Abstract

We study the transport properties in a narrow MOSFET device, which consists of a two-dimensional electronic waveguide, with an electric field applied in the transverse direction. Based on recent solutions of the Schrödinger equation for this system, we analyze the conductance fluctuations as a function of the electric field (the gate voltage), and of the Fermi energy. The statistical analysis of these fluctuations shows that the multichannel Poisson kernel description is valid also for this non-chaotic system. Besides the technological interest on MOSFET devices, it allows us to understand more fundamental quantities like the scattering properties of integrable systems.

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I. INTRODUCTION

In the past decades, electronic transport on submicron systems have attracted much attention because of the interesting phenomena of quantum origin at low temperatures\(^1\). Between these systems, narrow MOSFETs\(^2\), become an emblematic and exciting problem both in the chaotic and non-chaotic regimes. In the first case, random conductance fluctuations have been extensively studied in terms of random matrices and the Poisson’s kernel statistics\(^3-5\).

Clean systems characterized by elastic mean free paths larger than the system size, present interesting quantum interference effects. In these systems, electrons scatter on confining potential walls causing also random fluctuations on transport properties. At variance with truly chaotic systems, where the conductance fluctuations are of a universal character, the statistical behavior of this and other physical quantities in non-chaotic systems is different and much less understood. Nevertheless, some work has been done on the statistical behavior of confined particles\(^6,7\) and one-channel non-chaotic systems\(^8\). In the last case, it has been analytically proved that the \(S\)-matrix distribution is also given by the Poisson’s kernel.

In this paper, we use the analytical solutions for the scattering amplitudes of a two-dimensional multichannel electron gas\(^9\), to show that the clean MOSFET conductance fluctuates strongly as a function of the gate voltage. We then analyze the statistical conductance and transmission coefficients behavior and show that the Poisson kernel describes also this non-chaotic system.

Independently of the strong consequences in application problems, the MOSFET system and its solutions reveal the complex interference phenomena and become a useful tool to understand non-chaotic quantum systems. The paper is organized as follows. In section 2 we present a summary of the transfer matrix approach and the principal results required for the calculation of the electronic transport conductance. In Section 3 we present numerical simulation results and compare with the Poisson kernel description.

II. THE TRANSPORT PROBLEM IN THE TRANSFER MATRIX APPROACH

The system we are interested in is shown schematically in Fig 1. It consists of a 2D electronic waveguide along the \(z\)-axis with transverse width \(w_y\). In addition, an electric field \(F/e\) acts transversely in a layer of width \(w_z\). For a given Fermi energy \(E\), we have \(N\) propagating modes or channels.

In the scattering approach the electronic conductance is described by Landauer-Büttiker formula\(^10\):

\[
G = \frac{2e^2}{h}g \equiv \frac{2e^2}{h} \text{Tr } tt^\dagger
\]

where \(g\) is the dimensionless conductance and \(t\) is the \(N\times N\) transmission amplitude, which together with the reflection amplitude \(r\) defines the transport scattering matrix \(S\). For time reversal and
FIG. 1: Potential $V(y,z)$ for the MOSFET system. It is an electronic wave guide along the $z$-axis and the electrons are confined in the $y$-direction.

space inversion invariant systems, it takes the following structure$^4$:

$$S = \begin{pmatrix} r & t \\ t & r \end{pmatrix}$$

(2)

The matrix $t$ can also be obtained from transfer matrices. For the particular case of interest here, the transfer matrix $W$ has the form$^9$

$$W(w_z) = \begin{pmatrix} \cosh w_z u & u^{-1} \sinh w_z u \\ u \sinh w_z u & \cosh w_z u \end{pmatrix} = \begin{pmatrix} \vartheta & \mu \\ \nu & \chi \end{pmatrix},$$

(3)

where $u^2 = 2m^*(V - E)/\hbar^2 + k_T^2$ is an $N \times N$ symmetric matrix, $k_T$ is an $N \times N$ diagonal matrix whose elements are the transverse momenta $k_{Tj}$ in the $j$-th channel, with $j = 1 \ldots, N$, and $V$ is a matrix whose elements are:

$$V_{j,i}(z) = V_0 - \frac{F \omega_y}{2}, \quad \text{for} \quad i = j = 1, 2, ..., N,$$

$$V_{j,i}(z) = 0, \quad \text{for} \quad i \neq j = 1, 2, ..., N, \quad i + j \quad \text{even},$$

$$V_{j,i}(z) = \frac{8ijF \omega_y}{(i^2 - j^2)^2 \pi^2}, \quad \text{for} \quad i \neq j = 1, 2, ..., N, \quad i + j \quad \text{odd.}$$

(4)

(5)

The transmission amplitude in terms of the $W$ matrix elements is given by$^9$

$$t = 2\kappa^{1/2}(\vartheta^T + \kappa \chi^T \kappa^{-1} - i(\kappa \mu^T - \nu^T \kappa^{-1})^{-1}\kappa^{-1/2},$$

(6)

where $\vartheta, \chi, \nu$ are given in Eq. (3), and $\kappa$ is a $N \times N$ diagonal matrix whose elements are the longitudinal wave numbers.

