renormalized into the light-scalar mass parameter, $m^2(\mu)$, along with its other UV-divergent contributions.

The renormalized parameter m^2 acquires a renormalization-group (RG) running that at one loop is of order

$$\mu\frac{\partial m^2}{\partial\mu} \simeq \Big[c_1\,g^2 M^2 + (c_2\lambda_\phi + c_3\,g^2)m^2\Big], \quad (2.3)$$

where c_i are dimensionless constants (into which factors of $1/(4\pi)^2$ are absorbed), and the m^2 term arises from the graph of Fig. 2 using the quartic ϕ^4 interaction, and from the (wave-function)

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[1]We deliberately do not phrase this discussion in terms of a momentum cutoff, Λ, as is often done, for reasons described in more detail elsewhere [2]. Rather than meaning that naturalness arguments are irrelevant, it means as this section shows, that they can be usefully recast in terms of ratios of renormalized mass parameters.

renormalization of the kinetic terms. This may be integrated to give

$$
\begin{aligned}
m^2(t) &= e^{\int_0^t dx \, (c_2 \lambda_\phi + c_3 g^2)} \left[m_0^2 + c_1 \int_0^t dx \, g^2 M^2 e^{-\int_0^x du \, (c_2 \lambda_\phi + c_3 g^2)} \right] \\
&\simeq m_0^2 \left(\frac{\mu}{\mu_0} \right)^{c_2 \lambda_\phi + c_3 g^2} - \frac{c_1 g^2 M^2}{c_2 \lambda_\phi + c_3 g^2} \left[1 - \left(\frac{\mu}{\mu_0} \right)^{c_2 \lambda_\phi + c_3 g^2} \right],
\end{aligned} \tag{2.4}
$$

where $m_0^2 = m^2(\mu = \mu_0)$ and $t = \ln(\mu/\mu_0)$. Here the second, approximate, equality neglects the μ dependence of λ_ϕ, g^2 and M^2.

This expression shows that $m^2(\mu)$ can only be much smaller than M^2 at scales $\mu \ll M$ if $m^2(M)$ is large – of order $gM^2/(4\pi)^2$ – in order to very precisely cancel with the evolution from $\mu = M$ to $\mu \ll M$. A parameter like this, whose small size cannot be understood at any scale μ where one chooses to ask the question, is called 'technically unnatural.'[2] Although we know of many hierarchies of mass in Nature, we know of none for which the small scale emerges in this kind of technically unnatural way.[3] Instead, either m^2 really is measured to be of order $gM^2/(4\pi)^2$ (and so is not unnaturally small), or there exists a (possibly approximate) symmetry [3] that ensures that corrections to the parameter m^2 are never larger than of order m^2 itself.

The goal of this and later sections is to estimate the size of these loop corrections to scalar masses from several sources in extra-dimensional models, viewing them as lower bounds on the masses of the physical scalars that can naturally emerge from such models.

Relevant and irrelevant interactions

In the previous example the coupling between light and heavy sectors was through a marginal interaction, described by the dimensionless coupling g. But there are also other kinds of dangerous interactions that can generate large corrections to small scalar masses. One class of these consists of super-renormalizable – or relevant, in the RG sense – interactions, for which the corresponding couplings have dimension of a positive power of mass (in units where $\hbar = c = 1$).
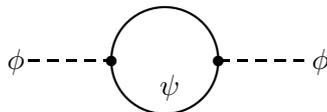

Figure 2: A graph contributing to a scalar mass through cubic couplings.

For instance, if one were to supplement the above theory with super-renormalizable cubic interactions, that break the symmetry $\phi \to -\phi$,

$$
-\Delta\mathcal{L} = \frac{1}{6} \left[\xi_\phi \phi^3 + 3h \, \phi \, \psi^2 \right], \tag{2.5}
$$

---

[2]Just how repulsive this is depends on how detailed the cancellation must be. Although reasonable people can differ on how repelled they are by cancellations of 1 part in 100 or 1000, most would agree that cancellations of 5 decimal places or more would be unprecedented.

[3]A possible exception is the 4D cosmological constant, for which no completely convincing technically natural proposal has been made. We regard the jury to be out on whether a technically natural solution to this particular problem is possible.

then evaluating the graph of Fig. 2 gives the new contribution

$$\delta m^2 \simeq h^2 \int \frac{\mathrm{d}^n k}{(2\pi)^n} \left(\frac{1}{k^2 + M^2}\right)^2 + \cdots, \tag{2.6}$$

and so

$$\mu \frac{\partial m^2}{\partial \mu^2} \simeq c' \left(\frac{h^2}{16\pi^2}\right) + \cdots. \tag{2.7}$$

Barring cancellations one expects m^2 to be larger than either $(h/4\pi)^2$ or $(gM/4\pi)^2$, whichever is largest.

In general the underlying theory could also include non-renormalizable – or irrelevant, in the RG sense – effective interactions, such as

$$-\Delta\mathcal{L}_{\mathrm{nr}} = \frac{1}{48\Lambda^2} \phi^2 \psi^4, \tag{2.8}$$

where $\Lambda \gg M \gg m$ is some still-higher scale that is already integrated out to obtain our starting lagrangian, eq. (2.1). This kind of interaction generates a contribution to the light scalar mass (see Fig. 3) that is of order

$$\delta m^2 \simeq \frac{1}{\Lambda^2} \left[\int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{1}{k^2 + M^2}\right]^2 \propto \left(\frac{1}{16\pi^2}\right)^2 \frac{M^4}{\Lambda^2}, \tag{2.9}$$

and so contributes contributions to m^2 that are small relative to those already considered.
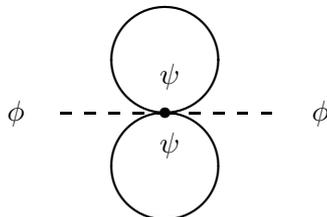

Figure 3: A large mass correction to a light scalar from an irrelevant coupling.

2.2 Extra dimensions without SUSY

Two new issues that arise when considering naturalness in extra-dimensional models (see refs. [4] for other discussions of loops in extra dimensions) are higher-dimensional kinematics and symmetries. An extra-dimensional generalization of the two-scalar model described above provides the simplest context for discussing the first of these, while higher dimensional gravity provides the most commonly encountered framework for the second. We therefore consider each of these examples in turn.

Scalar fields

Start first with a light and heavy scalar field in $D = 4 + d$ dimensions, with lagrangian

$$-\mathcal{L} = \frac{1}{2}\left[(\partial\Phi)^2 + (\partial\Psi)^2\right] + \frac{1}{2}\left[m^2\Phi^2 + M^2\Psi^2\right] + \frac{g_3}{2} \Phi\Psi^2 + \frac{g_4^2}{4} \Phi^2\Psi^2 + \cdots, \tag{2.10}$$

and use (for now) a flat background metric

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu + L^2 \delta_{mn}\, dy^m dy^n \,, \tag{2.11}$$

whose extra-dimensional volume is $V = L^d$. Since a canonically normalized scalar field in D dimensions has engineering dimension $(\text{mass})^{(D-2)/2}$, the cubic and quartic couplings generically are RG-irrelevant, having negative mass dimension

$$g_3 \sim \left(\frac{1}{\Lambda}\right)^{(D-6)/2} \quad \text{and} \quad g_4 \sim \left(\frac{1}{\Lambda}\right)^{(D-4)/2} \,. \tag{2.12}$$

We next integrate out the massive field Ψ, assuming the masses satisfy the hierarchy[4] $M^2 \gg m^2 \gg M_{KK}^2 := 1/L^2$. In this limit the sum over discrete KK modes is well described by a continuous integral, and so the contribution of the cubic interaction (from the graph of Fig. 2) is of order

$$\delta m^2 \simeq g_3^2 \int \frac{d^n k}{(2\pi)^n} \left(\frac{1}{k^2 + M^2}\right)^2 \propto g_3^2 M^{D-4} \simeq \left(\frac{M}{\Lambda}\right)^{D-6} M^2 \,, \tag{2.13}$$

where this time $n = D - 2\epsilon \to D$ rather than 4. The result obtained from using the quartic interaction in Fig. 1 is similarly estimated to be

$$\delta m^2 \simeq g_4^2 \int \frac{d^n k}{(2\pi)^n} \left(\frac{1}{k^2 + M^2}\right) \propto g_4^2 M^{D-2} \simeq \left(\frac{M}{\Lambda}\right)^{D-4} M^2 \,. \tag{2.14}$$

These both differ by the factor $(M/\Lambda)^{D-4} = (M/\Lambda)^d$ relative to the corresponding 4D example (for which $d = 0$), and these extra powers of M arise ultimately due to the additional phase space available for the extra-dimensional loop momenta.[5] If $M \simeq \Lambda$ then the corrections are generically of order M^2 in any dimension, but a proper exploration of contributions at Λ should really be done using whatever UV completion kicks in at this scale. The main lesson here is that extra-dimensional kinematics make most interactions irrelevant in the technical sense, leading to higher powers of the heavy scale M in the generic contribution to δm^2.

