THE RELATION BETWEEN SOLUTION OF BBGKY HIERARCHY
OF KINETIC EQUATIONS AND THE SOLUTION
OF WIGNER EQUATION

M.Yu. Rasulova\textsuperscript{1}

\textit{Institute of Nuclear Physics, Academy of Sciences Republic of Uzbekistan,}
\textit{Ulughbek, Tashkent 702132, Uzbekistan}
and
\textit{The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy}

and

T. Hassan\textsuperscript{2}

\textit{Faculty of Science, International Islamic University of Malaysia (IIUM),}
\textit{Gombak, Kuala Lumpur 53100, Malaysia.}

Abstract

The solution of BBGKY hierarchy of quantum kinetic equations is defined through the
solution of the Wigner equation.

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\textsuperscript{1}mrasulova@yahoo.com
\textsuperscript{2}torla@iium.edu.my
Introduction


The evolution of many particle systems are described by Liouville equation for many particle distribution functions or density matrices in classical or quantum physics, respectively.

Best chain of equations, which is equivalent to Liouville equation and, at the same time, allows in the first approximation to obtain the Boltzmann or Vlasov equations for one-particle distribution functions, is the Bogoluibov-Born-Green-Kirkwood-Yvon’s (BBGKY) chain of kinetic equations [10]-[15].

Soon after the formulation of BBGKY chain in 1946, the series methods based on the physical approximations were suggested [16]-[21], making it possible to find the first two equations, describing the evolution of one and two particles [5],[16]. Likewise the Boltzmann equation [22]-[30], BBGKY chain has been the object of detailed mathematical analysis [31]-[39].

In 1977 [35] the first author of present paper on the basis of the algebraic method Ruelle [40], methods of Liboff [41],[42], and Ishimary [16] developed a new method for plasma physics, which allows one to find all terms of series, which is the solution of BBGKY’s chain of kinetic equations.

There are some unsolved problems in quantum kinetic theory:

1. So far in many papers the motion of electrons, ions or phonons, which are objects of quantum physics, is described by classical Boltzmann or Vlasov equations for distribution functions. These distribution functions depend from coordinate and momentum variables. In the meantime, according to Heisenberg’s principles of quantum theory, it is impossible to exactly define coordinate and momentum at the same time.

2. The quantum dynamics of many-body systems are often modelled by nonlinear, time irreversible (quantum) Boltzmann equation, which exhibits a particle-like behavior [43].

Therefore to describe the motion of the quantum object one needs to construct the reversible, positivity-preserving quantum Boltzmann or Vlasov equations for density matrices, based on time reversible Schrödinger equation, quantum Liouville equation or BBGKY’s chain of quantum kinetic equations. Last time some results have been received in this direction [43]-[46].

However, the most appropriate method to describe the quantum objects is the method based on “quantum distribution function”, which is the solution of quantum Wigner equation (analogue of classical Vlasov equation) or quantum Boltzmann equation for quantum distribution function. The interest of the quantum kinetic equation in the Wigner formalism [47], in particular, has been generated by necessity to model and simulate submicron semiconductor devices [48]. This formalism allows an easy comparison with corresponding classical results. Series of results have been received concerning the solution of Wigner equation by Prof. P.Marcowich and his
colleagues [6],[7],[48]-[54]. However, these results are useful for describing the motion of one particle.

3. Since matter consists of infinitely interacting and moving particles, to describe dynamics of this matter it is necessary to find from quantum BBGKY’s chain the chain of kinetic equations for quantum correlation functions and to solve it.

To solve this problem in this paper in the frame of Wigner formalism the method of solution of quantum BBGKY chain of kinetic equations in the form of series, with the first term coinciding with solution of the Wigner equation describing the motion of electron in the vacuum, is suggested.

Hereby, for the case, when the interaction potential is in the form of Dirac’s delta function it was proved that the solution of Wigner equations can be found through soliton solution of nonlinear Schrödinger equation.