In Fig. 2(a) we show $g$ for $N = 4$ as a function of $F$ (at fixed energy) and, in the inset, as a function of $E$ (for fixed $F$). As $F$ approaches zero $g$ tends to 4 as it should be. In that case we have
FIG. 2: The resonances of the dimensionless conductance \( g \) occur when the bounded states in the continuum coincide with the Fermi energy \( E \). (a) \( g \) is plotted as a function of \( F \); (b) \( T_1 \) fluctuates as a function of \( F \), for a field window centered at \( F = 9.1 \ \text{eV/nm} \), above the fourth energy threshold \( E_4 \approx 0.89 \ \text{eV} \).

a clean electronic waveguide with \( N = 4 \) open channels. When \( F \) is not zero, electrons feel the triangular potential well, which depth is determined by \( F \). Thus, the conductance \( g \) decreases in magnitude. For strong electric fields, the well approaches to infinite square potential well. Varying \( F \), the bounded states in the continuum move also, giving rise to resonant values of \( g \) when the fixed Fermi energy coincides with the bounded states. Also \( g \) fluctuates with the Fermi energy for fixed \( F \), but the fluctuations are weaker, as can be seen in the inset of Fig. 2(a). In Fig. 2(b) we show a similar behavior for the transmission coefficient \( T_i = |t_{ii}|^2 \) for \( i = 1 \).

Our purpose is to characterize the conductance fluctuations which, as we can see in Fig. 3(a), are non universal. In this figure we plot the square modulus of the average of \( S_{11} \) and we observe that it increases with the size of the averaging window. This is typical of regular systems without universal statistics\(^\text{11} \). These fluctuations are quite interesting because they allow us to understand much of the scattering properties of integrable systems. The analysis is led in terms of the scattering matrix distribution.
III. STATISTICAL ANALYSIS

Let us restrict ourselves to the $N = 4$ case as before. The $S$ matrix of Eq. (2) can be block diagonalized as follows:

$$S = R_o^T \begin{pmatrix} S_1 & 0_N \\ 0_N & S_2 \end{pmatrix} R_o \quad \text{with} \quad R_o = \frac{1}{\sqrt{2}} \begin{pmatrix} I_N & -I_N \\ I_N & I_N \end{pmatrix}$$

where $0_N$ and $I_N$ are the null and unit matrices of dimension $N$, $S_1$ and $S_2$ are $N \times N$ most general unitary and symmetric scattering matrices, $R_o$ is the generalization to $2N \times 2N \pi/4$ rotation matrix.

In terms of the reduced $S_i$ matrices, the transmission amplitude matrix is $t = (S_1 - S_2)/2$. To characterize the statistics of this quantity, we assume that $S_1$ and $S_2$ are also distributed according with the Poisson kernel. For the transmission coefficient $T_1 = |t_{11}|^2$, we obtain, in the large $N$ limit:

$$P(T_1) \propto \exp \left[ -\frac{N}{(1 - S_{11}^2)^2} T_1 \right]$$

Here $S_{11}$ is the field average of the 1,1-element of $r = (S_1 + S_2)/2$.

We compare, in figure 3(b), the prediction of this distribution with the numerical evaluation of the equation above for two different values of $S_{11}$, one of them corresponding to figure 2(b). The agreement is good and improves as $S_{11}$ increases. It is so because our numerical results are not realistic for large $N$. Since large values of $S_{11}$ mimic large values of $N$, the agreement has to improve for larger $S_{11}$. 
IV. CONCLUSIONS

We have shown with a simple example, soluble within the approach approximations, that even in the non-chaotic systems the well known Poisson kernel provides the right statistical description of the scattering and ensuing transport properties. This result is interesting independently of the strong consequences in application problems: the MOSFET system considered here is an appealing and soluble system. It exhibits complex interference phenomena and becomes a useful example to understand interesting fluctuating properties of non-chaotic quantum systems.

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References

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