Gravity

In practice, the fields of most interest in higher dimensions are various types of gauge and gravitational fields. The crucial difference between these and the scalar fields described heretofore is the existence of local gauge symmetries (or general covariance) that keep these fields massless in the extra-dimensional theory. Their only masses in 4D are therefore those due to the KK reduction. We now explore the sensitivity of these masses to integrating out particles that are both heavier and lighter than the KK scale.

To see the effects of integrating out a heavy field in this case consider a massive scalar coupled to gravity,

$$-\frac{\mathcal{L}}{\sqrt{-g_{(D)}}} = \lambda_0 + \frac{1}{2\kappa_0^2}\,\mathcal{R} + \frac{1}{2}(\partial\Psi)^2 + M^2\Psi^2 \,, \tag{2.15}$$

---

[4]We do not attempt to track factors of 2π, so ignore these in the definition of M_{KK}.

[5]Alternatively these additional powers of M relative to the 4D results found earlier can be regarded as being due to the necessity to sum over the tower of KK modes (see Appendix A).

where $\mathcal{R} = g^{MN}\mathcal{R}_{MN}$ denotes the metric's Ricci scalar, λ_0 is a (bare) cosmological constant, and $\kappa_0^2 = 8\pi G_D := M_g^{-(D-2)/2}$ is the (bare) reduced gravitational coupling constant, which also defines the higher-dimensional Planck scale, M_g. A semiclassical treatment [5] assumes we require $\lambda \ll M_g^D$ and that the heavy particle mass satisfies $M \ll M_g$.

As before, integrating out the massive field Ψ leads to many new effective interactions for the remaining light field, which are local so long as the heavy particle's Compton wavelength, is much shorter than the size of the extra dimensions, $M \gg M_{KK}$. These can be organized in order of increasing dimension, with the coefficient of each involving the appropriate power of M, as required on dimensional grounds. Because in the present case the low-energy field is the metric, the form of these interactions is strongly restricted by general covariance to take the form of a curvature expansion,

$$-\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g_{(D)}}} = \lambda + \frac{1}{2\kappa^2}\mathcal{R} + a_2 M^{D-4}\mathcal{R}_{MNPR}\mathcal{R}^{MNPR} + \cdots, \tag{2.16}$$

where all possible terms involving curvatures and their derivatives are contained in the ellipses, while λ and κ^2 are appropriately renormalized

$$\lambda = \lambda_0 + a_0 M^D \quad \text{and} \quad \frac{1}{2\kappa^2} = \frac{1}{2\kappa_0^2} + a_1 M^{D-2}, \tag{2.17}$$

with a_i dimensionless constants, and so on.

In general, the condition $M \ll M_g$ implies $\kappa^2 \simeq \kappa_0^2$, making the Einstein term largely insensitive to the effects of integrating out heavy particles. The same is usually not true for λ and λ_0 because the Einstein equations imply $\mathcal{R} \simeq \mathcal{O}(\kappa^2\lambda)$, and calculability demands $\mathcal{R} \simeq 1/L^2 = M_{KK}^2$ be much smaller than M_g^2. For non-supersymmetric theories taking $M \gg 1/L$ and $\kappa^2\lambda \simeq \mathcal{O}(M_{KK}^2)$ usually means fine-tuning λ_0 to ensure that $\lambda \sim \mathcal{O}(M_{KK}^2 M_g^{D-2}) \ll M^D$.

The upshot is this: integrating out a field with mass $M \gg M_{KK}$ just leads to the addition of local interactions to the higher-dimensional action. These do not qualitatively change the low-energy consequences so long as it is the lowest-dimension interactions that are of physical interest (like the Einstein action) and provided that the most general interactions are included in the action from the start. This is because the condition for the validity of the semiclassical treatment in terms of an extra-dimensional field theory requires $M \ll M_g$, ensuring that the new contributions are swamped by those involving M_g. An understanding of the contributions at scale M_g then should be done using the UV completion at the gravity scale, which within string theory would potentially involve a full string calculation.

Integrating out KK modes

The effective theory becomes four-dimensional at energies of order M_{KK} and below, so once these scales are integrated out we can no longer use higher-dimensional symmetries (like extra-dimensional general covariance) to restrict the form of the result.

We wish to track how the integrating out of modes with masses at the KK scale and below depends on the underlying scales M_{KK} or M_g. Within the semiclassical approximation any such an integration arises as an expansion about a background field, $\overline{g}_{MN}$, so that

$$g_{MN} = \overline{g}_{MN} + \kappa h_{MN} \,. \tag{2.18}$$

It is also useful to explicitly scale out the local linear size of the extra dimensions, $e^{u(x)}$, (measured using the background geometry) from the total metric

$$g_{MN}\mathrm{d}x^M\mathrm{d}x^N = \omega\, e^{-du}\, \hat{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{2u}\hat{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n + \text{off-diagonal terms}\,, \tag{2.19}$$

where the factor pre-multiplying the 4D metric is chosen to ensure no u-dependence in the 4D Einstein action (*i.e.* the 4D Einstein frame).

The factor $\omega := (M_p/M_g)^2$ numerically converts to 4D Planck units, and the vacuum value $\langle e^u \rangle \propto M_g L$ provides a dimensionless measure of the extra-dimensional linear size[6] (and so $\langle e^{du} \rangle \simeq (M_g L)^d := \mathcal{V}$). Recall the 4D Planck scale is related to the dimensionless volume, $\mathcal{V} = V M_g^d$, by $M_p^2 = V M_g^{D-2} = \mathcal{V} M_g^2$, and so $\omega = M_p^2/M_g^2 = \mathcal{V}$, and $M_g \simeq M_p/\mathcal{V}^{1/2}$. Using $\sqrt{-g_{(D)}} = \sqrt{-\hat{g}_{(4)}}\, \sqrt{\hat{g}_{(d)}}\, \omega^2 e^{-du}$ and $\int \mathrm{d}^d y \propto M_g^{-d}$, we find the 4D Einstein term becomes

$$
\begin{aligned}
M_g^{D-2} \int \mathrm{d}^d y \sqrt{-g_{(D)}}\, g^{\mu\nu}\mathcal{R}_{\mu\nu} &= \omega\, M_g^2 \sqrt{-\hat{g}_{(4)}}\, \hat{g}^{\mu\nu}\hat{R}_{\mu\nu} + \cdots \\
&= M_p^2 \sqrt{-\hat{g}_{(4)}}\, \hat{g}^{\mu\nu}\hat{R}_{\mu\nu} + \cdots \,, \tag{2.20}
\end{aligned}
$$

as required.

Expanding the action in powers of fluctuations and focussing on the 4D scalar KK modes contained within $h^m{}_p := \overline{g}^{mn} h_{np}$ — generically denoted φ^i — similarly gives the following dimensionally reduced kinetic terms

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{kin}} \simeq \frac{1}{2}M_g^{D-2} \int \mathrm{d}^d y \sqrt{-g_{(D)}}\, g^{\mu\nu}\mathcal{R}_{\mu\nu} &= \frac{\omega}{2} M_g^2 \int \mathrm{d}^d y \sqrt{-\hat{g}_{(4)}}\, \hat{g}^{\mu\nu}\mathcal{H}^{mn}{}_{pq}\, \partial_\mu h^p{}_m \partial_\nu h^q{}_n + \cdots \\
&:= \frac{M_p^2}{2} \sqrt{-\hat{g}_{(4)}}\, \hat{g}^{\mu\nu}\mathcal{G}_{ij}(\varphi)\partial_\mu\varphi^i\partial_\nu\varphi^j + \cdots \,. \tag{2.21}
\end{aligned}
$$

where $\mathcal{H}^{mn}{}_{pq}$ are a set of coefficients depending on the $h^m{}_p$ (but not their derivatives), while $\mathcal{G}_{ij}(\varphi)$ denotes the target-space metric for the dimensionless 4D fields φ^i. The detailed form of $\mathcal{G}_{ij}$ is not important beyond the fact that it contains no additional dependence on $\mathcal{V}$, and so is generically $\mathcal{O}(1)$ in the large-$\mathcal{V}$ limit.