For this purpose, in the first Section of paper the chain of quantum kinetic equations has been derived for quantum correlation functions. In Section 2 the solution of this chain for quantum distribution functions is reduced to solution of the homogeneous and non-homogeneous Wigner equations. In Section 3 for the system of particles, interacting by Dirac’s delta function pair potentials in the form of Dirac’s delta function, the equivalence of the Wigner equation and the nonlinear Schrödinger equation (NLSE) is shown. So, the solution of BBGKY chain has been defined through soliton solution of NLSE.

1. Derivation of Chain of Kinetic Equations for density matrices

Suppose there is a system of particles. Suppose that the particles interact through a two-body potential $\phi$. In the framework of quantum statistical physics, we consider for the given system the problem of solving the chain of BBGKY quantum kinetic equations [10]:

$$i \frac{\partial}{\partial t} f_n(t) = [H_n, f_n(t)] + \frac{1}{v} \int \sum_{1 \leq i \leq n} [\phi(x_i - x), f_{n+1}(t)] dx$$  \hspace{1cm} (appendix 1), \hspace{1cm} (1)

where $f_n$ is the density matrix, $x$ is the particle coordinate, $[,]$ is the Poisson bracket, $m = 1$, $h = 1$. $0 \leq t$ is time, $n \in N$, $N$ is the number of particles, $V$ - the volume of the system, $N \to \infty, V \to \infty$, $v = \frac{V}{N} = const$ is volume per particle, and $H$ is the Hamiltonian:

$$H_n = \sum_{1 \leq i \leq n} T_i + \sum_{1 \leq i < j \leq n} \phi(x_i - x_j), \hspace{1cm} T_i = -\frac{\partial^2}{2\partial x_i^2}$$

Introducing the notation

$$(H f)_n = [H_n, f_n]: \hspace{1cm} (D_x f)_n (x_1, \cdots, x_n; x'_1, \cdots, x'_n) = f_{n+1} (x_1, \cdots, x_n, x; x'_1, \cdots, x'_n, x');$$

$$(A_x f)_n = \frac{1}{v} \sum_{1 \leq i \leq n} [\phi(x_i - x), f_n];$$

$$f(t) = \{f_1(t, x_1; x'_1), \cdots, f_n(t, x_1, \cdots, x_n; x'_1, \cdots, x'_n), \cdots\}, \hspace{1cm} n = 1, 2, \cdots$$

we can rewrite Eq.(1) in the form

$$i \frac{\partial}{\partial t} f(t) = H f(t) + \int A_x D_x f(t) dx.$$  \hspace{1cm} (2)
Proposition 1. The chain of kinetic equations for the density matrices has the form

\[ i \frac{\partial}{\partial t} \phi(t) = \mathcal{H} \phi(t) + \frac{1}{2} W(\phi(t), \phi(t)) + S p_x A_x D_x \phi(t) + S p_x A_x \phi(t) \star D_x \phi(t), \]  

(3)

where [40],[35]:

\[ f(t) = \Gamma \phi(t) = I + \varphi(t) + \frac{\varphi(t) \star \varphi(t)}{2!} + \cdots + \frac{(\varphi(t))^n}{n!} + \cdots \]  

(appendix 2),

\[ \varphi(t) = \{ \varphi_1(t, x_1; x_1'), \ldots, \varphi(t, x_1, \ldots, x_n; x_1', \ldots, x_n') \}; \]

\[ (\varphi \star \varphi)(X; X') = \sum_{Y \subset X, Y' \subset X'} \varphi(Y; Y') \varphi(X \setminus Y; X' \setminus Y') \]  

