EFFECT OF THE STREAMED NEGATIVE IONS ON DUST ACOUSTIC WAVES

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Abstract

The propagation of streaming negative ions in the dust acoustic waves (DAWs) is investigated, including nonlinear effects such as dust charge fluctuation, dust temperature, and the negative ions. The streaming velocity of the negative ions played an essential and effective role in the DAWs characteristics. The dust charge fluctuation shows a remarkable decrease with the negative ions concentration decrement. Increasing of negative ions temperature causes an increase of the dust charge for different ratios of streaming velocity to thermal velocity ($\nu$). Also, both of the amplitude and the width of the soliton solutions have shown strong dependence on $\nu$ as well as on the negative ions concentration. The results of this study will be useful for a better understanding of crystallization in dust plasma, especially phase transition governed from fluid state to solid state, and in a variety of modern technology applications such as fabrication of the semiconductor.
1. Introduction

Dusty plasma theory started in the late 1970s -- early 1980s [1-4], when diverse and often surprising new facts concerning planetary rings and cometary environments were reported by the interplanetary missions. First, Voyager revealed intricate structure in the planetary rings. Later, the Galileo and Ulysses missions measured dust streams in the vicinity of Jupiter and rocket flights provided the observational evidence of dust layers in the Earth's ionosphere. Dusty plasmas are now also a subject of intensive laboratory investigations, especially after the experimental discovery of dust crystals. There has been a rapidly growing interest in physics research of dusty plasma, not only because of the dust being an omnipresent ingredient of our universe, but also because of its vital role in applied physics and modern technology such as in tokamak, low temperature glow discharges and in the fabrication of semiconductors using plasma aided processes, dusty Plasmas exhibit new and unusual behavior, and provide a possibility for modified or entirely new collective modes of oscillations and instabilities, as well as coherent nonlinear structures. Unlike usual plasmas, complex plasmas can easily be condensed to form crystalline structures, providing new tools to study waves and oscillations in strongly coupled systems [5-10]. The dust acoustic waves (DAW) that were first predicted theoretically in 1990 by Rao et al. [11], were experimentally observed in 1995 by Barkan et al. [12]. A number of theoretical and experimental investigations have been carried out for understanding the charging of dust grains in plasma containing Maxwellian distributions of electrons and ions [13 - 17]. The role of streaming negative ions on plasma properties is also very important for the potential control of plasma crystal, dust as a killer particle in semiconductor processing, astrophysics (nebulon) and modification of powders (technological application)[18-21]. Non-linear DAWs with negative ions have been studied theoretically and experimentally by many researchers as they are important and complementary to each other. It was found that the effects of negative ions density, charge and streaming speed could significantly affect the dust grain surface potential or dust grain charge [22-23]. Nakamura et al. [24] showed that when the concentration of negative ions is larger than a critical value, a small compressive pulse evolved into subsonic wave trains and a large pulse developed into a solitary wave. They also measured the threshold amplitude and the velocity of the solitary waves and compared the results with the predictions using the pseudopotential method. [25]. Paul et al. [26] has some new findings. They showed that the concentration of negative ions and stream velocity have a major role on the instability of ion acoustic wave (IAW). From the dispersion relation, they showed that IAW has six modes of propagation and some modes are unstable. In this paper, a theoretical study is carried out for the effects of streaming velocity of negative ions on the fluctuated charges of dust grains in plasma composed of Boltzmannian electrons and positive ions, the effect of the negative ions' properties on the DAW characteristics, the Korteweg-de Vries (K-dV) solution of the soliton and K-dV-type will be also studied. The results of this work are
also important for the study of the Nebulon structure (cloud-like structure observed in the milky way) [27].

2. Governing equations

The basic equations describing the propagation of DAWs in dimensionless are

\[
\frac{\partial n_d}{\partial t} + \frac{\partial (u_d n_d)}{\partial x} = 0, \tag{1a}
\]

\[
\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + 3 \sigma_d n_d \frac{\partial u_d}{\partial x} = Z_d \frac{\partial \phi}{\partial x}, \tag{1b}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} - Z_d n_d - \nu e^{\gamma \phi} + \mu e^{-\gamma \phi} - \alpha \mu e^{\gamma \phi} = 0 \tag{1c}
\]

The positive ions, electrons and negative ions are assumed to obey the Boltzmann distribution namely;

\[
n_i = \mu e^{-(\gamma \phi)} \tag{2a}
\]

\[
n_e = \nu e^{\gamma \phi} \tag{2b}
\]

\[
n_{ni} = \alpha \mu e^{\gamma \phi} \tag{2c}
\]

where

\[
\sigma = \frac{k_B T_d}{Z_{do} T_{eff}}, \quad s = \frac{1}{\mu(1 + \alpha \gamma) + \beta \nu}, \quad \beta = T_i / T_e, \quad \gamma = T_i / T_n
\]

where \(\mu\), \(\nu\) and \(\alpha\) \(\mu\) are the normalized ion, electron and negative ion number densities, respectively. The densities of electrons and ions are normalized by \(Z_{do} n_{do}\) and the space coordinate \(x\), and time \(t\), velocity \(u_d\), pressure \(P_d\), and the electrostatic potential \(\phi\) are normalized to the Debye length \(\lambda_{de} = [T_{eff} / (4\pi Z_{do} n_{do} e^2)]^{1/2}\), where \(T_{eff} = T_i / [\mu(1 + \alpha \gamma) + \nu \beta]\) is the effective temperature, the inverse of the dust plasma frequency \(\omega_{pd}^{-1} = (m_d / (4\pi Z_{do}^2 n_{do} e^2))^{1/2}\), the dust acoustic speed \(C_d = (Z_{do} T_{eff} / m_d)^{1/2}\), \(n_{do} K_B T_d\) and \(T_{eff} / e\), respectively.