By contrast, the contributions to the scalar potential for the φ^i instead scale with M_g and L

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[6]We take $V \simeq L^d$ and $R \simeq 1/L^2$ for the same scale L, and by so doing ignore features that might arise within strongly warped spacetimes (for which the low-energy sector can be strongly localized), or within some negatively curved spaces (for which the scales associated with volume and curvature can radically differ).

according to

$$-\mathcal{L}_{\text{pot}} \simeq M_g^{D-2} \int \mathrm{d}^d y \sqrt{-g_{(D)}} \, g^{mn} \mathcal{R}_{mn}$$

$$= \omega^2 \, M_g^2 \int \mathrm{d}^d y \, e^{-du} \sqrt{-\hat{g}_{(4)}} \, (e^{-2u} \hat{g}^{mn}) \mathcal{K}^{pq}{}_{rs} \, \partial_m h^r{}_p \, \partial_n h^s{}_q + \cdots$$

$$:= \frac{M_p^4}{\mathcal{V}^{1+2/d}} \sqrt{-\hat{g}_{(4)}} \, U(\varphi)$$

$$= M_{KK}^2 M_p^2 \sqrt{-\hat{g}_{(4)}} \, U(\varphi) \,, \tag{2.22}$$

which uses $e^{(d+2)u} = \mathcal{V}^{1+2/d}$ and $M_{KK}^2/M_p^2 = (M_{KK}^2/M_g^2)(M_g^2/M_p^2) \simeq \mathcal{V}^{-2/d}\mathcal{V}^{-1}$. Again, the detailed form of $\mathcal{K}^{pq}{}_{rs}$ and $U(\varphi)$ are not important, beyond the fact that they generically do not contribute to the $\mathcal{V}$ dependence of the result.

Once canonically normalized, $\varphi \propto \phi/M_p$, we find the following schematic mass terms, cubic and quartic interactions

$$-\frac{\mathcal{L}_2}{\sqrt{-g_{(4)}}} \simeq (\partial\phi)^2 + M_{KK}^2 \, \phi^2$$

$$-\frac{\mathcal{L}_3}{\sqrt{-g_{(4)}}} \simeq \frac{1}{M_p}\phi(\partial\phi)^2 + \frac{M_{KK}^2}{M_p} \, \phi^3 \tag{2.23}$$

$$\text{and} \quad -\frac{\mathcal{L}_4}{\sqrt{-g_{(4)}}} \simeq \frac{1}{M_p^2}\phi^2(\partial\phi)^2 + \frac{M_{KK}^2}{M_p^2} \, \phi^4 \,,$$

and so on. These show that the generic mass is $M_{KK} \simeq M_p/\mathcal{V}^{(1+2/d)/2}$ (as expected), and although the low-energy *derivative* interactions are Planck suppressed, those in the scalar potential have a universal additional suppression by a factor of $M_{KK}^2/M_p^2 = 1/\mathcal{V}^{1+2/d} = (M_g^2/M_p^2)^{1+2/d}$ relative to generic Planck size.

A similar analysis for the curvature-squared terms shows that these introduce three kinds of 4D interactions: (i) $\mathcal{O}(1)$ 4-derivative interactions, $\sim k_4(\varphi)(\partial\varphi)^4$; (ii) $\mathcal{O}(M_{KK}^2)$ two-derivative interactions, $\sim M_{KK}^2 k_2(\varphi)(\partial\varphi)^2$; and $\mathcal{O}(M_{KK}^4)$ potential terms, $\sim M_{KK}^4 k_0(\varphi)$. Each is therefore suppressed relative to its counterpart (if this exists) coming from the Einstein term by an additional factor of $M_{KK}^2/M_p^2 = 1/\mathcal{V}^{1+2/d}$.

Provided λ is chosen to allow classical solutions for which $\mathcal{R} \simeq \kappa^2\lambda \simeq 1/L^2 \sim M_{KK}^2$, as discussed above, the contributions of the cosmological constant term to the 4D dimensionally reduced interactions scale in the same way as do those coming from the Einstein term.

Naturality of the KK couplings

It is noteworthy that, from a 4D perspective, the couplings of the KK modes amongst themselves are *weaker* than Planckian, yet they are stable against radiative corrections.

As we've seen, the dominant contribution from the scales much larger than M_{KK} come from scales $M \gtrsim M_g$, which must be performed within the theory's UV completion. If this is a string theory, the contribution to the low-energy action from integrating out string states is what

gives the initial higher-dimensional gravity action, plus a variety of higher-derivative corrections involving powers of the curvature and other low-energy fields. Since our initial estimate for the size of the interactions comes from the higher-dimensional Einstein term, the leading corrections in the UV come from dimensionally reducing terms involving more derivatives than this. If it is a curvature-squared term that dominates, then we expect the largest corrections from these scales to be suppressed relative to the leading terms by of order M_{KK}^2/M_g^2.

The sole exception to this happy picture of insensitivity to higher-scale physics is the contribution to the extra-dimensional cosmological constant, although this is also not dangerous once $\lambda \simeq M_{KK}^2/\kappa^2 \simeq M_{KK}^2 M_g^{D-2}$ is tuned to be small enough to allow the extra dimensions to be large in the first place. As we shall see, even this problem need not arise when couched in a supersymmetric context, since in this case higher-dimensional supersymmetry usually requires $\lambda = 0$.

Finally, what of the naturality of the 4D scalar masses from loops with $M \lesssim M_{KK}$? Since these are not protected by higher-dimensional symmetries, they should be analyzed within the effective 4D theory. As shown above, it is the contributions of the relevant and marginal – i.e. cubic and quartic – interactions that are then the most dangerous. But because all of the interactions in the scalar potential are suppressed by M_{KK}^2/M_p^2 relative to Planck strength, their use in loop graphs generates interactions that are generically suppressed relative to those we start with by similar factors. This implies they are of similar order to the contributions of higher-derivative corrections in the extra-dimensional theory just considered.

Similarly, graphs using the derivative interactions give much the same result, since although these involve couplings unsuppressed (relative to M_p) by powers of $1/\mathcal{V}$, they are more UV divergent and so depend more strongly on the mass of the heaviest 4D state that can circulate within the loop. But this mass is again M_{KK}, since for masses much higher than this the restrictions of higher-dimensional general covariance limit the result to one of the local interactions considered above. For instance, using these estimates in Fig. 2 then gives

$$\delta m^2 \simeq \frac{1}{M_p^2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^4}{(k^2 + m^2)^2} \simeq \frac{c\mu^{2\epsilon}}{\epsilon} \left(\frac{M_{KK}^4}{M_p^2} \right), \qquad (2.24)$$

again implying corrections that are suppressed by M_{KK}^2/M_p^2.

These arguments indicate that the dominant corrections to the 4D scalar potential are of order $\delta V \simeq \mathcal{O}(M_{KK}^4)\delta U(\varphi)$. Notice that this implies the corresponding contributions to the 4D vacuum energy are $\delta V \simeq M_{KK}^4 \simeq 1/L^4$, in agreement with explicit Casimir energy calculations [7].

Moduli

It often happens that accidental symmetries in the leading order action imply that some of the KK scalars are massless once dimensionally reduced. This would happen for the volume modulus, e^u, in particular, if the equations of motion coming from the leading action were scale invariant

(as would be true for the Einstein equations in the absence of a cosmological term, or for the lowest-derivative terms in most higher-dimensional supergravities).

In this case the dimensionally reduced mass for the corresponding moduli comes from next-to-leading effects that break the relevant symmetry. These could be from loop effects in the low-energy 4D theory, in which case the above estimates indicate their masses would be expected to be of order $M_{\text{mod}} \simeq M_{KK}^2/M_p \ll M_{KK}$ rather than being precisely massless.[7] Alternatively, the dominant corrections could come from loop effects in the higher-dimensional theory, which we've seen are equivalent to use of sub-dominant terms in the derivative expansion (such as from curvature-squared or higher) when compactifying. If it is curvature-squared terms that dominate,[8] then the discussion above shows we again expect masses of order $M_{\text{mod}} \simeq M_{KK}^2/M_p$. Notice that masses these size are also no larger than the generic size of radiative corrections in the low-energy theory, based on the estimates given above.

2.3   Supersymmetric effects

For supersymmetric systems, the estimates just given can differ in several important ways.