(appendix 3);

\[ I \star \varphi = \varphi; \quad (\varphi^n) = \underbrace{\varphi \star \varphi \cdots \star \varphi}_{n \text{ times}}; \]

\[ X = (x_1, \ldots, x_n) = (x_{(n)}); \quad X' = (x'_1, \ldots, x'_n) = (x'_{(n)}) \]

\[ Y = (x_{(n')}), \quad Y' = (x'_{(n')}) \quad n' \in n; \quad n' = 1, 2, \cdots; \]

\[ \mathcal{U} \varphi_n = \left[ \sum_{1 \leq i < j \leq n} \phi(x_i - x_j), \varphi_n \right]; \]

\[ W(\varphi, \varphi) = \sum_{Y \subset X, Y' \subset X'} \mathcal{U}(Y; Y', X \setminus Y; X' \setminus Y') \varphi(Y; Y') \varphi(X \setminus Y; X' \setminus Y'). \]

Proof: To obtain (3), we substitute (4) in (2):

\[ i \frac{\partial}{\partial t} \Gamma \phi(t) = \mathcal{H} \Gamma \phi(t) + \int A_x D_x \Gamma \phi(t) dx. \]  

(5)

We have

\[ D_x \Gamma \phi(t) = D_x \varphi(t) \star \Gamma \phi(t), \]

(6)

\[ A_x \Gamma \phi(t) = A_x \varphi(t) \star \Gamma \phi(t), \]

(7)

\[ A_x D_x \Gamma \phi(t) = A_x D_x \varphi(t) \star \Gamma \phi(t) + A_x \varphi(t) \star D_x \varphi(t) \star \Gamma \phi(t), \]

(8)
\[ T\Gamma \varphi(t) = T\varphi(t) \ast \Gamma \varphi(t), \quad (9) \]

\[ U\Gamma \varphi(t) = U\varphi(t) \ast \Gamma \varphi(t) + \frac{1}{2} W(\varphi(t), \varphi(t)) \ast \Gamma \varphi(t), \quad (10) \]

\[ \frac{\partial}{\partial t} \Gamma \varphi(t) = \frac{\partial}{\partial t} \varphi(t) \ast \Gamma \varphi(t). \quad (11) \]

Substituting (6) – (11) in (5), multiplying both sides by \( \Gamma(\varphi(t)) \) we obtain (3). This proves the proposition.

2. The solution of the chain of kinetic equations for quantum correlation matrices

To investigate our system on the basis of arguments similar to those in [10],[16],[35] we can choose \( v \) as expansion parameter by setting

\[ \phi(x_i - x_j) = v\theta(x_i - x_j) \quad (12) \]

and substituting:

\[ \varphi_n(t) = v^{n-1} \psi_n(t) \quad (13) \]

On the basis of (12), (13) Eq.(3) for \( n \) particles can be represented as follows:

\[ \frac{\partial}{\partial t} \psi_n(t, X; X') = \left[ \sum_{1 \leq i \leq n} T_i, \psi_n(t, X; X') \right] + v \langle U\psi \rangle_n(t, X; X') \]

\[ + \frac{v}{2} \langle W\psi, \psi \rangle_n(t, X; X') + v^2 S_{px}(A_x D_x \psi)_n(t, X; X') \]

\[ + v S_{px}(A_x \psi \ast D_x \psi)_n(t, X; X') \quad (14) \]

In the 6n dimensional phase space of coordinates and momentum most full analogue to classical case will be the use of density matrices in the form suggested by Wigner and named as quasi-distribution function or quantum distribution function. The quantum distribution function is related with ordinary density matrix in the coordinate representation as:

\[ f_n(t, x_1, x_2, ..., x_n; p_1, p_2, ..., p_n, ...) = \frac{1}{(2\pi)^{3n}} \int \exp(-\sum_{1 \leq i \leq n} p_i \tau_i) \]

\[ f_n(t, x_1 - \frac{\tau_1}{2}, x_2 - \frac{\tau_2}{2}, ..., x_n - \frac{\tau_n}{2}; x_1 + \frac{\tau_1}{2}, x_2 + \frac{\tau_2}{2}, ..., x_n + \frac{\tau_n}{2}, ...) \bigcap_{1 \leq i \leq n} d\tau_i. \quad (15) \]
On the basis of equations (14), (15) we can find the equation which defines the evolution of the quantum correlation function. It can be presented as follows:

