We study the renormalization of (softly) broken supersymmetric theories at the one loop level in detail. We perform this analysis in a superspace approach in which the supersymmetry breaking interactions are parameterized using spurion insertions. We comment on the uniqueness of this parameterization. We compute the one loop renormalization of such theories by calculating superspace vacuum graphs with multiple spurion insertions. To perform this computation efficiently we develop algebraic properties of spurion operators, that naturally arise because the spurions are often surrounded by superspace projection operators. Our results are general apart from the restrictions that higher super covariant derivative terms and some finite effects due to non–commutativity of superfield dependent mass matrices are ignored. One of the soft potentials induces renormalization of the Kähler potential.
1 Introduction and Summary

It is known for a long time that supersymmetric theories possess many amazing ultra–violet (UV) properties. One of them being the absence of quadratic divergences. It is this property that has led to the development of the Minimal Supersymmetric Standard Model (MSSM). For renormalizable models the only exceptions to this mild UV behavior are quadratically divergent $D$–terms [1], but since those only arise when there are also mixed gravitation–gauge anomalies, they are not relevant for MSSM phenomenology. For string model building on the other hand these $D$–terms are very useful to reduce the rank of the gauge group. For non–renormalizable theories one can expect more sources of non–logarithmically divergences.

Almost all of the parameters of the MSSM encode possible ways that supersymmetry is broken in a soft way, while preserving the gauge symmetries of the MSSM. The soft parameters have essentially been classified in by Girardello and Grisaru [2]: They are either gaugino masses, Hermitian and complex mass matrices for the complex scalars or three–linear scalar couplings. In fact, there are some additional interactions that can be conditionally soft [3], see also discussion in [4,5]. These parameters can either be introduced on the component level of the supersymmetric theory, or described by spurion insertions, $\theta^2$ and $\bar{\theta}^2$. The latter approach is very powerful because it leaves most of the supersymmetric structure intact. A drawback of this approach is that the spurion parameterization is not unique [6]: A given soft supersymmetry breaking interaction can be parameterized in various different ways. (We return to this issue in more detail.) The renormalization group equations (RGE’s) of these soft parameters have been investigated up to the two loop level [7–12].

Most of these works focused on the MSSM or renormalizable models in general. However, when we see the MSSM as an effective description of a more fundamental theory, it seems unnatural to only restrict the attention to renormalizable models. String theory is often considered as a possible candidate for the UV completion of the MSSM. The effective low energy models that can be derived from String theory always seem to be non–renormalizable, therefore it is important to have a detailed analysis of quantum corrections of such effective supersymmetric models as well.

For exact globally supersymmetric models in four dimensions (renormalizable or not) there has been a lot of investigations to UV properties. Such a model with up to two derivatives is described by a real Kähler potential $K$ and two holomorphic functions, the superpotential $W$ and the gauge kinetic function $f_{IJ}$ of chiral multiplet $\phi^a$. Because of holomorphicity the superpotential and the gauge kinetic function are very much constraint. This is reflected in various non–renormalization theorems [13] for these functions and lead to impressive results to all order [14–16]. The works [17–19] show that even a lot of non–perturbative information can be obtained. In particular, in the certain $N = 2$ theories the full non–perturbative superpotential has been computed [20].
The situation for the Kähler potential receives quantum corrections at all orders in perturbation theory. The one loop Kähler potential in supersymmetric theories was first investigated by [21] and subsequently computed by many groups (see e.g. [22–27]), using dimensional reduction the result \(^3\) reads [28]

\[
K_{1L} = -\frac{1}{16\pi^2} \operatorname{tr}_A h^{-1} m_C^2 \left( 2 - \ln \frac{h^{-1} m_C^2}{\bar{\mu}^2} \right) + \frac{1}{32\pi^2} \operatorname{tr} M_W^2 \left( 2 - \ln \frac{M_W^2}{\bar{\mu}^2} \right),
\]

where \( \bar{\mu}^2 = 4\pi e^\gamma \mu^2 \) defines the \( \overline{\text{MS}} \) renormalization scale and \( h_{IJ} = (f_{IJ} + \bar{f}_{IJ})/2 \). The FP–ghost and superpotential mass matrices are defined by

\[
(m_C^2)_{IJ} = 2 \bar{\phi} T_I G T_J \phi, \quad M_W^2 = G^{-1} \overline{W} G^{-1T} W,
\]

respectively. Here \( T_I \) are the Hermitian generators of the gauge group and \( W, \overline{W} \) and \( G \) denotes the second holomorphic derivative of the superpotential: \( W = W_{,ab} \), the second anti–holomorphic derivative \( \overline{W}_{\bar{a}\bar{b}} = \overline{W}_{,\bar{a}\bar{b}} \) and the second mixed derivative of the Kähler potential: \( G_{\bar{a}a} = K_{,\bar{a}a} \), respectively. The renormalization of the Kähler potential has also been investigated at the two loop level [30–32]; complete results including gauge corrections can be found in [28]. These one and two loop results in [28] were obtained by using that the vacuum graphs in superspace precisely correspond to the graphs needed to compute the renormalization of the Kähler potential. This is an efficient approach because the number of topologies of vacuum graphs is limited, while computing diagrams with an arbitrary number of external legs is very involved.

The basic aim of this paper is to investigate the effective action of (softly) broken supersymmetric theories at the one loop level. To calculate the renormalization of softly broken supersymmetric models, we choose to use similar method which were used to compute the quantum corrections to the Kähler potential. In particular, to reduce the number of diagrams we would like to compute only vacuum supergraphs. In order to be able to do this, the use of spurion supersymmetry breaking proves crucial: The full supersymmetric and the supersymmetry breaking terms can then be represented as interactions in superspace, and their renormalization can be performed systematically by computing vacuum graphs in superspace only. As observed above the parameterization of supersymmetry breaking effects is far from unique. However, using some holomorphicity restrictions we give unique definitions for scalar functions \( \tilde{K}, \tilde{k} \) and \( \tilde{W} \).

The vacuum supergraphs, that give rise to the renormalization of soft supersymmetry breaking action, involve infinite sums of spurion insertions. To be able to manage this it is convenient to introduce spurion operators. These spurion operators in fact arise naturally because in the (quadratic) action the spurions find themselves surrounded by either chiral or vector superfield

\(^3\)When non–Abelian gauge interaction are included those results and ours [28, 29] differ, because the computation has been performed in different gauges: We have used the ’t Hooft–Feynman gauge, while the other computations have been performed in the Landau gauge. We have confirmed that this is a gauge artifact by using a gauge fixing that can interpolate between these two gauges.
projection operators: For the chiral multiplets these projection operators are hidden in the
definition of chirality of the superfields, while the full superspace representation for the vector
multiplets explicitly contains the vector superfield projector. These spurion operators possess
interesting algebraic properties because they can be shown to generate two dimensional Clifford
algebras. This observation greatly simplifies computations. To avoid having to explicitly com-
pute infinite sums of vacuum supergraphs we derive some results for computing logarithms of
functional determinants. Combining these results with the algebraic properties makes it concep-
tually straightforward to compute the one loop renormalization of softly broken supersymmetric
theories, which is parameterized by the one loop corrections of the functions $\tilde{K}$ and $\tilde{k}$.

Our results for the renormalization of (softly) broken supersymmetric theories are general
apart from the restrictions: We observe that for non–renormalizable theories, there are additional
supersymmetry breaking interactions possible that cannot be parameterized by the functions we
have introduced. These interactions, that involve higher super covariant derivatives, can also re-
ceive renormalization. Because in this paper we ignore all possible superspace derivatives on the
chiral superfield background, our computations are blind to quantum effects that generate such
operators. In addition, we encountered one technical problem: In general we consider theories
with many chiral multiplets, therefore one can encounter many field dependent mass matrices.
(We run into superpotential, complex and Hermitian scalar mass matrices, to name a few.) In
general these matrices do not all commute, as a consequence even seemingly straightforward one
loop scalar integrals can become difficult to compute exactly. For this reason we have preformed
approximations such that all effects due to non–commutativity of mass matrices are finite and
at least proportional to one commutator of some of these mass matrices.

As the main part of the investigations in this paper is rather technical, we have decided to
structure the paper as follows: In next section, section 2, we first give a concise definition of
the (soft) supersymmetry breaking functions, which we call soft potentials, to avoid ambiguities
that can arise when using the spurion description of supersymmetry breaking. After we have
introduced the necessary notation, we give our results of one loop computation of these soft
potentials. In section 3 we give some applications and illustrations of our general results. We
find the conditions for softness of general non–renormalizable theories. In subsection 3.2 we
give a simple example of the Wess–Zumino model with supersymmetry breaking which induces
renormalization of the Kähler potential, and discuss to what extend this is related to non–softness
of the theory. Finally we derive the soft potentials for Super Quantum Electrodynamics (SQED)
with soft breaking in subsection 3.3. The remainder of the paper is devoted to the details of the
one loop computation of these soft potentials. Section 4 lays the technical foundations for this:
We first develop the properties of spurion operators in subsection 4.1. Next the quadratic action
is derived from which all the one loop vacuum bubble supergraphs can be obtained. The final
subsection 4.3 gives the general expression for these vacuum supergraphs with chiral superfield
or vector superfields running around in the loop. The actual computation of the one loop
The renormalization of the soft potentials is performed in section 5, relying heavily on the material developed in the preceding section. In subsection 5.1 we first consider the gauge contributions to the renormalization of these functions, because they are technically easier than the ones that result from the chiral multiplets. Their contributions is described in detail in subsection 5.2. The paper is concluded with two appendices that discuss various one loop integrals we encountered. In appendix A the two basic integrals are calculated using dimensional regularization in which all our results will be expressed. Appendix B is devoted to three types of more complicated integrals that arise in section 5.

2 Results for One Loop Soft Potentials

We consider globally supersymmetric gauge theories with arbitrary holomorphic superpotential \( W(\phi) \) and gauge kinetic function \( f_{IJ}(\phi) \) and real Kähler potential \( K(\bar{\phi}, \phi) \) of the chiral superfields \( \phi^a \) and their conjugates \( \bar{\phi}_a \). We assume that all these functions are gauge invariant. And in particular, the gauge kinetic functions is proportional to the Killing metric. The vector superfield \( V = V^I T_I \) are contracted with Hermitean generators \( T_I \) of some (non–)Abelian group.

The supersymmetric action for this theory is given by

\[
S_{\text{susy}} = \frac{1}{2} \int d^8 z \left( K \left( e^{2V} \phi \right) + \int d^6 z \left( W(\phi) + \frac{1}{4} f_{IJ}(\phi) W^I W^J \right) \right) + \text{h.c.},
\]

where we use the full and chiral superspace measures, \( d^8 z = d^4 x \, d^4 \theta \) and \( d^6 z = d^4 x \, d^2 \theta \), respectively, and the Hermitean conjugation acts on all terms. In this action we have introduced the (non–)Abelian superfield strength

\[
W^I_a = -\frac{1}{8} \Theta^a \left( e^{-2V} D_a e^{2V} \right).
\]

To include generic (soft) supersymmetry breaking interactions we extend this theory by including the following soft action

\[
S_{\text{soft}} = \frac{1}{2} \int d^8 z \left\{ \theta^2 \theta^2 \bar{K}(\bar{\phi} e^{2V}, \phi) + \theta^2 \bar{k}(\bar{\phi} e^{2V}, \phi) \right\} + \int d^6 z \theta^2 \left\{ \bar{W}(\phi) + \frac{1}{4} \bar{f}_{IJ}(\phi) W^I W^J \right\} + \text{h.c.}
\]

This soft action is essentially identical to the supersymmetric action (3), except for the appearance of the spurions \( \theta^2 \) and \( \bar{\theta}^2 \). We may therefore refer to \( \bar{K}(\bar{\phi}, \phi) \) as the soft (Kähler) potential, and to \( \bar{W}(\phi) \) and \( \bar{f}_{IJ}(\phi) \) soft superpotential and soft gauge kinetic function, respectively. As their supersymmetric analogs, these soft functions are all assumed to be gauge invariant. The soft function \( \bar{K} \) can result in terms like the Hermitean scalar masses, while the soft superpotential \( \bar{W} \) can give rise to complex scalar masses and tri–linear scalar couplings. The constant part of the \( \bar{f} \) gives rise to gaugino masses. Notice that we also have introduce the soft potential \( \bar{k} \) in (5) which does not have an supersymmetric analog.