- *Extra-dimensional non-renormalization theorems:* For higher dimensional supergravity it is the expectation of one of the fields (the dilaton) that plays the role of the loop-counting parameter. Often its appearance in the leading derivative expansion of the action is restricted by supersymmetry, in which case extra-dimensional loops must necessarily contribute suppressed both by powers of the coupling constant *and* low-energy factors. For instance, in string theory it is often true that higher-order contributions in the string coupling, g_s, necessarily only arise for terms that are also sub-leading in powers of the string scale, $\alpha' = 1/M_s^2$ [9].

  It is also true that supersymmetry can raise the order in the derivative expansion of the first subdominant contributions to the action that arise even without loops. For instance, in the Type IIB models of later interest, the first higher-curvature corrections that arise at string tree level involve four powers of the curvature, as opposed to the generic expectation of curvature squared [10].

- *Four-dimensional non-renormalization theorems:* If the supersymmetry breaking scale should be much smaller than M_{KK}, then the effective 4D description can be written as a 4D supergravity, possibly supplemented by soft supersymmetry-breaking terms. In this case the most dangerous relevant and marginal scalar interactions appear in the holomorphic superpotential, $W(\phi)$, and so are protected by 4D non-renormalization theorems [11, 12, 13, 14].

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[7]Notice that if $M_{KK} \simeq 10^{-3}$ eV then $M_{\text{mod}} \lesssim 10^{-30}$ eV, showing that if the observed vacuum energy density could be arranged to be dominated by a KK energy, then the presence of moduli would produce a quintessence cosmology, with no additional tunings required to ensure small enough scalar masses [8].

[8]For an example where curvature-squared terms do *not* dominate, see the supersymmetric estimate below.

If supersymmetry is not broken these exclude corrections to all orders in perturbation theory. Corrections become possible once supersymmetry is broken, but are further suppressed by the relevant supersymmetry breaking scale. When computing explicit loops, these suppressions come about through the usual cancellations of bosons against fermions, together with mass sum rules like [15]

$$\sum_s (-)^{2s}(2s+1)\text{Tr } M_s^2 \simeq M_{3/2}^2 \,, \tag{2.25}$$

where $M_{3/2}$ is the 4D gravitino mass.

- *Additional extra-dimensional fields:* A third way in which supersymmetric theories can differ is through their extra-dimensional field content, which always involves more fields than just gravity. Interactions involving these other fields sometimes provide the dominant contributions to quantities like the masses of moduli. The most familiar example of this sort occurs when the background supergravity solution involves nonzero 3-form flux fields, G_{mnp}, whose presence gives some of the moduli masses (as in 10D Type IIB flux compactifications). In this case it is the term $\mathcal{L} = \sqrt{-g}\, G^{mnp}G_{mnp} \propto 1/\mathcal{V}^2$ that dominates these masses, leading to $M_{\text{mod}} \simeq M_p/\mathcal{V} \gg M_{KK}^2/M_p \simeq M_p/\mathcal{V}^{4/3}$.

3 Large-volume (LV) string models

The large-volume (LV) framework [1] is a scenario for moduli stabilization for Calabi-Yau compactifications within Type IIB string theory. In generic Type IIB flux compactifications, the presence of background 3-form fluxes alone can stabilize the dilaton and the complex structure moduli of the underlying Calabi-Yau geometry [17]. The Kähler moduli can then be stabilized by the non-perturbative effects localized on four cycles associated with various branes that source the geometries [18]. The LV scenario identifies an interesting subclass for which the volume modulus is naturally stabilized at exponentially large volumes,

$$\mathcal{V} \propto \exp\left(c\tau_s\right) \,, \tag{3.1}$$

where τ_s is the size of a (comparatively much smaller) blow-up cycle of a point-like singularity in the underlying geometry, while c is an order-unity constant.

The framework applies to a large class of Calabi-Yau compactifications since there are only two requirements for its implementation [19]: (i) there must be at least one of the Kähler moduli must be the blow-up mode, τ_s, of a point-like singularity; and (ii) the number of complex structure moduli have to be greater than the the number of Kähler moduli (in order for the Euler number of the Calabi-Yau space to have the sign required for the potential to have a minimum for large $\mathcal{V}$).

A key ingredient is the inclusion of the leading order α' correction to the 4D Kähler potential, since this is what generates the potential with a minimum with compactification volumes that

are exponentially large in τ_s. Furthermore, because τ_s scales as the inverse of the the value of the dilaton,

$$\tau_s \propto \frac{1}{g_s} , \tag{3.2}$$

and so is given as a ratio of integer flux quanta. Thus the framework naturally generates an exponentially large volume of compactification from integer flux quanta, with $\mathcal{V}$ passing through an enormous range of values as the fluxes range through a range of order tens.

In what follows we review some relevant aspects of these compactifications with an emphasis on the mass scales and strength of couplings.

3.1 Masses and couplings

LV models enjoy a rich pattern of particle masses and couplings, for which it is useful for many purposes to express in 4D Planck units. Our goal here is to express how these quantities scale as functions of the dimensionless extra-dimensional volume, expressed in string units: $\mathcal{V} \simeq \alpha'^3 \int \sqrt{g_6} \simeq (L/\ell_s)^6 \gg 1$.

Masses

In terms of $\mathcal{V}$ and the 4D Planck scale, M_p, the largest scale in the excitation spectrum is the string scale itself, $M_s^{-2} = \ell_s^2 \simeq \alpha'$, which (ignoring factors of g_s) is of order

$$M_s \simeq \frac{M_p}{\mathcal{V}^{1/2}} . \tag{3.3}$$

This characterizes the order of magnitude of the masses of generic string excitation levels.

If the linear sizes of the various extra dimensions are all of the same order, L, then since $\mathcal{V} \propto (L/\ell_s)^6$, the scaling with $\mathcal{V}$ of the masses of Kaluza-Klein (KK) excitations is

$$M_{KK} \simeq \frac{1}{L} \simeq \frac{M_s}{\mathcal{V}^{1/6}} \simeq \frac{M_p}{\mathcal{V}^{2/3}} . \tag{3.4}$$

The presence of fluxes stabilizes the complex structure moduli, or what would otherwise have been massless KK zero modes. These are systematically light compared with the KK scale because of the $\mathcal{V}$ suppression of the various fluxes, due to the quantization conditions of the form $\int F \simeq N\alpha'$, for an appropriate 3-form, F_{mnp}. Masses obtained from the flux energy, $\int \mathrm{d}^6 x \sqrt{-g_{10}}\, F^2 \propto \mathcal{V}^{-2}$, are of order

$$M_{cs} \simeq \frac{M_s}{\mathcal{V}^{1/2}} \simeq \frac{M_p}{\mathcal{V}} . \tag{3.5}$$

The Kähler moduli survive flux compactification unstabilized to leading order, and so naively might be expected to receive a mass that is parametrically smaller than those of the complex-structure moduli. For instance, small Kähler moduli like τ_s are stabilized by the nonperturbative terms, $W \simeq e^{-c\tau_s}$, in the low-energy 4D superpotential, and at the potential's minimum these are comparable to the α' corrections, implying the relevant part of the potential depends on the

volume like $1/\mathcal{V}^3$. However, because the Kähler potential for these moduli is $K = -2\ln\mathcal{V}$, their kinetic term is also proportional to $1/\mathcal{V}$, implying that the volume-dependence of the mass of these moduli is again of order

$$M_k \simeq \frac{M_s}{\mathcal{V}^{1/2}} \simeq \frac{M_p}{\mathcal{V}} \,. \tag{3.6}$$

The Kähler modulus corresponding to the volume of the compactification also receives its mass from the leading α' correction. However for this modulus $\partial\mathcal{V}$ depends nontrivially on $\mathcal{V}$ (unlike for the small moduli like τ_s), leading to a kinetic term of order $\partial\overline{\partial}K \simeq \partial\overline{\partial}\mathcal{V}/\mathcal{V}$ and a quadratic term in the potential that is of order $\partial\overline{\partial}\mathcal{V}/\mathcal{V}^4$. The result is a volume-modulus mass whose $\mathcal{V}$-dependence is of order

$$M_V \simeq \frac{M_s}{\mathcal{V}} \simeq \frac{M_p}{\mathcal{V}^{3/2}} \,. \tag{3.7}$$

For many compactifications Kähler moduli come with a range of sizes, and there are often many having volumes $\tau_i \gg \tau_s$. These tend to be exponentially suppressed in the superpotential, $W \simeq e^{-c_i\tau_i} \ll e^{-c\tau_s} \simeq 1/\mathcal{V}$, and so at face value also appear to have exponentially small masses when computed using only the leading α' corrections. However for any such moduli it is the sub-leading corrections to the scalar potential (like string loops or higher orders in α') that instead dominate their appearance in the potential. Consequently these states are systematically light relative to the generic moduli discussed above. The leading string-loop contribution turns out to be dominant, and so these moduli generically have masses of order

$$M \simeq \frac{M_p}{\mathcal{V}^{5/3}} \,. \tag{3.8}$$

Finally, the gravitino mass associated with supersymmetry breaking is itself of order

$$M_{3/2} \simeq \frac{M_s^2}{M_p} \simeq \frac{M_s}{\mathcal{V}^{1/2}} \simeq \frac{M_p}{\mathcal{V}} \,. \tag{3.9}$$

Notice for the purposes of the naturalness arguments that the masses of the volume modulus and the other large Kähler moduli are parametrically lighter than both the gravitino mass and the KK scale.