\[
\frac{\partial}{\partial t} \psi_n(t, X; P) = - \sum_{1 \leq i \leq n} p_i \frac{\partial}{\partial x_i} \psi_n(t, X; P) + \frac{1}{(2\pi)^3} \int \cap_{1 \leq i \leq n} \tau_i \{ \exp(- \sum_{1 \leq i \leq n} p_i \tau_i) [v(\mathcal{U}\psi)_n(t, X - \frac{\tau}{2}; X + \frac{\tau}{2}) + \frac{v^2}{2} (\mathcal{W}\psi, \psi)_n(t, X - \frac{\tau}{2}; X + \frac{\tau}{2}) + v^2 \mathcal{S}_{px}(\mathcal{A}_x \mathcal{D}_x \psi)_n(t, X - \frac{\tau}{2}; X + \frac{\tau}{2})] \} \}
\]

(16)

where

\[ P = \{p_1, p_2, \ldots p_n\}, \quad X \pm \frac{\tau}{2} = \{x_1 \pm \frac{\tau_1}{2}, x_2 \pm \frac{\tau_2}{2}, \ldots, x_n \pm \frac{\tau_n}{2}\}. \]

To solve Eq.(16), we apply the perturbation theory. We shall seek a solution in the form of the series [18],[35]

\[
\psi_n(t, X; P) = \sum_{\mu} \psi_{n\mu}^\mu(t, X; P), \quad n = 1, 2, 3, \ldots, \quad \mu = 0, 1, 2, \ldots
\]

(17)

Substituting the series (17) in Eq.(16) and equating the coefficients of equal powers of \( v \) we obtain

\[
\left( \frac{\partial}{\partial t} + \mathcal{L}_1 \right) \psi_0^0(t) = 0,
\]

(18)

\[
\left( \frac{\partial}{\partial t} + \mathcal{L}_1 + \mathcal{L}_2 \right) \psi_0^1(t) = S_0^0(t),
\]

(19)

\[
\vdots
\]

\[
\left( \frac{\partial}{\partial t} + \sum_{i=1}^{n} \mathcal{L}_i \right) \psi_0^n(t) = S_0^n(t),
\]

(20)

where we have introduced the notation

\[
\mathcal{L}_1 \psi_0^0(t, x_1, p_1) = p_1 \frac{\partial}{\partial x_1} \psi_0^0(t, x_1; p_1) - \frac{1}{(2\pi)^3} \int \exp(-i\tau_1 p_1) \times
\]

\[
\times \mathcal{S}_{px}[(\Theta(x_1 - x - \frac{\tau_1}{2}) - \Theta(x_1 - x + \frac{\tau_1}{2})) \psi_0^0(t, x_1 - \frac{\tau_1}{2}; x_1 + \frac{\tau_1}{2}) \psi(t, x, x)]d\tau_1,
\]

\[
\mathcal{L}_2 \psi_0^0(t, x_1, x_2; x_1', x_2') = p_1 \frac{\partial}{\partial x_1} \psi_0^0(t, x_1, x_2; p_1, p_2) - \frac{1}{(2\pi)^6} \int \exp(-i\tau_2 p_1 - i\tau_2 p_2) \times
\]

\[
\vdots
\]

\[
\mathcal{L}_n \psi_0^n(t, x_1, x_2; x_1', x_2') = p_1 \frac{\partial}{\partial x_i} \psi_0^n(t, x_1, x_2; p_1, p_2) - \frac{1}{(2\pi)^6} \int \exp(-i\tau_2 p_1 - i\tau_2 p_2) \times
\]

\[
\vdots
\]
\[ \times S_{p_x}[(\Theta(x_1 - x - \frac{\tau_1}{2}) - \Theta(x_1 - x + \frac{\tau_1}{2}))\psi_2^0(t, x_1, x_2; x_1 + \frac{\tau_1}{2}, x_2 + \frac{\tau_2}{2})\psi(t, x, x)]d\tau_1 d\tau_2, \]