This brings us to an important issue concerning the uniqueness of presenting the soft action using spurion superfields \( \theta^2 \) and \( \bar{\theta}^2 \) as is done in (5). This issue has been discussed before in
the literature: To obtain a unique definition ref. [6] advocates to represent all soft breaking as $D$-terms, while Yamada [8] gives spurion dependent transformations to bring the (divergent) quantum corrections back to its starting form. The ref. [6]'s choice to represent all supersymmetry breaking terms as Kähler terms works in general, but for our purposes this classification is not fine enough. Because Yamada was consider renormalizable theories, he was able to explicitly construct these transformations. Since we also would like to consider more complicated non–renormalizable models, we do not want to rely on the existence nor explicit construction of such transformations. To resolve these ambiguities in the definition of the soft action \((5)\) we note that the physics encoded in them is uniquely defined by the component action after the auxiliary fields have been eliminated. This mean one needs to compute the scalar potential $V_F$ from (3) and (5)

$$-V_F = \tilde{K} + \tilde{W} + \overline{\tilde{W}} - \left( \tilde{k}^a + \overline{\tilde{W}}^a \right) G^a \left( \tilde{k}^a + W^a \right). \tag{6}$$

The last term includes the standard $F$ term in supersymmetric models. In addition the chiral fermion masses depend on the functions $\tilde{k}$ and $k$:

$$L_{\text{ferm mass}} = - \frac{1}{2} \left( \tilde{k} + \overline{\tilde{k}} \right) ;_{ab} \psi^a \psi^b - \frac{1}{2} \left( \tilde{W} + \overline{\tilde{W}} \right) ;_{ab} \overline{\psi^a} \psi^b , \tag{7}$$

where the subscript $;_{ab}$ denotes the second Kähler covariant holomorphic derivative. From these expressions it is clear, that there are many different spurion representations that give rise to the same physics that is encoded in the scalar potential and the fermion mass terms. This explains the existence of transformations like the ones used by Yamada [8].

An additional complication is that there are more expressions, which one can write down, that can lead to scalar potentials and fermion masses similar to the ones quoted above. For example, one can consider the interaction

$$\int d^8 z \theta^2 \overline{\theta}^2 B D^2 A , \tag{8}$$

with $A$ and $B$ arbitrary functions of the chiral multiplets and their conjugates. It is not difficult to see that this in general gives both modification of the scalar potential (6) and the fermion mass term (7). If $B$ would be anti–chiral, one can partially integrate the $D^2$ to the $\theta^2$, and show that this can be absorbed into the function $\tilde{k}$. But in general such terms can not be absorbed into the functions we already defined. Since there is no obvious symmetry forbidding such interactions, one can expect that in general at the quantum level they will be generated. Because these terms are generated by diagrams with more super covariant derivatives on the external legs, the degree of divergence will be less than other diagrams in which all super covariant derivatives act on the superspace delta functions in the internal of the loop of the supergraph. If the theory is indeed soft, i.e. no quadratic divergences, therefore one expects that these terms will be finite. Technically computing the quantum corrections of such terms results in similar difficulties as computing higher derivative corrections to supersymmetric theories. In this work we ignore the
possibility of generating these interactions, by assuming that the background of chiral superfields can be treated as strictly constant.

A unique definition of these functions is obtained by rewrite the action (5) as

$$S_{\text{soft}} = \frac{1}{2} \int d^8z \theta^2 \bar{\theta}^2 \left( \tilde{K} + \tilde{W} + \bar{\tilde{W}} \right) + \int d^6\bar{z} \tilde{k} + \text{h.c.}. \quad (9)$$

From this we infer that we can define $\tilde{K}$ and $\tilde{W}$ uniquely by the requirement that $\tilde{K}$ does not contain a sum of purely holomorphic anti–holomorphic terms: The holomorphic part can be absorbed in $\tilde{W}$. Moreover, because $\tilde{k}$ has appeared under the $\int d^6\bar{z}$ integral, if it is anti–holomorphic, it can simply be absorbed into the anti–holomorphic superpotential $\tilde{W}$. On the other hand, it can be absorbed in $\tilde{K} + \tilde{W} + \bar{\tilde{W}}$ when its second anti–holomorphic derivative vanishes. A physical reason is that as long as it does not possess a second anti–holomorphic derivative, it does not contribute to the fermion mass term (7), hence only modifies the scalar potential (6): From that expression one can read of the modifications of the functions $\tilde{K}$ and $\tilde{W}$ that result in the same scalar potential. The holomorphicity constraints on the functions $\tilde{K}$, $\tilde{k}$ and $\tilde{W}$ are not respected by quantum corrections, as the calculations below will show. Of course it is possible to preform the same splitting on these corrections, we will not preform this explicitly here, as this becomes rather cumbersome.

The one loop corrections to the effective soft potentials include divergent contributions to $1/\epsilon$ and finite parts. Even though the aim of this work is to obtain finite contributions, the divergent parts of the one loop soft potentials are useful as well: They provide us with an important consistency check of our results, because from them we can obtain (part of) the well–known one beta functions [33–37] (for results including two loop beta functions see [7, 8]) for the soft parameters, if we restrict to renormalizable models. We obtain these beta functions by computing the renormalization of the parameters that appear in the scalar (6), and found exact agreement.

In the remainder of this section we give the results of analysis. First of all the superpotential mass $M^2_W$ defined in (2) is modified to

$$m^2_W = \bar{w}w, \quad w_{ab} = W_{ab} + \tilde{k}_{ab}, \quad (GwG^T)^{ab} = \bar{W}_{ab} + \tilde{k}_{ab}. \quad (10)$$

Strictly speaking we only find normal, not covariant, derivatives. This is a consequence of the fact that we assume that the background satisfy the background equations of motions, which means that various first derivatives are zero. The first effect of the one loop renormalization of the renormalization of the supersymmetry breaking action (5) is somewhat surprising: Because of the modification of the superpotential mass matrix in (10) the Kähler potential of the supersymmetric sector of the theory is modified to

$$K_{1L} = - \text{tr}_{\Lambda_\delta} L_1 (m^2_{\tilde{C}}) + \frac{1}{2} \text{tr} L_1 (m^2_W), \quad (11)$$
where the integral $L_1$ is defined in appendix A, which results from replacing $M_W^2$ by $m_W^2$ in (1) and modification of the counter term corresponding to the wavefunction renormalization. The reason for this effect is that $\tilde{k}$ gives similar terms as the anti–holomorphic superpotential $W$ as long as only anti–holomorphic derivatives are applied.

In addition to the mass $m_W^2$ defined in (10), we introduce the mass matrices:

$$m_V^2 = \frac{1}{2}(m_C^2 + m_C^2 T), \quad m_G^2 = 2Tj\phi h^{-1/2} \tilde{\phi} TjG, \quad m_S^2 = -G^{-1} \tilde{G}, \quad \tilde{m}^2 = (\tilde{w} \tilde{w})^{1/2},$$

(12)

where have used the notations

$$\tilde{G}_{ab} = \tilde{K}_{ab} - \tilde{k}_{ab} (G^{-1})^b_{\tilde{b}} \tilde{k}_{\tilde{a}}, \quad \tilde{w}_{ab} = \tilde{W}_{ab} + \tilde{K}_{ab} - \frac{1}{2} \left( w_{ac} (G^{-1})^c_{\tilde{c}} \tilde{k}_{\tilde{b}} + a \leftrightarrow b \right),$$

(13)

and $\tilde{w} = G^{-1} \tilde{w}^T G^{-1 T}$. The first two masses are the vector and Goldstone boson masses in the ‘t Hooft–Feynman gauge. The latter two matrices are soft masses. These definitions can be intuitively understood. However, we need to explain why the definition of $\tilde{G}$ also includes the second mixed derivatives of $\tilde{k}$, and why the expression for $\tilde{w}$ also contains the second holomorphic derivative of $\tilde{K}$ and the second mixed derivative of $\tilde{k}$: As explained above the definitions of the soft functions is not unique. When defining the quadratic action (which is used to obtain our results) similar ambiguities arise. We have resolved them by making choices, that are most convenient for the computation of the one loop corrections. Finally we define mass matrices $m_{V}^2$

$$m_{V}^2 = h^{-1} m_V^2 + \frac{1}{8} |h^{-1} \tilde{f}|^2 \pm \sqrt{(h^{-1} m_V^2 + \frac{1}{8} |h^{-1} \tilde{f}|^2)^2 - (h^{-1} m_V^2)^2},$$

(14)

which involve the gaugino mass $\tilde{f}$.

We present the corrections to the effective one loop soft potentials in the following: It is natural to split the contributions into terms that are proportional to $\theta^2$ (and their conjugates proportional to $\bar{\theta}^2$), and terms proportional to $\theta^2 \bar{\theta}^2$. The detailed derivation of these results in the next sections is performed up to the level that we obtain standard scalar integrals $J_0$, $J_1$ and $L_0$ defined in appendix A. In terms of these integrals we obtain for $\tilde{k}$ at one loop

$$\tilde{k}_{1L} = \frac{1}{2} \int d^4 \bar{v} \text{tr} \left[ \left\{ J_0(m_{v+}^2) - J_0(m_{v-}^2) + \frac{1}{2} m_S^2 J_1(m_{v+}^2) - \frac{1}{2} m_S^2 J_1(m_{v-}^2) \right\} \tilde{m}^2 \tilde{w} \right]$$

$$+ \frac{1}{2} \Delta R_0 - \frac{1}{4} \Delta R_1 + \frac{1}{2(D-1)} \text{tr}_\text{Ad} \left[ \frac{1}{m_+ - m_-} \left\{ m_+ J_0(m_{v}^2) - m_- J_0(m_{v}^2) \right\} h^{-1} \tilde{f} \right].$$

(15)

This result combines the expressions given in (102) and (81) on the first and second line, respectively. We have defined the mass matrices $m_{V}^2 = m_S^2 \pm v \tilde{m}^2 + v^2 m_W^2$, that depend on the integration variable $v$. For $\tilde{K}$ we find

$$\tilde{K}_{1L} = \text{tr} \left[ L_0(m_G^2) - L_0(m_G^2 + m_S^2) + L_0(m_S^2) - \frac{1}{2} L_0(m_S^2 + m_W^2 + \tilde{m}^2) - \frac{1}{2} L_0(m_S^2 + m_W^2 - \tilde{m}^2) + L_0(m_W^2) \right] - \frac{1}{2} \Delta K + \text{tr}_\text{Ad} \left[ L_0(m_{v+}^2) + L_0(m_{v-}^2) - 2 L_0(h^{-1} m_{v}^2) \right].$$

(16)
Here we have collected the contributions given in (86), (98), (99) and (79). In these expressions \( \Delta R_0 \), \( \Delta R_1 \) and \( \Delta K \) represent further corrections that only arise if the various mass matrices do not commute. In appendix B we have defined these functions such that they give only finite contributions and are proportional to at least one commutator of such matrices. In principle one might also expect that the gauge corrections result in similar corrections which are proportional to commutators containing \( \tilde{f} \). However, because we assume that \( \tilde{f} \) is gauge invariant and proportional to the Killing metric, all such commutators vanish.