Couplings

The potential term associated with the complex structure and Kähler moduli scales as $\mathcal{V}^{-2}$ as it arises by dimensionally reducing a higher-dimensional flux energy, $\sim \int \mathrm{d}^6 y \sqrt{g}\, G_{mnp}G^{mnp}$, rather than from a curvature-squared term.

Because the KK scale is larger than typical SUSY-breaking scales, like $M_{3/2}$, the effective 4D theory can be described within the formalism of $N = 1$ supergravity. In this context the suppression of the couplings relative to Planck strength reflects itself in the no-scale form of the Kahler potential,

$$K = -2\log\mathcal{V} \,, \tag{3.10}$$

since this suppresses all terms in the potential by a factor of $e^K W \propto W/\mathcal{V}^2$.

The volume dependence of the interactions of low-energy fields with Kaluza-Klein modes can be inferred using arguments [6] very similar to those used in section 2. Specializing the potential term obtained there to $D = 10$ and $d = 6$ implies that it scales as $\mathcal{V}^{-4/3}$.

The couplings of these fields to ordinary matter can be estimated if the ordinary matter is assumed to be localized on a space-filling brane somewhere in the extra dimensions. The strength of these couplings depends on the particular brane field of interest, but a representative coupling is to gauge bosons, which has the generic form

$$\mathcal{L}_{\rm int} = \frac{\phi}{f} \, F^{\mu\nu} F_{\mu\nu} \,, \tag{3.11}$$

where, for example, $f \simeq M_p$ for many of the Käher moduli (like the volume modulus itself), although some of the 'small' moduli couple with strength $f \simeq M_s \simeq M_p/\mathcal{V}^{1/2}$.

4 Radiative corrections in LV models

The previous sections summarize the rich pattern of scalar masses, related to one another by powers of $\mathcal{V}$, predicted by LV models at leading order:

$$M_s \simeq \frac{M_p}{\mathcal{V}^{1/2}}\,, \quad M_{KK} \simeq \frac{M_s}{\mathcal{V}^{1/6}} \simeq \frac{M_p}{\mathcal{V}^{2/3}}\,, \quad M_{\rm mod} \simeq M_{3/2} \simeq \frac{M_p}{\mathcal{V}}\,, \quad M_V \simeq \frac{M_p}{\mathcal{V}^{3/2}}\,. \tag{4.1}$$

The couplings among these states are also $\mathcal{V}$-dependent, and this is important when computing the size of the loop-generated masses. The contributions of wavelengths longer than the KK scale to these loop corrections can be evaluated within the context of an effective 4D theory, while those having shorter wavelength should be computed within the higher dimensions. Technical naturalness asks that both the 4D and the higher-dimensional contributions be smaller than the lowest-order mass of the light particle itself.

4.1 4D contributions

Suppose a light particle has a mass, $m_\phi \simeq M_p \mathcal{V}^{-a}$, that is suppressed by a particular negative power of $\mathcal{V}$. Suppose further that this light particle couples to a more massive particle having a mass $m_\psi \simeq M_p \mathcal{V}^{-b}$, with $b < a$. Imagine these particles to experience a relevant cubic coupling of the schematic form $\mathcal{L}_3 \simeq h \, \phi \, \psi \partial^{p_3} \psi$ whose coupling strength is of order $h \simeq M_p (1/M_p)^{p_3} \, \mathcal{V}^{-c_3}$, where the power of energy, E, would arise if the coupling were to involve the derivatives of the fields. Assuming 4D kinematics are relevant, repeating the steps of section 2 shows that the one-loop correction to the light scalar mass generated using this interaction in a graph like Fig. 2 is of order

$$\begin{aligned} \delta m_\phi^2 \simeq h^2 \int \frac{{\rm d}^4 k}{(2\pi)^4} \, \frac{k^{2p_3}}{(k^2 + m_\psi^2)^2} \quad &\simeq \quad \left(\frac{h \, m_\psi^{p_3}}{4\pi} \right)^2 \\ &\simeq \quad \frac{1}{(4\pi)^2} \left(\frac{1}{\mathcal{V}} \right)^{2(p_3 b + c_3)} M_p^2 \,. \end{aligned} \tag{4.2}$$

The mass of the light particle is larger than these radiative corrections provided $\delta m_\phi \lesssim m_\phi$, and so $b\,p_3 + c_3 \geq a$. In light of the discussion in §2, since the most massive states for which 4D kinematics applies have $m_\psi \simeq M_{KK} \simeq M_p/\mathcal{V}^{2/3}$, we make take as the worst case $b = \frac{2}{3}$. Furthermore, the discussion of section 2 implies that the couplings of such states is generically either a Planck-strength derivative coupling ($p_3 = 2$ and $c_3 = 0$), or a potential interaction whose strength is proportional to $M_{KK}^2/M_p^2 \simeq \mathcal{V}^{-4/3}$ ($p_3 = 0$ and $c_3 = \frac{4}{3}$). In both cases $b\,p_3 + c_3 = \frac{4}{3}$, implying corrections that are sufficiently small for all 4D moduli (for which $a = 1$) except for the volume modulus (for which $a = \frac{3}{2}$).

Alternatively, suppose the light particle couples to the massive one through a marginal quartic coupling $\mathcal{L}_4 \simeq g^2 \phi^2 \psi \partial^{2p_4} \psi$, whose coupling strength is of order $g \simeq (1/M_p)^{p_4}\,\mathcal{V}^{-c_4}$, where the power of energy, E, again arises as derivatives of the fields. In this case the one-loop correction (using 4D kinematics) to the light scalar mass is of order

$$\delta m_\phi^2 \simeq g^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^{2p_4}}{(k^2 + m_\psi^2)} \simeq \left(\frac{g\,m_\psi^{p_4+1}}{4\pi} \right)^2$$
$$\simeq \frac{1}{(4\pi)^2} \left(\frac{1}{\mathcal{V}} \right)^{2(p_4+1)b + 2c_4} M_p^2 , \qquad (4.3)$$

so this contribution to the mass is technically natural provided $b\,(1 + p_4) + c_4 \geq a$.

Again, the maximum mass appropriate to 4D kinematics is $m_\psi \sim M_{KK}$, corresponding to $b = \frac{2}{3}$ and there are two kinds of quartic interactions amongst the KK modes: Planck-suppressed derivative couplings ($p_4 = 1$ and $c_4 = 0$) and non-derivative interactions suppressed by M_{KK}^2/M_p^2 ($p_4 = 0$ and $c_4 = \frac{2}{3}$). Both of these choices satisfy $b\,(1 + p_4) + c_4 = \frac{4}{3}$, and so are the same size as those obtained from cubic interactions, and not larger than any of the moduli masses ($a = 1$), except for the volume modulus ($a = \frac{3}{2}$).

Supersymmetric cancellations

Apart from the volume modulus, we see that moduli masses tend to be generically stable against 4D radiative corrections. Naively this would seem to indicate the classical approximation is not a good one for the volume modulus itself, which should then be expected to be more massive than $\mathcal{O}(M_p/\mathcal{V}^{3/2})$. However to this point we have not yet availed ourselves of the cancellations implied by 4D supersymmetry, which survives into the 4D theory because $M_{3/2} \ll M_{KK}$.

Let us reconsider the 4D contributions in this light. As usual, since particle masses are driven by the form of the superpotential, they receive no corrections until supersymmetry breaks. At one loop an estimate of the leading supersymmetry-breaking effects obtained by summing the contributions of bosons and fermions, keeping track of their mass difference. For instance, keeping in mind the sum rule, eq. (2.25), gives

$$\delta m^2 \simeq g\left(m_B^2 - m_F^2 \right) \lesssim \left(\frac{M_{KK}^2}{M_p^2} \right) M_{3/2}^2 \simeq \left(\frac{1}{\mathcal{V}^{4/3}} \right) \frac{M_p^2}{\mathcal{V}^2} , \qquad (4.4)$$

implying the generic supersymmetry-breaking mass shift is $\delta m \simeq M_p / \mathcal{V}^{5/3}$. This is small enough not to destabilize the mass of the volume modulus, $M_V \simeq M_p / \mathcal{V}^{3/2}$.