\[ S_{p_0}^0(t) = \frac{1}{(2\pi)^n} \int \exp(-i\tau_1 p_1 - i\tau_2 p_2)[(\Theta(x_1 - \frac{\tau_1}{2} - x) - \Theta(x_1 + \frac{\tau_1}{2} - x)) \times \]

\[ \times \psi_1^0(t, x_1 - \frac{\tau_1}{2}; x_1 + \frac{\tau_1}{2})\psi_1^0(t, x_2 - \frac{\tau_2}{2}; x_2 + \frac{\tau_2}{2}) + S_{p_x}[(\Theta(x_1 - \frac{\tau_1}{2} - x) - \Theta(x_1 + \frac{\tau_1}{2} - x)) \times \]

\[ \times \psi_1^0(t, x_2 - \frac{\tau_2}{2}; x_2 + \frac{\tau_2}{2})\psi_1^0(t, x_1 - \frac{\tau_1}{2}; x_1 + \frac{\tau_1}{2}, x)]d\tau_1 d\tau_2, \]

\[ L_{t}\psi_{\mu}^n(t, X, P) = p_i \frac{\partial}{\partial x_i} \psi_{\mu}^n(t, X, P) - \frac{1}{(2\pi)^3} \int \exp(-i \sum_{1 \leq i \leq n} p_i \tau_i) \times \]

\[ \times [S_{p_x}(\Theta(x_1 - \frac{\tau_1}{2} - x) - \Theta(x_1 + \frac{\tau_1}{2} - x))\psi_{\mu}^n(t, X - \frac{\tau}{2}; X + \frac{\tau}{2})\psi_1^0(t, x, x) + \]

\[ + S_{p_x}(\Theta(x_1 - \frac{\tau_1}{2} - x) - \Theta(x_1 + \frac{\tau_1}{2} - x))\psi_1^0(t, x_i - \frac{\tau}{2}; x_i + \frac{\tau}{2}) \times \]

\[ \times \psi_{\mu}^n(t, (X - \frac{\tau}{2}, x) \setminus (x_1 - \frac{\tau_1}{2}; (X + \frac{\tau}{2}, x) \setminus (x_i + \frac{\tau_i}{2})]) \bigcap_{1 \leq i \leq n} \tau_i, \]

\[ S_{p_0}^n(t) = \frac{1}{(2\pi)^3} \int \bigcap_{1 \leq i \leq n} \tau_i \{\exp(-i \sum_{1 \leq i \leq n} p_i \tau_i)[(U^{\psi_{\mu}^n-1}(t))_n(t, X - \frac{\tau}{2}; X + \frac{\tau}{2}) + \]

\[ + \frac{1}{2} \sum_{\delta_1 + \delta_2 = \mu - 1} (V(\psi_{\delta_1}^{\delta_2})(t, X - \frac{\tau}{2}; X + \frac{\tau}{2}) + vS_{p_x}(A_x D_x^{\psi^{\mu-1}_{\delta_2}})_n(t, X - \frac{\tau}{2}, X + \frac{\tau}{2})) + \]

\[ + vS_{p_x} \sum_{\frac{\tau'}{2} X, Y \in \frac{\tau'}{2} X, Y} (A_x^{\psi_{\delta_1}})(t, Y - \frac{\tau'}{2}; Y + \frac{\tau'}{2})(D_x^{\psi_{\delta_2}})(t, X - \frac{\tau'}{2}; X + \frac{\tau'}{2} Y + \frac{\tau'}{2} Y)]}, \}