These expressions of these soft potentials can now be evaluated using dimensional regularization. The standard integrals \( J_0 \), \( J_1 \) and \( L_0 \) are evaluated in appendix A, see (A.2) and (A.5). In this scheme the finite parts of these soft potentials is obtained by making the substitutions

\[
J_1(m^2) \rightarrow J_1'(m^2) = \frac{1}{16\pi^2} \left[ 1 - \ln \frac{m^2}{\mu^2} \right], \quad L_0(m^2) \rightarrow L_0'(m^2) = -\frac{1}{2} \frac{m^4}{16\pi^2} \left[ \frac{3}{2} - \ln \frac{m^2}{\mu^2} \right],
\]

and using that within dimensional regularization \( J_0(m^2) = -m^2 J_1(m^2) \). We do not give the expression resulting from substituting (17) in (15) and (16), because they are somewhat lengthy and not very illuminating. Moreover, for simplicity we do not give expressions for the additional functions \( \Delta K \), \( \Delta R_0 \) and \( \Delta R_1 \) here, because they are difficult to compute explicitly. On the other hand, we see that –at least in these expressions– the existence of quadratic divergences is independent of the functions \( \tilde{k} \) and \( \tilde{W} \).

3 Illustrations and Examples

3.1 Conditions for Softness

Even though we call the spurion insertions in action (5) soft, these interaction might still lead to quadratic divergences, because we generically consider non–renormalizable theories. Therefore, we will here derive some simple criterion to ensure that the interaction terms (5) are indeed soft.

In this paper we predominantly use dimensional reduction to regularize our quantum corrections at one loop. For the purpose of classifying quadratic and logarithmic divergences this scheme is less useful because both divergences just result in a pole in \( \epsilon \). To perform the classification of the quadratic divergences we therefore here resort to the cut–off scheme, in which the loop momentum is integrated up to scale \( \Lambda \). Since we are only interested in one loop vacuum graphs, i.e. without external lines, there are no ambiguities in the definition of the cut–off scheme. Since we have represented all results as sums of three types of standard integrals \( J_0 \), \( J_1 \) and \( L_0 \), we simply have to give the representation of their divergent behavior in the cut–off scheme. They read

\[
J_0^{\text{div}}(m^2) = \frac{1}{16 \pi^2} \left[ \Lambda^2 - m^2 \ln \Lambda^2 \right], \quad J_1^{\text{div}}(m^2) = \frac{1}{16 \pi^2} \ln \Lambda^2, \quad L_0^{\text{div}}(m^2) = \frac{m^2}{16 \pi^2} \left[ \Lambda^2 - \frac{1}{2} m^2 \ln \Lambda^2 \right].
\]

(18)
Using these expressions to determine the quadratically divergent parts of (15) and (16) we find
\[
\tilde k_{\text{quad div}}^{1L} = \frac{\Lambda^2}{96 \pi^2} \text{tr}\text{Ad} h^{-1} \tilde f, \quad \tilde K_{\text{soft}}^{\text{soft}} = \frac{\Lambda^2}{16 \pi^2} \left\{ \frac{1}{4} \text{tr}\text{Ad} |h^{-1} \tilde f|^2 - \text{tr} m^2_S \right\},
\]
respectively. From these expression we conclude that if \( h^{-1} \tilde f \) and \( m^2_S \) are constants they give rise to constant quadratic divergent corrections to the soft potentials which are unobservable, otherwise quadratic divergence arise. In particular, as \( m^2_S \) is a function of \( \tilde K \) and \( \tilde k \), one can say that if these functions are non-trivial quadratic divergences will arise and softness is lost.

3.2 Renormalization of the Kähler Potential

In the previous section we showed that the soft potential \( \tilde k \), defined in (5) can induce a modification of the expression of the one loop Kähler potential. As the Kähler potential describes the supersymmetric part of the theory, this is a surprising result. We explain that this additional renormalization is generically accompanied by quadratic divergences, but one can consider models where those are absent.

Because our general results are rather involved, we would like to give a simple example in which these effects arise. To this end we consider the renormalizable Wess–Zumino model described by the Kähler and superpotential
\[
K = \Phi \Phi, \quad W = \frac{1}{2} m \Phi^2 + \frac{1}{6} \lambda \Phi^3.
\]
To make our illustration as simple as possible, we take the soft potential \( \tilde k \) non–vanishing
\[
\tilde k = \frac{1}{2} \ell \Phi^2 \Phi,
\]
Using (11) we find for the Kähler potential
\[
K_{1L} = \frac{1}{2} \frac{m^2_W}{16 \pi^2} \left[ \frac{1}{\epsilon} + 2 - \ln \frac{m^2_W}{\mu^2} \right], \quad m^2_W = |m + \lambda \Phi + \ell \Phi|^2.
\]
Notice that it is not possible to reproduce the mass \( m^2_W \) from a modified superpotential alone, because it contains terms with \( \Phi^2 \) which cannot be obtain from a superpotential alone. We can not remove \( \tilde k \) by a field redefinition of the scalars, because their kinetic terms are then always modified. Also a modified \( \theta^2 \) dependent transformation does not work because such a transformation necessarily has to violate chirality constraints. In the previous subsection we have seen that a non–trivial \( \tilde k \) generically induces quadratic divergences.

From this one might conclude that the renormalization due to supersymmetry breaking interactions only arises if these interactions are not soft. As we will now illustrate this generic conclusion is not always true: The quadratic divergences can be fine tuned away at least up to the one loop level. In particular, if we choose
\[
\tilde K = \frac{1}{4} |\ell|^2 \Phi^2 \Phi^2,
\]
we find that \( m^2_S \) vanishes identically, and there are no quadratic divergences. Hence renormalization of the Kähler potential is possible even in model which are soft.
3.3 Softly Broken SQED

In this section we consider SQED with renormalizable soft breaking as a particular illustration of the results described in this work. The supersymmetric part of SQED is given by

\[ S_{\text{SQED}} = \int d^8z \bar{\phi}_\pm e^{\pm 2V} \phi_\pm + \int d^6z \left( m \phi_+ \phi_- + \frac{1}{4g^2} \int d^6z W^2 \right) + \text{h.c.} , \]  

where \( m \) is the mass of the electron superfield and \( g^2 \) the gauge coupling. The notation \( \pm \) in the first term indicates that we sum over the kinetic terms of \( \phi_+ \) and \( \phi_- \). The Hermitian conjugation only acts on the chiral superspace integral part of this expression. The corresponding soft action reads

\[ S_{\text{soft SQED}} = -\int d^8z \theta^2 \bar{\theta}_2 m_0^2 \phi_\pm e^{\pm 2V} \tilde{\phi}_\pm + \int d^8z \theta^2 \left( M^2 \phi_+ \phi_- + \frac{1}{2g^2} m_g W^2 \right) + \text{h.c.} , \]  

where \( m_0^2 \) is a real soft scalar mass, \( M \) a complex scalar mass, and \( m_g \) is the gaugino mass. The factor in front of the gaugino mass has been chosen such that the normalization of the kinetic term of the gauginos is taken into account. In principle there can be two real scalar masses; different ones in front of the first (two) terms. However, when one assumes the discrete symmetry

\[ \phi_\pm \to \phi_\mp , \quad V \to -V , \]  

these two masses are necessarily equal. In any case since all the masses \( m \), \( m_0^2 \) and \( M^2 \) are constants, the chiral multiplets only give rise to constant corrections to the soft potentials \( \tilde{K} \) and \( \tilde{k} \) at one loop, and therefore do not give rise to observable renormalization.

We do receive corrections due to the gauge interactions, because we encounter field dependent mass matrices

\[ m_V^2 = 2g^2 (|\phi_+|^2 + |\phi_-|^2) , \quad m_0^2 = m_V^2 \Pi , \quad \Pi = \frac{1}{|\phi_+|^2 + |\phi_-|^2} \left( \frac{\phi_+}{\phi_-} \right) \left( \bar{\phi}_+ \bar{\phi}_- \right) , \]  

where \( \Pi \) is a projection operator. Notice that we have absorbed a factor \( h^{-1} = g^2 \) into the definitions of the mass \( m_V^2 \). Finally the masses \( m_\pm^2 \) take the form

\[ m_\pm^2 = m_V^2 + \frac{1}{2} |m_g|^2 \pm \sqrt{(m_V^2 + \frac{1}{2} |m_g|^2)^2 - m_0^4} . \]  

Using the general expression (16) we obtain for \( \tilde{K} \) the one loop expression

\[ \tilde{K}_{1L} = L_0(m_+^2) + L_0(m_-^2) - L_0(m_V^2) - L_0(m_0 + m_V^2) = -\frac{1}{32 \pi^2} \left( (m_0^2 + m_V^2)^2 \ln \left( 1 + \frac{m_0^2}{m_V^2} \right) \right) \]
\[ + \left\{ (m_+^2 - m_-^2)^2 + m_V^4 - (m_0 + m_V^2)^2 \right\} \left( \frac{3}{2} - \ln \frac{m_V^2}{\mu^2} \right) - \frac{m_+^4 - m_-^4}{2} \ln \frac{m_+^2}{m_-^2} . \]
For the other soft potential $\tilde{k}$ we find using (15)
\[
\tilde{k}_{1L} = -\frac{m_g}{48\pi^2}\left( (m_+^2 + m_-^2 + m_+ m_-) \left( \frac{5}{3} - \ln\frac{m_+ m_-}{\mu^2} \right) - \frac{1}{2} \frac{m_+^3 + m_-^3}{m_+ - m_-} \ln\frac{m_+^2}{m_-^2} \right). \tag{30}
\]

A few observations about these results are in order: These one loop corrections respect the discrete symmetry (26), showing that (at least) up to the one loop level this discrete symmetry is respected. Also we see that they do not dependent on the complex scalar mass $M$ at all, and $\tilde{k}_{1L}$ also does not depend on $m_0$.

A consistency check on these expressions is obtained when one considers the supersymmetric limit, i.e. the tree parameters $m_0$ and $m_g$ tend to zero, all the soft quantum corrections vanish. This is indeed the case as can be seen from the leading behavior of the soft potentials in this limit:
\[
\tilde{K}_{1L} = -\frac{m_g^2}{16\pi^2} \left[ m_g^2 \left( 1 - 2 \ln\frac{m_Y^2}{\mu^2} \right) - m_0^2 \left( 1 - \ln\frac{m_Y^2}{\mu^2} \right) \right], \quad \tilde{k}_{1L} = -\frac{m_g m_Y^2}{16\pi^2} \left( 1 - \ln\frac{m_Y^2}{\mu^2} \right), \tag{31}
\]
when $m_0, m_g \to 0$.

### 4 Preparing for One Loop Computations

In this section we lay the basis for our one loop computation of the soft potentials. This calculation can be thought of as an extension of our computation of the Kähler potential [28, 29] to the case where supersymmetry is softly broken.

The aim is to compute the effective soft potentials $\tilde{K}$ and $\tilde{k}$, see (5), using a background field method at the one loop level. This means that we will encounter one loop vacuum graphs in this work, i.e. determinants of the kinetic operators of the various quantum fields in the theory. For this reason we want to determine the quadratic action of the quantum superfields, in which the masses are functions of the background chiral multiplets. In this calculation it is crucial to distinguish whether the spurions find themselves surrounded by super covariant derivatives or not. When this is the case the resulting spurion operators have many interesting algebraic properties that we develop in detail in subsection 4.1. (The super propagators as given in [38] can be obtained using this treatment of the spurion insertions.) We can make use of these algebraic properties, because we assume, that the background chiral superfields $\phi^a$ are constant; the spurion operators only act on each other or on superspace delta functions. The limitation of this procedure is that we are not able to compute the possibility of additional higher super covariant derivative terms, like the ones giving in (8). However, for the computation of the one loop expressions of the soft potentials $\tilde{K}$ and $\tilde{k}$ this procedure is sufficient. Using the spurion operators we compactly represent the quadratic action including soft terms in subsection 4.2. To compute the effective action amounts to evaluating various functional determinants, in the final subsection we review a basic method to do this.
4.1 Algebras of Spurions

To describe supersymmetry breaking terms in a way that takes most advantage of the special properties of supersymmetric theories, we work in superspace and use spurions $\theta^2$ and $\bar{\theta}^2$ to parameterize supersymmetry breaking terms. In this subsection we want to develop some algebraic properties of spurion operators, that arise when spurions find themselves between supercovariant derivatives.