Higher-dimensional contributions

For scalar mass corrections, normally it is loops involving the heaviest possible particles that are the most dangerous, so it might be natural to expect the contributions from states having $M \gg M_{KK}$ to dominate the 4D estimates just obtained. In this section we argue this not to be the case, with the dominant contribution to modulus masses arising from loops involving states at the KK scale.

The argument proceeds as in section 2, with the recognition that the effects of particles above the Kaluza-Klein scale require the use of the higher-dimensional theory, where new symmetries like extra-dimensional general covariance come to our aid. In particular, since the largest mass scale in the higher dimensional theory is M_g, or for Type IIB models the string scale, $M_s \simeq g_s M_g$, the contributions of states this massive are summarized by local string-loop and α' corrections to the 10D action.

However, any such contributions are strongly constrained by 10D supersymmetry. The leading α' corrections arising at string tree level are known, and first arise at $\mathcal{O}(\alpha'^3)$, with four powers of the curvature. The volume dependence of this, and of the other α' corrections that arise at next-to-leading order were studied by refs. [1], with the conclusion that these first contribute to the 4D scalar potential at order

$$\delta_{\alpha'} U \simeq \frac{W_0^2}{\mathcal{V}^3} \, \mathcal{U}_{\alpha'}(\varphi) \,, \tag{4.5}$$

where $\mathcal{U}_{\alpha'}(\varphi)$ denotes some function of the dimensionless moduli (whose detailed form is not important for the present purposes). Given their Planck-scale kinetic terms, this contributes of order $\delta m \simeq \mathcal{O}(M_p / \mathcal{V}^{3/2})$ to the low-energy moduli masses. Indeed, it is these α' corrections that lift the mass of the volume modulus in the first place, showing why it is generically of order $M_p / \mathcal{V}^{3/2}$.

The $\mathcal{V}$-dependence of the leading string-loop effects has also been analyzed [20, 21, 19], with the result that it contributes at order

$$\delta_{\text{loop}} U \simeq \frac{W_0^2}{\mathcal{V}^{10/3}} \, \mathcal{U}_{\text{loop}}(\varphi) \,, \tag{4.6}$$

with $\mathcal{U}_{\text{loop}}$ another function of the dimensionless moduli. The resulting contribution is $\delta m \simeq M_p / \mathcal{V}^{5/3}$, in agreement with the above estimates for the size of supersymmetric cancellations in one-loop effects. This agreement indicates that it is states having masses of order M_{KK} that provide the dominant contribution to these string loops.

4.2 Checks

Large-volume string models have the advantage of having been studied well enough that there are several nontrivial checks on the above estimates.

Comparison with the 4D supergravity

One check comes from comparing the above estimates of the size of cancellations in one-loop supergravity amplitudes with the kinds of corrections that are allowed to arise within the effective 4D supergravity describing the moduli. In this supergravity the above loop effects must appear as volume-dependent corrections to the Kähler function, K, since this is the quantity that encodes the effects of high-energy loops. The leading contributions to δK arising up to one loop has been estimated [16], and has the form

$$K \simeq -2\ln\mathcal{V} + \frac{k_1}{\mathcal{V}^{2/3}} + \frac{k_2}{\mathcal{V}} + \frac{k_3}{\mathcal{V}^{4/3}} \cdots , \tag{4.7}$$

where the k_1 term first arises at the level of one string loop while k_2 contains the α' contributions at string tree level, and the k_3 term gives the next corrections at one string loop.

Recall that the leading contribution to the 4D potential varies as $U \simeq W_0^2/\mathcal{V}^2$. It happens that because the k_1 term is proportional to $\mathcal{V}^{-2/3}$, it completely drops out of the scalar potential [20, 21]. The k_2 term then gives the dominant correction to U, contributing at order $1/\mathcal{V}^3$. Finally, the k_3 term contributes a correction to U that is of order $1/\mathcal{V}^{10/3}$, all in agreement with the above estimates.

Comparison to explicit string calculations

The explicit form of one-loop effects associated with Kaluza-Klein (and string) modes are available for orbifolds and orientifolds of toroidal compactifications. We here briefly summarize the strength of these corrections and their interpretation in the low energy effective field theory. This illustrates that the truncation to the four dimensional effective field theory to be consistent and the quantum corrections to the mass of the volume modulus associated with Kaluza-Klein modes to be subleading in an expansion in the inverse volume.

We focus on $N = 1$ orientifold $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ analyzed by Berg, Haack and Kors in ref. [16]. The untwisted moduli in this model are the three Kähler and complex structure moduli $\{\rho^I, U^I\}$ axio-dilaton S, open string scalars A^I associated with position of the $D3$ branes, the model also has moduli associated with $D7$ brane positions which was set to zero in [16].

The tree-level Kähler potential, found using the disc and sphere string-worldsheet graphs, is given by

$$K = -\ln(S - \bar{S}) - \sum_{I=3}^{3} \ln\left[(\rho^I - \bar{\rho}^I)(U^I - \bar{U}^I) - \frac{1}{8\pi}(A^I - \bar{A}^I)^2 \right], \tag{4.8}$$

while the one-loop correction was computed in [16], and found to be

$$K^{(1)} = \frac{1}{256\pi^6} \sum_{I=1}^{3} \left[\frac{\mathcal{E}_2^{D3}(A^I, U^I)}{(\rho^I - \bar{\rho}^I)(S - \bar{S})} + \frac{\mathcal{E}_2^{D7}(0, U^I)}{(\rho^J - \bar{\rho}^J)(\rho^K - \bar{\rho}^K)}\bigg|_{K \neq I \neq J} \right] \tag{4.9}$$

where the superscripts $D3$ and $D7$ indicate contributions which originate from open string diagrams with boundaries on these branes. While the general expression for $\mathcal{E}_2^{D3}$ and $\mathcal{E}_2^{D7}$ are

complicated, a symmetric choice of the $D3$ brane positions (we refer the reader to [16] for details) the contribution from $D3$ branes vanishes and the contribution from $D7$ branes is an Eisenstein series

$$\mathcal{E}_2^{D7}(0, U) = 1920 \sum_{(n,m)\neq(0,0)} \frac{\mathrm{Im}(U)^2}{|n + Um|^4} \qquad (4.10)$$

The indices (m, n) can be interpreted as labelling the Kaluza-Klien momenta of the exchanged particles in the open-string loop diagrams. Note that the contribution from the higher KK modes is suppressed by inverse powers of the KK momentum, as a result the higher KK modes make only a small contribution. We note that while the contribution of a single mode running in loop correction to the mass would scale as the square of the mass (n^2), cancelations due to supersymmetry lead to an effective scaling of n^{-4}. We would like to emphasize that the resulting sum is ultraviolet finite. One can interpret this as the restoration of supersymmetry at the high scale.

The contribution of the one-loop Kähler potential to the scalar potential scales as $\mathcal{V}^{-10/3}$ which indeed is subleading in the large volume expansion. As mentioned earlier, this scaling of the volume precisely matches the estimations of the size of loop effects from KK modes in the low energy effective field theory [21].

The bottom line is that the combination of supersymmetry with the $\mathcal{V}$-suppressed couplings coming from extra dimensions ensures the stability of the masses of the moduli against radiative corrections in large-volume string models.

5   Scenarios

Given the robustness of the light scalar masses, for phenomenological purposes it is useful to see how large the above masses are for various choices for the string scale, in order to see precisely how light the relevant scalars can be. Since $\mathcal{V}$ varies exponentially sensitively with the parameters of the modulus-stabilizing flux potential, it varies over many orders of magnitude as potential parameters are changed only through factors of order 10. It can therefore be regarded as a dial that can be adjusted freely when exploring the model's implications.

Weak-scale gravitino mass

One attractive choice is to take $\mathcal{V} \simeq 10^{15}$, in which case $M_{3/2} \simeq M_p/\mathcal{V} \simeq 10^3$ GeV is of order the TeV scale and $M_s \simeq M_g \simeq M_p/\mathcal{V}^{1/2} \simeq 10^{11}$ GeV, corresponding to the intermediate-scale string [22].