Eq. (18) is the time-dependent Wigner equation for Hartree-Fock systems [6]. Thus, the solution of Eq. (16) is reduced to the solution of the homogeneous (18) and non-homogenous (19), (21) Wigner’s equations for \( \psi_1^0(t) \) and \( \psi_{\mu}^n(t) \), respectively.
As it was shown in [35] the series \( \psi_n(t, X; P) = \sum_\mu \psi^\mu_n(t, X; P) \), where \( \psi^\mu_n(t) \) is defined in accordance with the solution of Wigner equation and the \( \psi^\mu_n(t) \), which is determined on the basis of the formula

\[
\psi^\mu_n(t, X; P) = \int dY \int dP' \int_{-\infty}^t d't' S^\mu_n(t', Y, P') \bigcap_{1 \leq i \leq n} G(t - t', x_i, y_i, p_i, p_i'),
\]

are solutions of Eq.(16). Here \( G(t, x_i, y_i, p_i, p_i') \) is the solution of Cauchy’s problem [17],[36]:

\[
i\frac{\partial G(t - t', x_i, y_i, p_i, p_i')}{\partial t} = -\sum_{1 \leq i \leq 2} p_i \frac{\partial}{\partial x_i} G(t - t', x_i, y_i, p_i, p_i') + \frac{1}{(2\pi)^\nu} \int \exp(-i\tau p_i - i\tau' p_i') \times
\]

\[
\times \{Sp_x[\theta(x_i - \frac{\tau_i}{2} - x) - \theta(x_i + \frac{\tau_i}{2} - x)]\psi^0_1(t, x_i - \frac{\tau_i}{2}; x_i + \frac{\tau_i}{2})G(t - t', x_i - \frac{\tau_i}{2}; x_i + \frac{\tau_i}{2}) \}
\]

\[
+Sp_x[\theta(x_i - \frac{\tau_i}{2} - x) - \theta(x_i + \frac{\tau_i}{2} - x)]\psi^0_1(t, x; x)G(t - t', x_i - \frac{\tau_i}{2}; x_i + \frac{\tau_i}{2}) \}
\]

with the initial condition

\[
G(0; x_1, y_1, p_1, p_1') = \delta(x_1 - y_1)\delta(p_1 - p_1').
\]

3. Equivalence of the Wigner equation and the nonlinear Schrödinger equation

We will represent the Wigner equation (18) in the following form:

\[
\frac{1}{(2\pi)^3} \int \exp(-p_1 \tau_1) \{i\frac{\partial \psi^0_1(t, x_1 - \frac{\tau_1}{2}; x_1 + \frac{\tau_1}{2})}{\partial t} \} d\tau_1 = \frac{1}{(2\pi)^3} \int \exp(-p_1 \tau_1) \times
\]

\[
\times \{-\frac{1}{2}(\Delta(x_1 - \frac{\tau_1}{2})\psi^0_1(t, x_1 - \frac{\tau_1}{2}; x_1 + \frac{\tau_1}{2}) - \Delta(x_1 + \frac{\tau_1}{2})\psi^0_1(t, x_1 - \frac{\tau_1}{2}; x_1 + \frac{\tau_1}{2})) \}
\]

\[
+Sp_x[\theta(x_1 - \frac{\tau_1}{2} - x) - \theta(x_1 + \frac{\tau_1}{2} - x)]\psi^0_1(t, x_1 - \frac{\tau_1}{2}; x_1 + \frac{\tau_1}{2}) \}
\]

The expression (22) takes form of the von Neumann’s equation:

\[
i\frac{\partial \psi^0_1(t, x_1; x_1')}{\partial t} = -\frac{1}{2}(\Delta_{x_1}\psi^0_1(t, x_1; x_1') - \Delta_{x_1'}\psi^0_1(t, x_1, x_1')) +
\]

\[
+Sp_x[\theta(x_1 - x) - \theta(x_1' - x)]\psi^0_1(t, x_1; x_1') \psi^0_1(t, x; x).
\]

Define the correlation matrix as [10],[55],[37]:

\[
\psi^0_1(t, x_i; x_i') = \Phi_1(t, x_i)\Phi^0_1(t, x_i'),
\]