Because $\theta^\alpha$ and $\bar{\theta}^\dot{\alpha}$ are Grassmannian variables, we obviously have that $(\theta^2)^2 = (\bar{\theta}^2)^2 = 0$. However, as we will see in the next subsection, when these spurions appear in the action of chiral superfields, then they are surrounded by the chiral projectors

$$P_+ = \frac{D^2\bar{D}^2}{16\Box}, \quad P_- = \frac{D^2\bar{D}^2}{16\Box},$$

which leads to interesting algebraic structure. To show this in the most transparent way, we define the chiral spurion operators

$$\eta_\pm = \Box \hat{\theta}^2 P_\pm, \quad \bar{\eta}_\pm = \Box \hat{\bar{\theta}}^2 P_\pm.$$  

First, of all, we should realize that these objects are operators rather than simple Grassmannian numbers, as the original spurions $\theta^2$ and $\bar{\theta}^2$ are. By definition $\eta_\pm$ are left and right chiral operators from both side, so properties like $\eta_+\eta_- = \eta_+\bar{\eta}_- = 0$, follow immediately. Also the algebraic properties can be verified easily

$$\eta_+^2 = \bar{\eta}_+^2 = 0, \quad \eta_+\bar{\eta}_- = \eta_+, \quad \bar{\eta}_+\eta_- = \bar{\eta}.$$  

(Because identical properties can be obtained for $\eta_-$ and $\bar{\eta}_-$, we do not describe them here explicitly.) Moreover, the products $\eta_+\bar{\eta}_+$ and $\bar{\eta}_+\eta_+$ are not equal, as can be seen by writing both expressions in terms of the original spurions

$$\eta_+\bar{\eta}_+ = \frac{D^2\bar{D}^2}{-4} \hat{\theta}^2\hat{\bar{\theta}}^2 \frac{D^2\bar{D}^2}{-4}, \quad \bar{\eta}_+\eta_+ = \Box P_+ \hat{\theta}^2\hat{\bar{\theta}}^2 P_+.$$  

These properties imply that $\eta_+$ and $\bar{\eta}_+$ generate a Clifford algebra: $\{ \eta_+, \bar{\eta}_+ \} = \mathds{1}_{\eta_+}$, where the combination $\mathds{1}_{\eta_+} = \eta_+\bar{\eta}_+ + \bar{\eta}_+\eta_+$ plays the role of the identity, because $\eta_+\mathds{1}_{\eta_+} = \mathds{1}_{\eta_+}\eta_+ = \eta_+$, and similarly for $\bar{\eta}_+$. This means that we can identify $\bar{\eta}_+$ and $\eta_+$ with Pauli matrices $\sigma_+$ and $\sigma_-$, respectively. And therefore, we can interpret the combination

$$A_{\eta_+} = A_{11} \bar{\eta}_+\eta_+ + A_{12} \bar{\eta}_+ + A_{21} \eta_+ + A_{22} \eta_+\bar{\eta}_+$$  

as a $2 \times 2$ matrix

$$A_{\eta_+} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \eta_+,$$

where $A_{ij}$ are constant complex numbers. It is also straightforward to confirm that the product $A_{\eta_+}A_{\bar{\eta}_+}$ computed, either using the expansion (36) and the above given properties, or the matrix
expression (37), leads to the same result. Using the identity \( \eta_+ \) we can define the projection operator

\[
\bot_+ = P_+ - \eta_+ \bar{\eta}_+ - \bar{\eta}_+ \eta_+ ,
\]

which is perpendicular to any \( A_{\eta_+} \): From the expansion (36) it follows immediately that \( A_{\eta_+} = A_{\eta_+} \bot_+ = 0 \). This implies that

\[
\left( A_0 \bot_+ + A_{\eta_+} \right)^2 = (A_0)^2 \bot_+ + (A_{\eta_+})^2 ,
\]

where \( A_0 \) is a complex number, just like the other \( A_{ij} \).

In the following sections we will often use this identification of the expansion (36) with the matrix (37) and the multiplication property (39) to simplify computations. This is possible, because all statements made above, can be generalized to the case where \( A_0 \) and \( A_{ij} \) are \( N \times N \) matrices themselves rather than merely complex numbers. In that case we can consider the trace \( \text{tr} \) over such \( N \times N \) matrices. We generalize this notion to the matrices \( A_{\eta_+} \) in the following way:

\[
\text{tr} A_{\eta_+] = \begin{pmatrix} \text{tr} A_{11} & \text{tr} A_{12} \\ \text{tr} A_{21} & \text{tr} A_{22} \end{pmatrix}_{\eta_+] = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}_{\eta_+]}
\]

Moreover, we can define two notions of transposition: We denote the transposition of the \( N \times N \) matrices by \( A^T_{ij} \), and use the symbol \( A^t_{\eta_+] \) to refer to the transposition of the entries of the \( A_{\eta_+] \) matrix. In particular, on a matrix (37) of \( N \times N \) matrices these two types of transposition act as

\[
\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}_{\eta_+] = \begin{pmatrix} A^T_{11} & A^T_{12} \\ A^T_{21} & A^T_{22} \end{pmatrix}_{\eta_+] , \quad \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}_{\eta_+] = \begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix}_{\eta_+]}
\]

These definitions imply that \( \text{tr} A^T_{\eta_+] = \text{tr} A_{\eta_+] , \) but \( \text{tr} A^t_{\eta_+] \neq \text{tr} A_{\eta_+] . \)

For the vector superfields we will also encounter a special combination of spurions and super covariant derivatives, similar to what we have seen above for chiral superfields. We associate to the projector on the vector superfield degrees of freedom,

\[
P_V = \frac{D^\alpha \overline{D^2} D_\alpha}{-8 \Box} = \frac{\overline{D_\alpha D^2 \overline{D^2}}}{8 \Box} ,
\]

the spurion operators

\[
\eta_V = \frac{D^\alpha \theta^2 \overline{D^2} D_\alpha}{8 \Box^{1/2}} , \quad \bar{\eta}_V = \frac{\overline{D_\alpha \theta^2 D^2 \overline{D^2}}}{8 \Box^{1/2}} .
\]

They have been defined such that they have very similar properties as \( \eta_\pm \) and \( \bar{\eta}_\pm \). In particular, they satisfy

\[
\eta_V^2 = \bar{\eta}_V^2 = 0 , \quad \eta_V \bar{\eta}_V \eta_V = \eta_V , \quad \bar{\eta}_V \eta_V \bar{\eta}_V = \bar{\eta}_V ,
\]

\[
14
\]
and \( \eta_V \bar{\eta}_V \neq \bar{\eta}_V \eta_V \) because
\[
\eta_V \bar{\eta}_V = -\frac{i}{2} \sigma^m_{\alpha \beta} D^\alpha \eta_+ \bar{\eta}_- D^\beta \partial_\mu \, , \quad \bar{\eta}_V \eta_V = -\frac{i}{2} \sigma^m_{\alpha \beta} D^\alpha \bar{\eta}_- \eta_+ D^\beta \partial_\mu .
\]
(45)

Notice that any product between these vector spurion operators and \( \eta_\pm, \bar{\eta}_\pm \) vanish. As in the case of these chiral spurions, we can say that \( \eta_V \) and \( \bar{\eta}_V \) generate a Clifford algebra, and we can make the identification between functions of \( \eta_V \) and \( \bar{\eta}_V \) and \( 2 \times 2 \) matrices:
\[
A_V = A_{11} \eta_V \eta_V + A_{12} \eta_V + A_{21} \eta_V + A_{22} \eta_V \bar{\eta}_V = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}_V .
\]
(46)

This identification is consistent in the sense that the product \( A_V A_V \) computed as functions of \( \eta_V \) and \( \bar{\eta}_V \) or as matrices lead to the same result. In particular, we have the identity matrix \( \mathbb{1}_V = \eta_V \bar{\eta}_V + \bar{\eta}_V \eta_V \). Finally, we can also define the projector
\[
\perp_V = P_V - \eta_V \bar{\eta}_V - \bar{\eta}_V \eta_V ,
\]
(47)

which is perpendicular to \( A_V : \perp_V A_V = A_V \perp_V = 0 \). Therefore as chiral spurion operators we find:
\[
\left( A_0 \perp_V + A_V \right)^2 = (A_0)^2 \perp_V + (A_V)^2 .
\]
(48)

Also here we can generalize all properties to matrices \( A_0 \) and \( A_{ij} \). This completes our exposition of the algebraic properties of the spurion operators \( \eta_\pm \) and \( \eta_V \).

We close this subsection by giving some relations to simplify these spurion operators when they are sandwiched between two superspace delta functions \( \delta_{21} \) as naturally happens in the evaluation one loop supergraphs. When they are placed between two superspace delta functions \( \delta_{21} \) and are integrated over the full double superspace, then these spurion operators reduce to simple spurion insertions. For expression with both spurion operators of a given type inserted, we find
\[
\int d^8 z_{12} \delta_{21} \left[ F \bar{\eta}_V \eta_V \right]_2 \delta_{21} = \int d^8 z_{12} \delta_{21} \left[ F \bar{\eta}_\pm \bar{\eta}_\pm \right]_2 \delta_{21} = \int d^4 x_{12} d^4 \theta \theta^2 \bar{\theta}^2 \delta_{21}^4 \delta_{21}^4 ,
\]
(49)
\[
\int d^8 z_{12} \delta_{21} \left[ F \eta_V \bar{\eta}_V \right]_2 \delta_{21} = \int d^8 z_{12} \delta_{21} \left[ F \eta_\pm \eta_\pm \right]_2 \delta_{21} = -2 \int d^4 x_{12} d^4 \theta \theta^2 \bar{\theta}^2 \delta_{21}^1 \delta_{21}^1 ,
\]
(50)

where \( \delta_{21} = \delta^4 (x_2 - x_1) \delta^4 (\theta_2 - \theta_1) \) and the subscript 1 or 2 on the square brackets \([\ldots]\) and a given superspace operator \( F \) denote that the corresponding expression is defined in (superspace) coordinate system 1 or 2. When only a single spurion operator of a given type is inserted, we instead obtain
\[
\int d^8 z_{12} \delta_{21} \left[ F \eta_\pm \right]_2 \delta_{21} = \int d^4 x_{12} d^4 \theta \theta^2 \bar{\theta}^2 \delta_{21}^4 \left[ F \frac{1}{\Box_+^{1/2}} \right]_2 \delta_{21}^4 ,
\]
(51)
\[
\int d^8 z_{12} \delta_{21} \left[ F \eta_V \right]_2 \delta_{21} = 2 \int d^4 x_{12} d^4 \theta \theta^2 \delta_{21}^1 \left[ F \frac{1}{\Box_+^{1/2}} \right]_2 \delta_{21}^4 ,
\]
(52)

and similar relations are obtained for the conjugate spurion operator insertions.
4.2 Quadratic Action

To be able to compute the one loop effective soft potentials we expand the chiral multiplets around a non-trivial background of chiral multiplets. To implement this we perform the background quantum splitting by the shift \( \phi \to \phi + \Phi \), where \( \phi \) refers to the classical background and \( \Phi \) is the quantum fluctuation. We assume that the classical background is strictly constant throughout the computation. This assumption is sufficient for our purposes because we are interested in the computation of the soft potentials \( \tilde{K} \) and \( \tilde{k} \) which do not involve any supercovariant derivatives. Treating \( V \) also as a quantum fluctuation we end up with the following quadratic action

\[
S_{\text{quadr}} = \int d^8z \left\{ \overline{\Phi} \left( G + \tilde{G} \theta^2 \bar{\theta}^2 \right) \Phi + 2 \bar{\phi} V (G + \tilde{G} \theta^2 \bar{\theta}^2) \Phi + \frac{1}{4} \left( f_{IJ} + \tilde{f}_{IJ} \theta^2 \right) W^I \alpha W^J \alpha \right\} + \text{h.c.},
\]

where \( W^I \alpha = -\mathcal{T}^2 D_\alpha V^I / 4 \), and the Hermitian conjugation only acts on the \( \int d^8z \) integral. Furthermore, we have used the notation introduced in (13) which arises for the following reasons: The functions \( \tilde{K} \) and \( \tilde{k} \) can possess non-vanishing second holomorphic derivatives, resulting in the definitions of \( w \) and part of \( \tilde{w} \). The last term of \( \tilde{w} \) in (13) results from first eliminating the auxiliary fields in this quadratic theory and then only reintroducing them for the modified superpotential \( w \). In this notation we keep the dependence on the background chiral multiplets \( \phi \) implicit.