In this case the generic moduli also have masses at the TeV scale, while the volume modulus is interestingly light, being of order $M_V \simeq M_p/\mathcal{V}^{3/2} \simeq 10^{-3}$ GeV $\simeq 1$ MeV. Even though very light, such a scalar would be very difficult to detect, given its gravitational couplings to matter. It is not light enough to mediate a measurable force competing with gravity, and so is not constrained by the observational tests of gravity in the lab or in astrophysics. Even though this

contradicts the standard lore that moduli masses are of the same order as the gravitino mass after supersymmetry breaking, it actually makes the cosmological moduli problem [23] more severe since a gravitationally coupled scalar field with mass ~ 1 MeV tends to dominate the energy density of the universe through its coherent oscillations after inflation. A low-energy mechanism to dilute this field, such as a second period of inflation, may be needed to avoid this problem. The physical implications of this scenario are explored in more detail in [24].

Intermediate scale gravitino mass and soft terms

In [25] a novel framework for supersymmetry breaking is put forward in the context of the large volume scenario. Since the main source of supersymmetry breaking is the F-term of the volume modulus and, since the $4D$ supergravity is approximately of the no-scale type, its contributions to soft terms can be highly suppressed in powers of the volume, the F-terms of the other Kähler moduli dominate the structure of the soft terms. If the brane that hosts the standard model does not wrap the dominant cycle for supersymmetry breaking, then the soft terms are hierarchically suppressed with respect to the gravitino mass $\Delta m \sim M_p/\mathcal{V}^q$ with $q = \frac{3}{2}, 2$ depending on potential cancellations. For $\Delta m \sim 1$ TeV, the gravitino mass can be as high as 10^{10} GeV. Again we have a situation in which the soft terms, in particular the masses of the scalar partners of the standard model fields, are much smaller than the gravitino mass and loop corrections could in principle destabilize these masses. However, the same arguments as before imply that these loop corrections are at most proportional to $M_{KK}M_{3/2}/M_p \simeq M_p/\mathcal{V}^{5/3}$ (see also [25, 26]).

As a result squarks and sleptons much lighter than the gravitino mass are naturally stable. A similar estimate can be made for the other soft terms, such as A-terms and gaugino masses. This implies that as long as the contributions to soft terms from the volume modulus are suppressed, including anomaly mediated contributions [27] (see however [28]), the gravitino mass can be hierarchically heavier than the TeV scale. Notice that this ameliorates the cosmological moduli problem mentioned above since the volume modulus mass, which is of order $M_p/\mathcal{V}^{3/2}$, in this scenario can be as heavy as 1 TeV or heavier, instead of being order MeV as in the previous scenario.

Weak-scale strings and SUSY breaking on the brane

The largest possible volume within this scenario that is consistent with experience is $\mathcal{V} \simeq 10^{30}$, for which $M_s \simeq M_p/\mathcal{V}^{1/2} \simeq 10^3$ GeV is at the weak scale, while the moduli and gravitino are extremely light: $M_{\mathrm{mod}} \simeq M_{3/2} \simeq M_p/\mathcal{V} \simeq 10^{-12}$ GeV $\simeq 10^{-3}$ eV. In this case the volume modulus would appear to be astrophysically relevant, since $M_V \simeq M_p/\mathcal{V}^{3/2} \simeq 10^{18}$ eV.

More care is needed in this particular case, however, because the supersymmetry breaking scale, $M_{3/2}$, is so very low. Because it is so low, another source of breaking must be introduced for the model to be viable, to accommodate the absence of evidence for supersymmetry in accelerator experiments. The simplest such source of supersymmetry breaking is hard breaking by anti-

branes, with all of the observed particles living on or near these branes so that they are not approximately supersymmetric.

There are dangers to breaking supersymmetry in such a hard fashion, however. In particular, for a viable scenario one must check that the potential energy of the anti-brane — which we shall find is of order $M_s^4 \simeq M_p^4/\mathcal{V}^2$ — does not destabilize the LV vacuum.[9] Notice that this is a more serious problem than occurs if anti-branes are introduced to uplift the previous cases so that their potentials are minimized to allow flat spacetime on the branes. It is worse in this case because the anti-brane must be the dominant source of SUSY breaking, and so the value of its tension cannot be warped down to small values without also making the mass splittings amongst supermultiplets — which are of order M_s on the brane — too small. In what follows we assume that this issue has been dealt with, either through inspired modelling or through fine-tuning.

Since the splitting between the masses of the bosons and fermions localized on the brane is of the order M_s, it is loops involving these brane states that are the most dangerous corrections to M_V in this picture. We now argue that these loops of brane states generate mass corrections of order $M_p/\mathcal{V}$, and so in this particular case lift the volume modulus to be comparable in mass to the masses of the other moduli.

The estimate begins with the Born-Infield action on the brane and the ten dimensional Einstein action in the bulk

$$S = M_g^8 \int d^{10}x \sqrt{g_{10}} \mathcal{R}_{10} - T_3 \int d^4x \sqrt{\det[G_{\alpha\beta}]} \tag{5.1}$$

where $T_3 \simeq \mathcal{O}(M_s^4)$ is the brane tension and $G_{\alpha\beta}$ is the pullback of the ten dimensional metric to the brane world volume. Neglecting powers of the string coupling, we take the 10D Planck scale of order the string scale: $M_g \simeq M_s$.

To carry out the dimensional reduction we take, as before, the ten dimensional metric to be of the form

$$ds_{10}^2 = \omega e^{-6u(x)} g_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)} g_{mn} dy^m dy^n, \tag{5.2}$$

where, as before, $\omega = (M_p/M_s)^2$, $u(x)$ is the volume modulus and g_{mn} is the metric of a Calabi-Yau of unit volume in string units. Working with static embedding coordinates for the brane the effective action for the volume modulus $u(x)$ and the transverse scalars y^m is

$$-24M_p^2 \int d^4x \sqrt{-g_4} \, \partial_\mu u \partial^\mu u - \hat{T}_3 \int d^4x \sqrt{-g_4} e^{-12u(x)} \sqrt{\det[(\delta^\alpha_{\ \beta} + e^{8u(x)} \partial^\alpha y^m \partial_\beta y^n g_{mn})]} \tag{5.3}$$

where M_p^2 is the four dimensional Planck mass and $\hat{T}_3 = \omega^2 T_3 \simeq \mathcal{O}(M_p^4)$.

Next we make field redefinitions which bring their kinetic terms to the canonical form, $u(x) = u_0 + \ell(x)/4\sqrt{3}M_p$ and $y^m = e^{2u_0}\phi^m/\sqrt{\hat{T}_3}$, where u_0 is the v.e.v. of the field $u(x)$ and related to

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[9]We thank Joe Conlon for emphasizing this point.

the volume of the compactification in string units by $e^{6u_0} = \mathcal{V}$. This brings (5.3) to the form

$$-\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g_4}\partial_\mu\ell\partial^\mu\ell - \frac{\hat{T}_3}{\mathcal{V}^2}\int \mathrm{d}^4x\sqrt{-g_4}e^{-\sqrt{3}\ell/M_p}\sqrt{\det\left[\left(\delta^\alpha{}_\beta + \frac{\mathcal{V}2}{\hat{T}_3}e^{2\ell/\sqrt{3}M_p}\partial_\alpha\phi^m\partial_\beta\phi^n g_{mn}\right)\right]}$$

$$(5.4)$$

We note that the derivative expansion for interactions involving the fields ϕ^m is controlled by the scale $\Lambda = (\hat{T}_3/\mathcal{V}^2)^{1/4}$. At the two-derivative level interactions between ℓ and ϕ^m are Planck suppressed, the leading order term being

$$\frac{1}{M_p}\int \mathrm{d}^4x\sqrt{-g_4}\, g_{mn}\partial^\alpha\phi^m\partial_\alpha\phi^n\ell(x), \tag{5.5}$$

The graph for the one loop correction to the mass of the volume modulus involving this interaction has two inverse powers of M_p from the interaction vertices. The relevant integral over the virtual momenta has four powers of momenta from the integration measure, four from the two interaction vertices and four inverse powers from the propagators. This leads to a loop correction to the mass squared of the volume modulus of the order of Λ^4/M_p^2. The above estimate in fact holds for all contributions arising from virtual loops of the fields ϕ^m. This is most easily seen from dimensional analysis. The associated graph involves two legs of the field $\ell(x)$, and since the field always appears in the combination of ℓ/M_p this leads to two factors of $1/M_p$. Next note that for processes involving loops of ϕ^m, all the vertices and loop momenta are powers of $\hat{T}_3/\mathcal{V}$. This implies that the mass correction is of the form $\delta m^2 \propto (\hat{T}_3/\mathcal{V}^2)^p/M_p^2$, where on dimensional grounds the value of p must be $p = 1$. Since $\hat{T}_3 \sim \mathcal{O}(M_p^4)$, the scale $\Lambda \sim M_p/\mathcal{V}^{1/2} \sim M_s$ is of order the string scale, so the size of the mass correction is

$$\delta m \propto \frac{\Lambda^2}{M_p} \simeq \frac{M_s^2}{M_p} \simeq \frac{M_p}{\mathcal{V}}. \tag{5.6}$$

This implies the mass of the volume modulus is naturally of order $M_{\mathrm{mod}} \sim M_{3/2} \sim M_p/\mathcal{V}$ when supersymmetry is badly broken on a brane, removing the hierarchy between the volume modulus and other moduli.