8
Substituting (24) into (23) and taking $\theta(x_i - x_j)$ in the form of delta $\delta(x_i - x_j)$ we get [56],[37]:
\[
i \frac{\partial}{\partial t} \Phi_1(t,x_i) = - \frac{\partial^2}{\partial x_i^2} \Phi_1(t,x_i) + 2c \Phi_1(t,x_i) | \Phi_1(t,x_i) |^2, \quad \Phi_1(t,x_i) |_{t=0} = \Phi_1(x_i). \tag{25}
\]

\[
i \frac{\partial}{\partial t} \Phi_1^*(t,x'_i) = - \frac{\partial^2}{\partial x'_i^2} \Phi_1^*(t,x'_i) + 2c \Phi_1^*(t,x'_i) | \Phi_1^*(t,x'_i) |^2, \quad \Phi_1^*(t,x'_i) |_{t=0} = \Phi_1^*(x'_i) \tag{26}
\]

Equations (25),(26) are nonlinear Schrödinger’s equations. If we know the solution (25),(26), we shall be able to solve the von Neumann’s equation (23). It is known [55], that for the one dimensional D=1 case, at $c > 0$ the solution of (25) has the following form
\[
\Phi_1(t,x_i) = \sqrt{\frac{2}{c \lambda + i \nu}} + \exp(2\nu((x_i - x_0 - 2\lambda t))) \tag{27}
\]
where $\nu$ is the velocity and parameter $\lambda$ characterizes the amplitude. The velocity $\nu$ is expressed via parameter $\lambda$ as $\nu = \sqrt{1 - \lambda^2}$.

It should be noted that two relations are valid:
\[
\frac{c}{2} |\Phi_1(t,x_i)|^2 = 1 - \frac{\nu^2}{\hbar^2 \nu(x_i - x_0 - 2\lambda t)}
\]
and
\[
\int |\Phi_1(t,x_i)|^2 dx_i = N,
\]
where $N=1$ - number of particles in a system.

Thus, by the soliton solution (27) of nonlinear Schrödinger equations (25) a solution of von Neumann’s equation $\psi_1^0(t)$ can be defined as:
\[
\psi_1^0(t,x_i;x'_i) = \Phi_1(t,x_i)\Phi_1^*(t,x'_i),
\]

Accordingly the solution of Wigner equation can be defined as:
\[
\psi_1^0(t,x_i;p_i) = \frac{1}{(2\pi)^3} \int \exp(-i\tau_ip_i)(\Phi_1(t,x_i - \tau_i/2)\Phi_1^*(t,x_i + \tau_i/2))d\tau_i,
\]
where
\[
\Phi_1(t,x_i - \tau_i/2) = \sqrt{\frac{2}{c \lambda + i \nu}} + \exp(2\nu((x_i - x_0 - 2\lambda t)))
\]

Further, $\psi_n(t,X;X')$ can be defined by (21),(17), (13) and, by using (4), the density matrix $f_n(t,X;X')$ can be obtained, which is a soliton solution of BBGKY’s’ chain of quantum kinetic equations.
Conclusion

In the present paper to describe of the dynamics of quantum system particles, interacting through pair potential, based on BBGKY chain of quantum kinetic equations the chain of kinetic equations for quantum correlation "functions" (q.c.f) - the analogue classical correlation functions (c.c.f) is obtained.

The method of solution of this chain of equations for q.c.f. in the form of series is suggested. The solution of chain of equations for q.c.f. is reduced to solution of homogeneous and non homogeneous Wigner equations. It is shown, that in 0th term the solution of quantum kinetic equation for one particle q.c.f. reduces to the solution of the Wigner equation - analogue of classical Vlasov equation. The solution of the Wigner equation is defined through soliton solution of nonlinear Schrödinger equation. The definition of all other terms of the series, which is the solution of quantum kinetic equations for q.c.f., through this soliton solution, has been shown.

The structure of the first two quantum kinetic equations for q.c.f. coincides with known kinetic equations for classical correlation functions, used in plasma physics. This gives the possibility to generalization of the classical results to quantum case.