An important distinction is now made depending on whether the spurions find themselves surrounded by supercovariant derivatives are not. If they are not surrounded by such derivatives, they can at most be inserted a single time in a diagram, otherwise the expression for the diagram vanishes identically. Because, the superpotential and the gauge kinetic terms are governed by holomorphicity, the spurions are naturally surrounded by chiral projectors, therefore, the only two point interaction which are not surrounded by super covariant derivatives in (53) are given by

\[
S_{\text{single}} = 2 \int d^8z \theta^2 \bar{\theta}^2 \left\{ \bar{\phi} V \tilde{G} V \phi + \bar{\phi} V \tilde{G} \Phi + \overline{\Phi} \tilde{G} V \phi \right\}.
\]

We explain at the end of this subsection that these interactions do not lead to quantum correction to the soft potentials.

In the remaining terms of (53) the gauge superfields appear in the same way as in the supersymmetry preserving theory, hence we can use the gauge fixing action [39] (see also [28])

\[
S_{\text{G.F.}} = - \int d^8z \Theta^I h_{IJ} \overline{\Theta}^J, \quad \Theta^I = \sqrt{2} \mathcal{D}^I \left[ V^I + h^{-1} \mathcal{T}^I G T^I \phi \right]
\]

as if supersymmetry is unbroken: This gauge fixing is uniquely defined by requiring that the mixing between the chiral and vector quantum superfields is absent at the quadratic level, and...
that the $V$ propagator does not contain any $D$’s or $\overline{D}$’s. This implies that the FP–ghost sector is the same as in the supersymmetric theory in the Feynman–t’Hooft gauge [28,39]. Therefore, in this gauge the ghosts only give standard corrections to the Kähler potential (which are part of the result in (1)), but not to the soft potentials. Adding this gauge fixing action to the remaining terms in (53) (that do not appear in (54)), show that all mixing between the vector and chiral supermultiplets disappear. This allows us to separately give the quadratic actions for the chiral and vector superfields. Written as a full superspace integral, the action for the chiral superfields becomes after gauge fixing (55)

$$
S_\Phi = \int d^8 z \left\{ \overline{\Phi} G \left( P_+ - m_\Phi^2 \frac{1}{\Box} P_+ - m_\Phi^2 \frac{1}{\Box} \eta_+ \eta_+ \right) \Phi + \frac{1}{2} \Phi^T \left( \tilde{w} P_- + \tilde{\bar{w}} \frac{1}{\Box^{1/2}} \eta_- \right) \Phi + \frac{1}{2} \Phi^T \left( \tilde{\bar{w}} P_+ + \tilde{w} \frac{1}{\Box^{1/2}} \eta_+ \right) \Phi \right\} . 
$$

(56)

Here we have made use of the chiral spurion operators $\eta_{\pm}$ and $\bar{\eta}_{\pm}$ defined in section 4.1. Similarly, using the spurions $\eta_V$ and $\bar{\eta}_V$ defined there, we can write the quadratic vector superfield action after gauge fixing as

$$
S_V = - \int d^8 z V^I \left\{ h_{IJJ} \Box - m_V^2 \eta_{IJJ} - \frac{1}{2} \tilde{f}_{IJJ} \Box^{1/2} \eta_V - \frac{1}{2} \tilde{f}_{IJJ} \Box^{1/2} \bar{\eta}_V \right\} V^J . 
$$

(57)

The quadratic superfield actions (56) and (57) will be the starting points of the computation of the effective soft potentials $\mathcal{K}$ and $\tilde{K}$ in section 5.

We close this subsection by explaining why the two point interactions given in (54) do not lead to corrections to the soft potentials. As we already observed because these terms have spurions without having covariant derivatives or projectors surrounding them, they can be inserted at most a single time in a diagram. At the one loop level this means, that the only possible diagrams contain a single insertion of these operators and a single propagator closing the diagram. Now it is important that in the gauge (55) there are no propagators that interpolate between a vector multiplet $V$ and chiral or anti–chiral multiplets, $\Phi$ or $\overline{\Phi}$ (as can be seen from the quadratic actions (56) and (57)), therefore the last two interactions in (54) cannot give a contribution at one loop. The first term in (54) also gives a vanishing contribution: From (57) we can determine the full propagator with the spurion supersymmetry breaking [38]

$$
\Delta_V = \left[ 1 - \frac{1}{2} \left( \frac{|\tilde{f}|^2 \Box}{(h \Box - m_V^2)^2} \right)^{-1} \left\{ \frac{1}{h \Box - m_V^2} \Pi_V + \frac{1}{2} \left( \frac{1}{h \Box - m_V^2} \right)^2 \left( \tilde{f} \Box^{1/2} \eta_V + \tilde{f} \Box^{1/2} \bar{\eta}_V \right) \right\} \right] . 
$$

(58)

Because there is no mixing between the chiral and the vector multiplets, there is only one diagram we can consider in which this propagator is closed on the first interaction in (54). However, all possible contributions vanish because of the explicit spurion appearance:

$$
\delta_{21} [\theta^2 \bar{\theta}^2 F \eta_V]_2 \delta_{21} = \delta_{21} \left[ \theta^2 \bar{\theta}^2 \frac{D^\alpha D^\beta}{8 \Box} \overline{D}^\alpha D_\alpha \right]_2 \delta_{21} ,
$$

$$
\delta_{21} [\theta^2 \bar{\theta}^2 F \eta_V \bar{\eta}_V]_2 \delta_{21} = - \frac{i}{2} \sigma^m_{\alpha\dot{\alpha}} \delta_{21} \left[ \theta^2 \bar{\theta}^2 \frac{D^\beta}{4} \overline{D}^\alpha \overline{D}^\beta D_\alpha \overline{D}_m \right]_2 \delta_{21} ,
$$

(59)
see (43) and (45), using similar notation as in the reduction formulae (49)–(52). In both equations we see, that there are not enough super covariant derivatives hitting the last spurions and the delta–function on the right, hence both expressions are zero. We have therefore shown, that the interactions (54) do not give any contributions to the computation of the soft potentials.

4.3 One Loop Functional Determinants

To determine the one loop effective soft potentials we develop some general formalism to compute the corresponding functional determinants efficiently. We will perform the calculation for both chiral and vector multiplets, and aim to arrive at formulae, that can be used both in the standard supersymmetry preserving as well as the soft supersymmetry breaking situations.

We begin with chiral multiplets $\Phi^a$. Their effective action at one loop is given by

$$i\Gamma_{\Phi} = \int |D\Phi|^2 e^{iS_{\Phi}} \big|_{\text{conn}}, \quad S_{\Phi} = \frac{1}{2} \int d^8z \left( \Phi^T \Phi \right) K \left( \Phi \Phi^T \right),$$

(60)

the connected part of the Gaussian path integral. The quadratic operator $K = P + L$ is decomposed into a standard free part

$$P = \begin{pmatrix} 0 & P_- \\ P_+ & 0 \end{pmatrix},$$

(61)

and an arbitrary perturbation $L$. Depending on the precise form of $L$ this might be a rather involved computation.

To facilitate this computation we introduce a continuous parameter $0 \leq \lambda \leq 1$, and define $K_\lambda = P + \lambda L$. Because the Gaussian integral of the standard free quadratic operator $P$ is absorbed in the definition of the path integral, we can infer, that

$$i\Gamma_{\Phi} = \frac{1}{0} \frac{d}{d\lambda} i\Gamma_{\lambda},$$

(62)

where $\Gamma_{\lambda}$ is defined as $\Gamma_{\Phi}$ in (60) but with the interpolating quadratic operator $K_\lambda$ instead of $K$. The point of this exercise is, that the $\lambda$ derivative of $i\Gamma_{\lambda}$ is technically simpler to compute than the original functional determinant. Indeed, we find

$$\frac{d}{d\lambda} i\Gamma_{\lambda} = \int |D\Phi|^2 e^{iS_{\lambda}} \frac{i}{2} \int d^8z \left( \Phi^T \Phi \right) L \left( \Phi \Phi^T \right) \big|_{\text{conn}},$$

(63)

corresponds to a single one loop diagram in the theory defined by the interpolating kinetic operator $K_\lambda$ with a single insertion of the two point interaction defined by $L$.

Therefore to compute this we only need to determine the propagator in the interpolating theory. By rewriting the action of the interpolating action as

$$S_{\lambda} = \frac{1}{2} \int d^8z \left( \Psi^T \Psi \right) \frac{\nabla^2 \Psi}{-4} K_\lambda \frac{\nabla^2 \Psi}{-4} \left( \Psi^T \Psi \right),$$

(64)
using the field redefinitions

\[
\begin{pmatrix}
  \Phi \\
  \bar{\Phi}^T
\end{pmatrix} = \nabla^2 - 4 \begin{pmatrix}
  \Psi \\
  \bar{\Psi}^T
\end{pmatrix}, \quad \nabla^2 - 4 = \begin{pmatrix}
  0 & \bar{T}^2 \\
  D^2 & 0
\end{pmatrix},
\]

(65)

where \( \Psi(\bar{\Psi}) \) is (anti–)chiral, it is straightforward to confirm, that the propagator can be cast into the form

\[
\Delta_\lambda = \frac{\nabla^2}{-4\Box} \left( \mathbb{1} + \lambda P^T L \right)^{-1} P^T \nabla^2 - 4 \Delta_\lambda .
\]

(66)

Using this propagator and (63) we obtain

\[
\frac{d}{d\lambda} i \Gamma_\lambda = -\frac{1}{2} \int d^8 z_{12} \delta_{21} \text{Tr} \left[ P^T L(\mathbb{1} + \lambda P^T L)^{-1} \right] \delta_{21} .
\]

(67)

The notation Tr refers to the trace over the chiral multiplets in the real basis defined by the vector \((\Phi^T, \bar{\Phi})^T\). This expression is easily integrated over the parameter \( \lambda \), so that the final result compactly reads

\[
i \Gamma_\Phi = -\frac{1}{2} \int d^8 z_{12} \delta_{21} \text{Tr} \ln \left[ P^T K \right] \delta_{21} .
\]

(68)

Because the FP–ghost are described by anti–commuting chiral superfields, their one loop effective action is identical to the one above except that the overall sign is opposite.

For the vector multiplets we can perform a very similar analysis to compute the functional determinant

\[
i \Gamma_V = \int \mathcal{D}V e^{iS_V} |_{\text{conn}}, \quad S_V = -\int d^8 z V^I H_{IJ} V^J ,
\]

(69)

and we obtain

\[
i \Gamma_V = -\frac{1}{2} \int d^8 z_{12} \delta_{21} \text{tr}_{Ad} \ln \left[ H \right] \delta_{21} ,
\]

(70)

where \( \text{tr}_{Ad} \) denotes the trace in the adjoint representation.