If the string scale is ~ 1 TeV, then all of these light scalar masses are of order $M_p/\mathcal{V} \simeq 10^{-3}$ eV, making them just on the edge of relevance to laboratory tests of the gravitational force law. Because it is so close, corrections can be important, and – as shown in ref. [24] – the couplings between brane matter and bulk fields give an additional logarithmic suppression to the volume modulus mass, which then scales as $M_p/(\mathcal{V}\ln\mathcal{V})$. If this correction pushes the volume modulus into the range probed by terrestrial fifth-force experiments, it could make the effects of this field detectable. The existence of such a scalar as a robust consequence of weak-scale string models within the large-volume picture provides additional motivation for more detailed calculations of its mass and properties in the presence of anti-branes.

TeV string scenarios could have spectacular experimental implications at LHC (see for instance [29], and references therein), so it is of great interest that they might be viable within the

large volume scenario for which control over issues like modulus stabilization and supersymmetry breaking allows a detailed prediction of the low-energy spectrum. The existence within this spectrum of a very light volume modulus was the main obstacle to serious model building, so the existence of mass generation by explicit breaking of supersymmetry on the brane is of particular interest.[10]

6   Conclusions

In this paper we show that extra dimensions and supersymmetry can combine to to protect scalar masses from quantum effects more efficiently than either can do by itself. Supersymmetry by itself would not necessarily protect masses lighter than the gravitino mass and extra dimensions in principle need not protect scalar masses lighter than the KK scale. But both together allow scalar masses to be hierarchically smaller than both the KK and gravitino masses. We call this kind of unusual scalar hierarchy *über-natural.*

New mechanisms for keeping scalar masses naturally light are interesting because they are both rare and potentially very useful, both for applications to particle physics and cosmology. Because light scalars tend to have many observable consequences, their existence can help identify those models to which the ever-improving tests of general relativity on laboratory and astrophysical scales are sensitive. From a model builder's perspective, light scalars are also useful because they can cause problems, such as the cosmological moduli problem, and thereby focus attention on those models that can deal with these problems.

Our power-counting estimates show that über-naturally light scalars ultimately remain light because their masses and interactions are systematically suppressed by powers of $1/\mathcal{V} = M_g^2/M_p^2$, showing that they are light because the gravity scale is lower than the 4D Planck scale. But because scalars are potentially so UV sensitive, to properly establish that their masses are naturally small requires knowing the UV completion of gravity, within a framework that includes an understanding of modulus stabilization and supersymmetry breaking. The possibility of studying this quantitatively has only recently become possible, within the large-volume vacua of Type IIB flux compactifications in string theory.

The large volume scenario predicts clear hierarchies in the low-energy spectrum of scalar states, within a framework of calculational control that allows us to be explicit about the size of quantum corrections. It also allows several sub-scenarios, depending on the size of the volume and the location of the standard model within the extra dimensions. We discuss a few of the preliminary implications of our results for several of these.

- We find that soft supersymmetry breaking terms ($\Delta m \propto 1/\mathcal{V}^{3/2}, 1/\mathcal{V}^2$) much smaller than the gravitino mass $M_{3/2} \propto 1/\mathcal{V}$ can be stable against quantum corrections, since these are smaller or equal to $1/\mathcal{V}^{5/3}$. This is important for the stability of those recent models where

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[10]We thank I. Antoniadis, G. Dvali and D. Lüst for useful conversations on this issue.

the soft supersymmetry breaking relevant to weak scale particle physics is parametrically small compared with the gravitino mass, such as happens when the main contribution to supersymmetry breaking comes from cycles different from the cycle wrapped by the standard model brane.

- We find that TeV scale string models not only can be obtained from the large volume scenario, but that their main potential observational obstacle – the existence of an extremely light volume modulus – might not be such a problem, making them much more appealing. The volume modulus need not be a problem because the relatively large mass corrections it receives from the strong breaking of supersymmetry on the brane that is required in such models.

More generally, *über-naturalness* provides the mechanism that underlies many of the attractive features of LV models that have proven valuable for phenomenology. For instance, LV models ultimately bring the news of supersymmetry to ordinary particles through a form of gravity mediation, yet avoid the normal pitfalls (such as with flavour-changing neutral currents) of gravity mediation for low-energy phenomenology [30]. *Über-naturalness* provides the framework for the stability of this process against loop corrections.

Similarly, inflationary models have been constructed within the LV scenario with the inflaton being a volume modulus [31] or another Kähler modulus [32], with the latter achieving a slow roll by virtue of the inflaton taking large field values, rather than requiring a tuning of parameters in the potential. These scenarios profit from the *über-natural* protection of the potential within the LV picture, indicating that extra-dimensional symmetries like general covariance can provide an alternative to global shift symmetries (see for instance [33]) for addressing the η-problem.

Further implications can well be envisaged. In particular, the suppressed corrections to the masses of light moduli may be useful for cosmology by providing new and better models of inflation. Perhaps new models of dark energy could become possible with this new naturalness concept in mind. We hope that our results will at least serve to stimulate the search, in as explicit a manner as possible, for further suppression mechanisms for scalar fields in theories with supersymmetric extra dimensions.

Acknowledgments

A   Extra-dimensional kinematics vs KK sums

In this appendix we discuss the loops correction in the higher dimensional theory from the perspective of the lower dimensional theory. Consider a massless scalar field in D dimensions with a quartic coupling of strength of the order of the higher dimensional cutoff

$$-\mathcal{L}_{\mathcal{D}} = -\frac{1}{2}(\partial\Phi)^2 + \frac{g_4}{24}\Phi^4\,, \tag{A.1}$$

with

$$g_4 = \left(\frac{1}{\Lambda}\right)^{(D-4)}. \tag{A.2}$$

Upon dimensional reduction on a d-torus

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + L^2\delta_{mn}\mathrm{d}y^m\mathrm{d}y^n \tag{A.3}$$

and canonical normalization this gives the lower dimensional action for the KK modes of the form

$$-\mathcal{L}_4 = -\frac{1}{2}\sum_{n_i}\left[(\partial\phi_{n_i})^2 + \frac{1}{2}m_{n_i}^2\phi_{n_i}^2\right] + \frac{g_4}{L^d}\sum_{n_i,n_j,n_k,n_l}c_{n_in_jn_kn_l}\phi_{n_i}\phi_{n_j}\phi_{n_k}\phi_{n_l}\,, \tag{A.4}$$

where $i,j,k,l = 1..d$, $m_{n_i}^2 = \frac{1}{L^2}\sum_i n_i^2$ and the interaction coefficients $c_{n_in_jn_kn_l}$ are of order one if the KK charge associated with the vertex is vanishing.

Now let us consider the loops in the above theory, the loop contribution due to a KK of mass m_{n_i} is

$$\delta m_{n_i}^2 \approx \frac{g_4}{L^d}m_{n_i}^2\,. \tag{A.5}$$

In order to estimate the effect of the entire KK tower we sum the loop contributions of all KK modes up to the scale Λ i.e we restrict the KK momenta to $|n_i| < N_{\max} = \Lambda L$. This gives

$$\sum_{n_i}\delta m_{n_i}^2 \approx \frac{g_4}{L^{2+d}}(\Lambda L)^{2+d} = \Lambda^2\,, \tag{A.6}$$

in agreement with the discussion in section 2.

We note that the estimate assumed no cancellations in the sum over the loop contributions in (A.6), as is appropiate for a scalar. But, as emphasized in section 2 if one considers gravity in higher dimensions, the restrictions on the form of higher dimensional action imposed by general covariance necessarily implies cancellations between the loop contributions of various particles in the lower dimensional theory. Such cancellations can lower the size of loop corrections to scales parametrically below the cut off scale of the higher dimensional theory.

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