In conclusion, we hope that new equations for q.c.f. will, in future, allow us to construct the quantum Boltzmann equation in Wigner formalism, giving the opportunity to describe the dynamics of a quantum object through quantum Boltzmann equation.

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Appendices

Appendix 1. For n=1:

\[ \frac{\partial f_1(t, x_1; x'_1)}{\partial t} = -\frac{1}{2}(\Delta_{x_1} f_1(t, x_1; x'_1) - \Delta_{x'_1} f_n(t, x_1, x'_1)) + \frac{1}{v}Sp_x(\phi(x_1 - x) - \phi(x'_1 - x))f_2(t, x_1, x; x_1, x), \]

For n=2:

\[ \frac{\partial f_2(t, x_1, x_2; x'_1, x'_2)}{\partial t} + \sum_{1 \leq i \leq 2} \left( \frac{\partial^2}{\partial x_i^2} f_2(t, x_1, x_2; x'_1, x'_2) - \frac{\partial^2}{\partial x_i^2} f_2(t, x_1, x_2; x'_1, x'_2) \right) + \]

\[ + (\phi(x_1 - x_2) - \phi(x'_1 - x'_2))f_2(t, x_1, x_2; x'_1, x'_2) + \sum_{1 \leq i \leq 2} Sp_x(\phi(x_i - x) - \phi(x'_i - x)) \times \]

\[ \times f_3(t, x_1, x_2; x'_1, x'_2, x), \]

So we can find other equations (1), for n=3,4,5,...... .

Appendix 2. For n=1:

\[ f(x_1; x'_1) = \Gamma \varphi(x_1; x'_1) = \varphi(x_1; x'_1), \]

For n=2

\[ f(x_1, x_2; x'_1, x'_2) = \Gamma \varphi(x_1, x_2; x'_1, x'_2) = \varphi(x_1, x_2; x'_1, x'_2) + \varphi(x_1; x'_1) \varphi(x_2; x'_2), \]

For n=3

\[ f(x_1, x_2, x_3; x'_1, x'_2, x'_3) = \Gamma \varphi(x_1, x_2, x_3; x'_1, x'_2, x'_3) = \varphi(x_1, x_2, x_3; x'_1, x'_2, x'_3) + \]

\[ + \varphi(x_1; x'_1) \varphi(x_2, x_3; x'_2, x'_3) + \varphi(x_2; x'_2) \varphi(x_1, x_3; x'_1, x'_3) + \varphi(x_3; x'_3) \varphi(x_1, x_2; x'_1, x'_2) + \]

\[ + \varphi(x_1; x'_1) \varphi(x_2; x'_2)(x_3; x'_3), \]

and others.

Appendix 3. For n=1:

\[ (\varphi_1 \ast \varphi_2)(x_1, x_2; x'_1, x'_2) = \varphi_1(x_1; x'_1) \varphi_2(x_2; x'_2) + \varphi_1(x_2; x'_2) \varphi_2(x_1; x'_1), \]

For n=2

\[ (\varphi_1 \ast \varphi_2)(x_1, x_2, x_3; x'_1, x'_2, x'_3) = \varphi_1(x_1; x'_1) \varphi_2(x_2, x_3; x'_2, x'_3) + \varphi_1(x_2; x'_2) \varphi_2(x_1, x_3; x'_1, x'_3) + \]

\[ + \varphi_1(x_3; x'_3) \varphi_2(x_1, x_2; x'_1, x'_2) + \varphi_1(x_1, x_2; x'_1, x'_2) \varphi_2(x_3; x'_3) + \varphi_1(x_1, x_3; x'_1, x'_3) \varphi_2(x_2; x'_2) + \]

\[ \varphi_1(x_2, x_3; x'_2, x'_3) \varphi_2(x_1; x'_1), \]

and others.
References


23. Toyota Kola, Introduction to Kinetic Theory Stochastic Processes in Gaseous systems,