### 5 Computation of the One Loop Soft Potentials

In this section we present the details of the one loop computation of the soft potential using the material developed in the previous section which leads to the results quoted in section 2. The computations are somewhat involved, especially for the chiral multiplets. Therefore we first present in subsection 5.1 the contributions due to gauge multiplets, because they are technically a little easier than the ones that are due to the chiral multiplets. They are present in the second subsection. In the way we have setup the calculations we not only compute the soft potentials but also parts of the supersymmetry preserving Kähler potential. The well–known result of the one loop Kähler potential we quoted in (1) in the introduction, and we will have used it as one of our cross checks on our computations.
5.1 One Loop Soft Potentials due to Gaugino Masses

We compute the one loop corrections to the soft potentials due to the gaugino mass insertions. To this end we employ (70) of subsection 4.3 to the quadratic action of the vector multiplets, given in (57) of subsection 4.2, from which the matrix $H$ can be read off. Using the spurion operators $\eta_V$, $\bar{\eta}_V$ and the projector $\perp_V$ defined in subsection 4.1, we can decompose this matrix as

$$H = (h \Box - m_V^2) \perp_V + (h \Box - m_V^2) (\mathbb{I}_V + Z_V), \quad (71)$$

where the vector multiplet mass matrix is given in (12). The matrix $Z_V$ has only off–diagonal block entries

$$Z_V = -\frac{1}{2} (h \Box - m_V^2)^{-1} \left( \begin{array}{cc} 0 & \tilde{f} \Box \frac{1}{2} \\ \tilde{f} \Box \frac{1}{2} & 0 \end{array} \right). \quad (72)$$

Here we emphasize with the subscript $V$ on the matrix, that we are using the matrix notation for the corresponding expression in terms of the spurions $\eta_V$ and $\bar{\eta}_V$ defined in (46). With $H$ written in this form it follows directly from standard properties of taking $\text{tr} \ln$ of matrices, that the effective action, given in (70), becomes

$$i \Gamma_V = -\frac{1}{2} \int d^8z_{12} \delta_{12} \left( \text{tr} \ln \left[ (h \Box - m_V^2) \right] + \text{tr} \ln \left[ \mathbb{I}_V + Z_V \right] \right) \delta_{21}. \quad (73)$$

The first term does not give a contribution because there will be no super covariant derivatives acting on the superspace delta functions, so a standard supergraph theorem says that the result vanishes.

To evaluate the second term directly is still difficult, therefore we will use the same method as in subsection 4.3 to compute the effective actions in general: We introduce a fictitious parameter $0 \leq \lambda \leq 1$, and define $Z_\lambda = \lambda Z_V$. We can differentiate the expression of (73), in which we substitute $Z_V \rightarrow Z_\lambda$, w.r.t. $\lambda$ to obtain

$$\frac{d}{d\lambda} i \Gamma_\lambda = -\frac{1}{2} \int d^8z_{12} \delta_{12} \text{tr} \ln \left[ (Z_V - \lambda Z_\lambda^2) \left( \mathbb{I}_V - \lambda^2 Z_\lambda^2 \right)^{-1} \right] \delta_{21}. \quad (74)$$

The reason for this rewriting is, that now the whole expression is written in terms of $Z_\lambda^2$, expect for only the first term in the first factor, so that we can now easily split the effective action in “even” and “odd” contributions:

$$\frac{d}{d\lambda} i \Gamma_\lambda^{\text{even}} = \frac{1}{2} \lambda \int d^8z_{12} \delta_{12} \text{tr} \ln \left[ Z_\lambda^2 \left( \mathbb{I}_V - \lambda^2 Z_\lambda^2 \right)^{-1} \right] \delta_{21}, \quad (75)$$

$$\frac{d}{d\lambda} i \Gamma_\lambda^{\text{odd}} = -\frac{1}{2} \int d^8z_{12} \delta_{12} \text{tr} \ln \left[ Z_V \left( \mathbb{I}_V - \lambda^2 Z_\lambda^2 \right)^{-1} \right] \delta_{21}. \quad (76)$$

Because the matrix $Z_V$, given in (72), is block off–diagonal it follows that its square is block diagonal. Therefore, the even (odd) contributions leads to (off–)diagonal contributions. In other
words, using the identification (46) only the odd contributions lead to terms proportional to \( \eta_V \) or \( \tilde{\eta}_V \), while the even ones give rise to effects proportional to products of these two spurion operators.

Note that it is straightforward to integrate the even part (75), and we obtain the compact expression

\[
\Gamma_{\text{even}} = -\frac{1}{4} \int d^8 z \delta_{12} \tr \ln \left[ \left( \bar{1}_V - Z_V^2 \right)_2 \delta_{21} \right]. \tag{77}
\]

Because the expression for the square of \( Z_V \) is a matrix with two blocks on the diagonal, which are build out of the same matrices but in opposite order, we find that the trace of both these blocks give the same contributions. Moreover, using the identity for reducing vector spurion operators between superspace delta functions (50), we can write this as

\[
\Gamma_{\text{even}} = \int d^8 z \theta^2 \bar{\theta}^2 \int \frac{d^D p}{(2\pi)^D} \mu^{D-4} \tr \ln \left[ \left( \bar{1} + \frac{1}{4} p^2 \frac{1}{p^2 + h^{-1} m_V^2} h^{-1} \tilde{f} \frac{1}{p^2 + h^{-1} m_V^2} h^{-1} \tilde{f} \right) \right]. \tag{78}
\]

If the matrices \( h^{-1} m_V^2 \) and \( h^{-1} \tilde{f} \) do not commute, this expression is rather difficult to evaluate exactly. However, because throughout this work we have assumed that the classical action is gauge invariant, and in particular, that both the gauge kinetic function \( f_{IJ} \) and its soft analog \( \tilde{f}_{IJ} \) are proportional to the Killing metric, it follows that they do commute. Therefore, from now on we can simply assume that the matrices \( h^{-1} m_V^2 \) and \( h^{-1} \tilde{f} \) have already been diagonalized simultaneously. In this case this integral becomes a sum of standard integral \( L_0 \) defined in (A.3)

\[
\Gamma_{\text{even}} = \int d^8 z \theta^2 \bar{\theta}^2 \tr \left\{ L_0(m_+^2) + L_0(m_-^2) - 2 L_0(h^{-1} m_V^2) \right\}, \tag{79}
\]

where we have used the mass matrices \( m_{\pm}^2 \) given in (14) and reduction formula (50).

For the odd part of the effective action due to the soft gauge kinetic function we make the same simplifying assumption on the mass matrices \( h^{-1} m_V^2 \) and \( h^{-1} \tilde{f} \), so that are already diagonalized the odd part of the effective action can be written as

\[
\Gamma_{\text{odd}} = \frac{1}{2} \int d^8 z \int_0^1 d\lambda \int \frac{d^D p}{(2\pi)^D} \mu^{D-4} \tr \ln \left[ \frac{p^2 + h^{-1} m_V^2}{(p^2 + h^{-1} m_V^2)^2 + \frac{1}{4} \lambda |h^{-1} \tilde{f}|^2 p^2} h^{-1} \tilde{f} \theta^2 \right] + \text{h.c.}, \tag{80}
\]

making use of (52). This double integral has defined in (B.1) and is evaluated in appendix B.1. We obtain

\[
\Gamma_{\text{odd}} = \frac{1}{2(D-1)} \int d^8 z \tr \left\{ \frac{1}{m_+ - m_-} \left( m_+ J_0(m_+^2) - m_- J_0(m_-^2) \right) \right\} h^{-1} \tilde{f} \theta^2 + \text{h.c.}, \tag{81}
\]

where the mass matrices \( m_{\pm}^2 \) are defined in (14). This completes the computation of the corrections to the soft potentials \( \tilde{K} \) and \( \tilde{k} \) due to gauge interactions.
5.2 One Loop Soft Potentials due to Chiral Multiplets

We now turn to computation of the effective soft action due to chiral multiplets. From the quadratic action for the chiral multiplets after the gauge fixing (56) we read off its kinetic operator $K$, which can be used in the functional determinant calculation for chiral multiplets discussed in section (4.3). We can represent it in the following block matrix form

$$P^T K = \begin{pmatrix} A_+ & \overline{C} \\ C & A_- \end{pmatrix},$$

where the matrices $A_+$, $A_-$, $C$ and $\overline{C}$ are given by

$$A_+ = \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} - m_G^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) P_+ - m_S^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \eta_+ \eta_+, \hspace{1cm} C = \left( w P_- + \bar{w} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \eta_+ \right) \frac{D^2}{-4 \square},$$

$$A_- = \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} - m_G^2 T \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) P_- - m_S^2 T \begin{pmatrix} 1 \\ 0 \end{pmatrix} \eta_- \bar{\eta}_-, \hspace{1cm} \overline{C} = \left( \bar{w} P_+ + w \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{\eta}_+ \right) \frac{\overline{D}^2}{-4 \square}.$$ (83)

Here we have used the notations introduced below (10) and in (12). Notice that $A_- = A_+^T$ and that $\overline{C} = C^\dagger$. Using the expression for the one loop effective action for general chiral multiplets, eq. (68), we obtain

$$i \Gamma_\Phi = -\frac{1}{2} \int d^8 z_{12} \delta_{21} \left\{ \text{tr} \ln A_+ + \text{tr} \ln A_- + \text{Tr} \ln \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + Z \right] \right\} \delta_{21}, \hspace{1cm} Z = \begin{pmatrix} 0 & A_-^{-1} \overline{C} \\ A_-^{-1} C & 0 \end{pmatrix}. \hspace{1cm} (84)$$

Next we compute the different parts of this expression separately.

Notice that the contributions of $A_+$ and $A_-$ are the same, because these matrices are each others transposed, and there is a trace $\text{tr}$ over the whole expression. These contributions are easily computed by realizing that we can write

$$A_+ = (P_+ - \bar{\eta}_+ \eta_+) \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} - m_G^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) + \bar{\eta}_+ \eta_+ \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} - m_S^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right).$$ (85)

Using this expression it is easy to understand that the sum of the contributions due to $A_+$ and $A_-$ gives

$$\Gamma_A = \int d^8 z \text{tr} \left[ L_1(m_G^2) + \theta^2 \bar{\theta}^2 \left\{ L_0(m_G^2) - L_0(m_G^2 + m_S^2) \right\} \right], \hspace{1cm} (86)$$

where the integrals $L_0$ and $L_1$ are defined in (A.3) of appendix A, and we have used (49). The first term in this expression can be combine with the contribution of the FP–ghost to the effective action to

$$(\Gamma_A + \Gamma_{FP})_{\text{SUSY}} = -\frac{1}{16 \pi^2} \int d^8 z \text{tr} \left[ m_G^2 \left( \frac{1}{\epsilon} + 2 - \ln \frac{m_G^2}{\mu^2} \right) \right]. \hspace{1cm} (87)$$

Notice this reproduces the result of the effective Kähler potential in a supersymmetric theory given in (1) (using that $\text{tr} m_G^{2n} = \text{tr}_{Ad}(h^{-1} m_C)^{2n}$), and can therefore be ignored when computing the effective soft potentials.
To compute the contribution due to the last term in (84) is more work. We proceed in a similar fashion as before: Introduce an extra parameter $0 \leq \lambda \leq 1$ in front of the off-diagonal terms (the ones which are proportional to $C$ and $\overline{C}$), and then differentiate w.r.t. this parameter. Next, we split the contributions into ones with even and odd powers of $Z$. None of the odd powers of $Z$ give a contribution: Because $Z^2 = \left( A_+^{-1} \overline{C} A_-^{-1} C \right) \left( 0 \begin{array}{c} 0 \\ A_-^{-1} C A_+^{-1} \overline{C} \end{array} \right) \left( A_+^{-1} \overline{C} A_-^{-1} C \right)$ is block diagonal, it follows that odd powers of $Z$ are necessarily off-diagonal in the real basis of the chiral multiplets. But the trace Tr in this basis is the sum of the traces in the complex basis of the block diagonal parts, hence the odd powers do not contribute. The even part is again readily integrated: Because all the contributions are block diagonal in the real basis, the trace Tr in the real basis reduces to the traces in the complex basis

$$i \Gamma_C = -\frac{1}{4} \int d^8z_{12} \delta_{21} \text{tr} \left[ \ln \left( P_+ - A_+^{-1} \overline{C} A_-^{-1} C \right) + \ln \left( P_- - A_-^{-1} C A_+^{-1} \overline{C} \right) \right] \delta_{21} .$$

To evaluate this further we use the matrix notation introduced section 4.1 to write

$$P_- - A_-^{-1} C A_+^{-1} \overline{C} = \perp_- \left( 1 - \frac{w \overline{w}}{\mp} \right) + \mp_- - E_- ,$$

where

$$E_- \equiv \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \begin{array}{cc} 1 & 0 \\ 0 \mp -m_T^2 \end{array} \end{pmatrix} \begin{pmatrix} \frac{w}{\mp -m_T^2} \overline{w} & \frac{w}{\mp -m_T^2} \overline{w} \\ \frac{\overline{w}}{\mp -m_T^2} \frac{1}{\mp -m_T^2} & \frac{\overline{w}}{\mp -m_T^2} \frac{\overline{w}}{\mp -m_T^2} \end{pmatrix} \begin{pmatrix} \begin{array}{c} \alpha \\ \gamma \end{array} \end{pmatrix} \eta_-.$$

The matrix $E_-$ has similar properties as the matrix $A_{\eta_+}$ described in subsection (4.1). (The matrix notation after the first equal sign is introduced here for later use below.) A similar expression for the first combination in (89) in terms of

$$E_+ = \left\{ \begin{pmatrix} 0 & \mp -m_T^2 T \\ 1 \mp -m_T^2 \end{pmatrix} E_- \begin{pmatrix} 0 & 1 \\ \mp -m_T^2 & 0 \end{pmatrix} \right\}^T ,$$

where we would like to remind the reader that $T$ denotes transposition in the complex matrix basis, while $t$ denotes transposition in the $\eta_\pm$–matrix basis, see (41). These expressions suggest how we can proceed in calculating the soft potentials: We first compute the effect of the matrix (…)$_{\eta_-}$ and its conjugate, and after that concentrate on the contributions due to the terms proportional to $\perp_-$. The computation of the soft potentials due to the $\eta_-$–matrix (90) (and its conjugate) is rather involved, and we again resort to introducing a continuous parameter $0 \leq \lambda \leq 1$ in front of the $\eta_-$–matrix part of $A_+^{-1} \overline{C} A_-^{-1} C$. This gives

$$\frac{d}{d\lambda} i \Gamma_{\eta_\mp}^{\eta_\pm} = \frac{1}{4} \int d^8z_{12} \delta_{21} \text{tr} F[E_{\pm}]_2 \delta_{21} ,$$

23
with $F[E] = E(\mathbb{1} - \lambda E)^{-1}$. (For notational simplicity we have not made the $\lambda$ dependence explicit in $F$.) Using the expressions for $E_-$ and $E_+$ in eqs. (91) and (92), after a somewhat lengthy manipulations with these matrices, we can show that the sum of $\Gamma^\eta_{\tilde{\eta} \tilde{\eta}}$ is given by
\[
\frac{d}{d\lambda} \Gamma^\eta_{\tilde{\eta} \tilde{\eta}} = \frac{1}{2} \int d^4x_{12} d^4\theta \delta_{21} \text{tr} \left[ \left( F_{11}(E_-) + F_{22}(E_-) \right) \theta^2 \tilde{\theta}^2 + \frac{1}{2} \left( \square - m^2_T \right) + 1 \right] \left( \frac{1}{2} \text{tr} F_{11}(E_-) \theta^2 \right) \left( \frac{1}{2} \text{tr} F_{22}(E_-) \theta^2 \right),
\]
using the identities (49) and (51). This expression shows that we can distinguish between contributions that go proportional to $\theta^2 \tilde{\theta}^2$ and those that only involve either $\theta^2$ or $\tilde{\theta}^2$.

We first focus on the $\theta^2 \tilde{\theta}^2$ part. Denoting the components of the matrix $E_-$ in the $\eta_-$-basis by $\alpha$, $\beta$, $\gamma$ and $\delta$, as given in (91), we can show after some algebra that
\[
\text{tr} \left[ F_{11}(E_-) + F_{22}(E_-) \right] = -\frac{d}{d\lambda} \text{tr} \left[ \ln(1 - \lambda \alpha) + \ln \left( 1 - \lambda \delta - \gamma \frac{\lambda}{1 - \lambda \alpha} \beta \right) \right].
\]
This means that for this contribution it is now straightforward to integrate over $\lambda$. By noting that
\[
1 - \delta - \gamma \frac{1}{1 - \alpha} \beta = \frac{1}{\square - m^2_T} \left\{ \square - m^2_T - \tilde{w} \bar{w} \right. - \tilde{w} \bar{\tilde{w}} \left. \right\},
\]
it follows that the $\theta^2 \tilde{\theta}^2$ contribution takes the form
\[
\Gamma^\eta_{\theta^2 \tilde{\theta}^2} = \int d^8z \theta^2 \tilde{\theta}^2 \left\{ \text{tr} \left[ L_0(m^2_S) - L_0(m^2_S + m^2_W) \right] \right\} - \frac{1}{2} K(m^2_S + m^2_W, \bar{w}).
\]
The simple matrix valued logarithmic integral $L_0$ is defined in (A.3), while the more complicated logarithmic integral $K(m^2_S + m^2_W, \bar{w})$ is given in (B.6). The latter integral is very difficult to evaluate exactly if the mass matrices $m^2_S$, $m^2_W$ and $\tilde{w}$ do not commute. As is shown in appendix B.2 we can approximate $K$ by $K_0$, given in (B.10): The error $\Delta K$ is finite and at least proportional to a single commutator of these mass matrices. We obtain
\[
\Gamma^\eta_{\theta^2 \tilde{\theta}^2} = \int d^8z \theta^2 \tilde{\theta}^2 \text{tr} \left[ L_0(m^2_S) - \frac{1}{2} L_0(m^2_S + m^2_W + \tilde{m}^2) - \frac{1}{2} L_0(m^2_S + m^2_W - \tilde{m}^2) \right] - \frac{1}{2} \Delta K,
\]
where the mass matrix $(\tilde{m}^2)^2 = \bar{w} \tilde{w}$ has been introduced. Notice that the second term in (97) is canceled by a contributions from $K_0$.

The term proportional to $\perp_-$ can be treated independently because of property (39) and results in an standard logarithmic integrals (A.3). A similar contribution comes from the second term in (89), therefore we find the total contribution due to $\perp_+$ in (90) is given by
\[
\Gamma_+ = \int d^8z \text{tr} \left[ \frac{1}{2} L_1(m^2_W) + L_0(m^2_W) \theta^2 \tilde{\theta}^2 \right].
\]
The first term here corresponds to the supersymmetry preserving correction to the one loop Kähler potential due to the superpotential. This coincides with previous results [23, 27, 28],
except that the mass $m_W^2$ here is the modified superpotential mass (10) rather then the true supersymmetric mass $M_W^2$ given in (2). This means that in general even the supersymmetric Kähler potential is modified by the effect of the soft potential $\tilde{\kappa}$. Notice that the second term also only depend on $m_W^2$, therefore in the supersymmetric limit it has to be canceled by other corrections. These other contributions are provided by the second and third terms in (98): In the limit of vanishing soft parameters they precisely cancel $L_0(m_W^2)$.

The last contribution we encounter comes from the off-diagonal parts $F_{21}$ and $F_{12}$ in (94). They in the end give rise to $\theta^2$ and $\bar{\theta}^2$ contributions. As one can show that these contributions are each others complex conjugates, we only describe the computation of the $\theta^2$ part explicitly here. By substituting the expression (91) for $E_-$ we obtain

$$\Gamma_{\eta}^{\theta^2} = \frac{1}{4} \int \left. \frac{d}{d\lambda} \Gamma_{\theta^2,\lambda} \right| - \frac{i}{1} \int d^4x_1 d^4\theta \theta^2 \delta_{21}^4 \text{tr} \left[ \left( 2 - \frac{m_W^2}{v} \right) \left( 1 - \lambda \delta - \frac{\lambda^2}{1 - \lambda \alpha} \right)^{-1} \frac{1}{1 - \lambda \alpha} \frac{1}{\Box} \right] \delta_{21}^4.$$  

(100)

Using the definitions of $\alpha, \ldots, \delta$ given in (91) and the expression (96) extended to include $\lambda$, we can put the integrated contribution in the form

$$\Gamma_{\eta}^{\theta^2} = \frac{1}{4} \int d^4x d^4\theta \theta^2 \left\{ 2 R_0(m_S^2, \bar{w}, w) - R_1(m_S^2, \bar{w}, w) \right\},$$  

(101)

where the function $R_0$ and $R_1$ are defined in (B.12) in appendix B.3. As for the $\theta^2 \bar{\theta}^2$ contribution computed above, the integrals are difficult to perform explicitly, therefore we again split finite terms that are only proportional to commutators of combination of mass matrices and denote them by $\Delta R_0$ and $\Delta R_1$. Using the results for $R_0$ and $R_1$ given in (B.16), we can write the results compactly like

$$\Gamma_{\eta}^{\bar{\theta}^2} = \frac{1}{4} \int d^4x d^4\theta \bar{\theta}^2 \left\{ \int_0^1 dv \text{tr} \left[ \left\{ 2 J_0(m_{v+}^2) - 2 J_0(m_{v-}^2) + m_S^2 J_1(m_{v+}^2) - m_S^2 J_1(m_{v-}^2) \right\} \bar{m}^2 \frac{1}{v} \bar{w} \right] + 2 \Delta R_0(m_S^2, \bar{w}, w) - \Delta R_1(m_S^2, \bar{w}, w) \right\}.$$  

(102)

This result is written with an extra $v$ integration. To perform this integration is not conceptually difficult, but will make the expression rather lengthy even if when the mass matrices reduce to mere (complex) numbers. This completes the description of the details of the one loop contributions to the soft potentials $\tilde{K}$ and $\tilde{k}$ due to the gauge interactions. The final results are collected in section 2.

A Basic One Loop Scalar Integrals

This appendix is devoted to the evaluation of the basic one loop scalar integrals, which arise in the main text of this paper. We compute these scalar integrals in the $\overline{\text{MS}}$ scheme: We evaluate the integrals in $D = 4 - 2\epsilon$ dimensions, and we introduce the renormalization scale $\mu$ such that
all $D$ dimensional integrals have the same mass dimensions as their divergent four dimensional counter parts.

The first basic type one loop integral is given by

$$J_n(m^2) = \int \frac{d^D p}{(2\pi)^D \mu^{D-4}} \frac{1}{p^2} \frac{1}{p^2 + m^2} = \frac{(m^2)^{1-n}}{16\pi^2} \left( \frac{4\pi \mu}{m^2} \right)^2 \left( p^2 + M^2 \right)^{-\frac{D}{2} - 1} \frac{1}{\Gamma(D/2)} \frac{\pi}{\sin \pi(D/2 - n)},$$

(A.1)

for $n = 0, 1$. In the applications in the main text we need to expand this to the zeroth order in $\epsilon$ including the pole $1/\epsilon$. In particular, we encounter

$$J_0(m^2) = -\frac{m^2}{16\pi^2} \left[ \frac{1}{\epsilon} + 1 - \ln \frac{m^2}{\mu^2} \right], \quad J_1(m^2) = \frac{1}{16\pi^2} \left[ \frac{1}{\epsilon} + 1 - \ln \frac{m^2}{\mu^2} \right].$$

(A.2)

The second class of integrals we encounter frequently in this work are integrals over a single logarithm

$$L_n(m^2) = \int \frac{d^D p}{(2\pi)^D \mu^{D-4}} \frac{1}{p^2} \ln \left( 1 + \frac{m^2}{p^2} \right).$$

(A.3)

In fact, this class of integrals and the previous ones are related to each other via differentiation w.r.t. the mass parameter

$$\frac{\partial}{\partial m^2} L_n(m^2) = J_n(m^2) \Rightarrow L_n(m^2) = \frac{m^2}{D/2 - n} J_n(m^2),$$

(A.4)

for $n = 0, 1$. In the second equation we have directly integrated the general result for $J_n(m^2)$ given in (A.1). In the main part of the paper we need the following two integral results

$$L_0(m^2) = -\frac{1}{2} \frac{m^4}{16\pi^2} \left[ \frac{1}{\epsilon} + \frac{3}{2} - \ln \frac{m^2}{\mu^2} \right], \quad L_1(m^2) = \frac{m^2}{16\pi^2} \left[ \frac{1}{\epsilon} + 2 - \ln \frac{m^2}{\mu^2} \right].$$

(A.5)

B Complicated One Loop Scalar Integrals

In this appendix we collect the computation of a number of one loop integrals, which are more difficult to obtain directly, so that we can avoid having length digressions on their computations in the main text.

B.1 $O(m^2, M^2)$ Integral

The first integral we consider here is defined by

$$O(m^2, M^2) = \int_0^1 d\lambda \int \frac{d^D p}{(2\pi)^D \mu^{D-4}} \frac{p^2 + M^2}{(p^2 + M^2)^2 + 2 \lambda^2 m^2 p^2}. \quad (B.1)$$

After changing the order of integration, the integral over the parameter $\lambda$ can be rewritten as

$$O(m^2, M^2) = \frac{\sqrt{2}}{m} \int_0^\infty dp \frac{d^{D-1}}{(4\pi)^{D/2} \Gamma(D/2) \mu^{D-4}} \frac{\tilde{v}(p)}{1 + v^2}, \quad \tilde{v}(p) = \frac{\sqrt{2} m p}{p^2 + M^2}, \quad (B.2)$$
using the change of coordinates $\sqrt{2} \lambda mp = (p^2 + M^2) v$ (keeping $p$ fixed). Notice that only the upper limit is $p$ dependent, hence after performing an integration by parts, and rewriting the resulting expression as a $D$ dimensional integral again, we obtain

$$O(m^2, M^2) = \frac{1}{D - 1} \int \frac{d^Dp}{(2\pi)^{D-1}} \frac{p^2 - M^2}{(p^2 + M^2)^2 + 2m^2 p^2}. \quad (B.3)$$

Using the factorization

$$\frac{p^2 - M^2}{(p^2 + M^2)^2 + 2m^2 p^2} = \frac{1}{m_+ - m_-} \left\{ \frac{m_+}{p^2 + m_+^2} - \frac{m_-}{p^2 + m_-^2} \right\}, \quad (B.4)$$

with $m_\pm = m^2 + M^2 \pm \sqrt{(m^2 + M^2)^2 - (M^2)^2}$, it is not difficult to confirm that the integral can be written in terms of the simple integral $J_0$ as

$$O(m^2, M^2) = \frac{1}{D - 1} \frac{1}{m_+ - m_-} \left\{ m_+ J_0(m_+^2) - m_- J_0(m_-^2) \right\}. \quad (B.5)$$

### B.2 $K(m^2, M^2)$ Integral

This appendix is devoted to the computation of the integral

$$K(m^2, M^2) = \int \frac{d^Dp}{(2\pi)^{D-1}} \ln \left[ \mathbb{1} - \frac{1}{p^2 + m^2} M^2 \frac{1}{p^2 + m^2} \overline{M^2} \right]. \quad (B.6)$$

where $m^{2\dagger} = m^2$ is a Hermitian matrix, and $\overline{M^2} = M^{2\dagger}$ is the Hermitian conjugate of a complex matrix $M^2$. The main difficulty of this integral is that these matrices not necessarily commute. We will show that we can do the integral exactly if the matrices commute, and that the effects due to non-commutativity of these matrices only gives additional finite contributions.

We first explain how to determine the maximally commuting contribution of this integral. Because $m^2$ is Hermitian, it can be diagonalized by a unitary transformation $U^{-1} = U^\dagger$. We can define

$$m^2 = U^{-1} m_D^2 U, \quad M^2 = U^T M_D^2 U, \quad \tilde{m}_D^2 = (M_D^2 \overline{M_D^2})^{1/2}, \quad (B.7)$$

where the matrix $m_D^2$ is diagonal, but $M_D^2$ is not necessarily diagonal. (Of course, if also $M_D^2$ is diagonal it implies that the original matrices $m^2$ and $M^2$ commuted.) The matrix $\tilde{m}_D^2$, defined using the formal power series of the square root, is introduced so that we can write

$$K(m^2, M^2) = \int \frac{d^Dp}{(2\pi)^{D-1}} \ln \left[ \left( \mathbb{1} + \frac{m_D^2}{p^2} \right)^{-1} \left( \mathbb{1} + \frac{\tilde{m}_D^2}{p^2} \right)^{-1} \left( \mathbb{1} + \frac{\tilde{m}_D^2 - m_D^2}{p^2} \right) \right]$$

$$- \frac{1}{p^2 + m_D^2} \left\{ \tilde{m}_D^2 \frac{m_D^2}{p^2(p^2 + m_D^2)} \tilde{m}_D^2 - M_D^2 \frac{m_D^2}{p^2(p^2 + m_D^2)} \overline{M_D^2} \right\}. \quad (B.8)$$

We can now consider the expansion in the second term. The zeroth order of this expansion we denote by $K_0(m^2, M^2)$ and the rest of the expansion we call $\Delta K(m^2, M^2)$. The integral $K_0(m^2, M^2)$ can be evaluated in terms of the logarithmic integral $L_0$ as

$$K_0(m^2, M^2) = \text{tr} \left[ L_0(m_D^2 + \tilde{m}_D^2) + L_0(m_D^2 - \tilde{m}_D^2) - 2L_0(m_D^2) \right]. \quad (B.9)$$
because the tr ln of a product is equal to the sum of the tr ln of the factors. Because of the overall trace, we can rotate back to the original basis in which \( m^2 \) is not necessarily diagonal, we then obtain

\[
K_0(m^2, M^2) = \text{tr} \left[ L_0(m^2 + \tilde{m}^2) + L_0(m^2 - \tilde{m}^2) - 2L_0(m^2) \right],
\]

where we have defined \( \tilde{m}^2 = (M^2 M^2)^{1/2} \).

To conclude we show that the remaining part \( \Delta K(m^2, M^2) \) is finite. To see this we write

\[
\Delta K(m^2, M^2) = \int \frac{d^4p}{(2\pi)^4} \text{tr} \ln \left[1 + \frac{1}{p^2 + m_D^2 - \tilde{m}_D^2} (p^2 + m_D^2) \right] \times \frac{1}{p^2 + m_D^2 + \tilde{m}_D^2} \times \left\{ \tilde{m}_D^2 \frac{m_D^2}{p^2(p^2 + m_D^2)} \tilde{m}_D^2 - M_D^2 \frac{m_D^2}{p^2(p^2 + m_D^2)} M_D^2 \right\}.
\]

(B.11)

It is now easy to confirm that expanding this expression to any order gives an integral which converges. In particular to first order, we see that the integrant scales as \( 1/(p^2)^3 \) for large \( p^2 \), and hence is convergent.

### B.3 \( R_n(m_S^2, M^2, \omega) \) Integral

Let \( m_S^2 \) be a Hermitian matrix, and \( M^2 \) and \( \omega \) complex matrices. We define for \( n = 0, 1 \) the integrals

\[
R_n(m_S^2, M^2, \omega) = \int_0^1 d\lambda \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[ \left( \frac{m_S^2}{p^2} \right)^n \left( 1 - \lambda \frac{1}{p^2 + m_\lambda^2} M^2 \right)^{-1} \times \frac{1}{p^2 + m_\lambda^2} \left( \frac{1}{p^2 + m_\lambda^2} \omega \right) \right], \quad m_\lambda^2 = m_S^2 + \lambda m_W^2.
\]

(B.12)

We can follow a similar strategy as in the previous subappendix: First go to a basis in which the propagators are diagonal, then compute in that basis and finally rotate back. Because \( m_\lambda^2 \) is a Hermitian matrix, there exist unitary \( U_\lambda \) such that \( m_{D\lambda}^2 = U_\lambda m_\lambda^2 U_\lambda^\dagger \) is diagonal. We define \( m_S^2 = U_\lambda m_{S\lambda}^2 U_\lambda, M^2 = U_\lambda M_{\lambda D}^2 U_\lambda \) and \( \omega = U_\lambda^T w_{D\lambda} U_\lambda \). (We use notation, like \( w_{D\lambda} \) to indicate that \( \omega \) is evaluated in the basis in which \( m_\lambda^2 \) is diagonal. But since this matrix and its diagonalization depend on \( \lambda \), also \( \omega \) in this basis is \( \lambda \) dependent.) We can then write

\[
R_n(m_S^2, M^2, \omega) = \int_0^1 d\lambda \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[ \left( \frac{m_{S\lambda}^2}{p^2} \right)^n \left( \lambda - \left[ \tilde{m}_{D\lambda}^2(p^2 + m_{D\lambda}^2) \right]^2 \times \frac{1}{p^2 + m_{D\lambda}^2} \left( \tilde{m}_{D\lambda}^2(p^2 + m_{D\lambda}^2) \right)^{-1} \tilde{m}_{D\lambda}^2(p^2 + m_{D\lambda}^2) \right) \right].
\]

(B.13)

If the matrices commute the last two terms under the big inverse cancel; when they do not commute, the difference must be proportional to a commutator. Clearly, if we Taylor expand in
this difference

\[ \Delta R_n(m^2_S, M^2, w) = R_n(m^2_S, M^2, w) - R^0_n(m^2_S, M^2, w) . \] (B.14)

we obtain convergent integrals, except for the zeroth order. The zeroth order contribution of (B.13) can be rewritten as

\[ R^0_n(m^2_S, M^2, w) = \int_0^1 \frac{d\lambda}{2\lambda^{1/2}} \int \frac{d^D p}{(2\pi)^D} \mu^{D-4} \text{tr} \left[ \left( \frac{m_D^2}{p^2} \right)^n \left( \frac{1}{p^2 + m_{D\lambda-}^2} - \frac{1}{p^2 + m_{D\lambda+}^2} \right) \tilde{m}_{D\lambda}^2 M_{D\lambda}^2 \tilde{w}_{D\lambda} \right] , \] (B.15)

where \( m_{D\lambda\pm}^2 = m_{D\lambda}^2 \pm \lambda^{1/2} \tilde{m}_{D\lambda}^2 \). Transforming back to the original basis, and making a change of variables \( x = \sqrt{\lambda} \), we obtain

\[ R^0_n(m^2_S, M^2, w) = \int_0^1 dv \text{tr} \left[ m_S^2 \left( J_n(m_{v-}^2) - J_n(m_{v+}^2) \right) \tilde{m}_{v} \frac{1}{M^2} \tilde{w} \right] , \] (B.16)

with \( m_{v\pm}^2 \) defined below (15).

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**References**