Let us consider a dusty plasma consisting of four components, extremely massive and highly negatively charged dust grains, electrons, ions and negative ions. The charge neutrality at equilibrium requires

\[
Z_{do} + n_{eo} - n_{io} + n_{no} = 0, \tag{3}
\]
where \( n_{io}, n_{no}, n_{io}, \) and \( n_{do} \) are the unperturbed positive ion, negative ion, electron and dust number densities, respectively, and \( Z_{do} \) is the unperturbed number of charge residing on the dust grain measured in the unit of electron charge.

After renormalization, equation (3) leads to

\[
\nu = \mu - a\mu - 1
\]

Now, the dust charge will be determined by the following charge current balance equation,

\[
\frac{\partial Q_d}{\partial t} + u_d \frac{\partial Q_d}{\partial x} = I_e + I_i + I_n,
\]

where, \( Q_d \) is the dust charge variable.

The charge currents originating from electrons, positive ions and negative ions reach the grain surface. Thus, the current – balance equation reads

\[
I_e + I_i + I_n = 0
\]

where \( I_i, I_e \) and \( I_n \) are ion, electron and negative ion current following into the dust grain surface.

According to the well known orbit-motion-limited probe model the electron, ion and negative ion currents for spherical dust grains with radius \( r \), normalized by \( e\pi r^2(8T_e/\pi m_e)^{1/2} \), are given by

\[
I_e = -n_e e^{(e\Phi/T_e)}, \quad I_i = n_i (1 - (e\Phi/T_i)) \text{ and } I_n = n_n [F_1 + F_2 (e\Phi/T_n)]
\]

where, \( \Phi \) denotes the dust grain surface potential relative to the plasma potential \( \phi \), for \( \delta = \mu/\nu \), \( \mu_i = m_i/m_e \), \( F_1 = (\sqrt{\pi} / 4u)(1 + 2u^2)Erf[u] + (1/2)e^{-u^2} \) and \( F_2 = (\sqrt{\pi} / 4u)Erf[u] \) the current balance equation becomes

\[
e^{i[\beta(\phi + \psi)]} - \sqrt{\beta / \mu} \delta e^{i\beta \phi} (1 - s \psi) + \alpha \sqrt{\beta / \mu} \delta [F_1 + F_2 \gamma s \psi] = 0,
\]

where, \( \psi = e\phi/T_{ef} \). Equation (7) is important for determining the dust charges \( Q_d = C \Phi \), \( C \) is the capacitance of a dust grain, i.e. \( -eZ_d = (r\psi/T_{ef} / e) \). We have the normalized dust charges \( Z_d = \psi / \psi_o \), where, \( \psi_o = \psi (\phi = 0) \), is the dust surface floating potential with respect to the unperturbed plasma potential at infinite place. \( \psi_o \) can be determined from the following transcendental equation.
\( e^{i\beta \varphi} - \delta \rho (1-s \varphi) + \alpha \rho \sqrt{1/\gamma} \delta [F_1 + F_2 \varphi s \varphi] = 0, \quad (8) \)

where, \( \rho = \sqrt{\beta/\mu_i} \).

As can be seen, the dust charge is very sensitive to a small disturbance of \( \varphi \) around the unperturbed states. This is very important in explaining how the variable dust charge influences the shape of the soliton and solitary waves.

Figure 1 shows the relation between fluctuated charge and the potential of dust grains with variable density of negative ion, at low effect of streaming velocity of negative ion, \( v=2 \), where \( v \) is the ratio between streaming velocity to thermal velocity.

Figure 2 shows the relation between fluctuated charge and the potential of dust grains with variable density of negative ion at \( v \) equal 3.5.

Figure 3 shows the relation between fluctuated charge and the potential of dust grains with variable density of negative ion at \( v = 5 \).

Figure 4 shows the relation between fluctuated charge \( Z_d \) and the ratio of the density of negative charge to positive ion \( \alpha \), where, \( v = \{2, 3.5, 5\} \).

Figure 5 shows the relation between \( Z_d \) and the inverse ratio \( \gamma \) (Temperature of positive ion to negative ion), where, \( v = \{2, 3.5, 5\} \) and \( \alpha = .01 \).

Figure 6 shows the relation between \( Z_d \) and the inverse ratio \( \gamma \), where, \( v = \{2, 3.5, 5\} \) and \( \alpha = .1 \).

Figure 7 shows the relation between \( Z_d \) and the inverse ratio \( \gamma \), where, \( v = \{2, 3.5, 5\} \) and \( \alpha = .4 \).

3. Nonlinear dust acoustic (DA) wave

To study dust acoustic (DA) in the presence of the variation of dust charges with negative ions where the streaming velocity of negative ion is much larger than the thermal velocity, we derive an evolution equation from the system of Eqs. (3), employing a reductive perturbation technique (RPT) by introducing the stretched coordinate [27]

\[ \zeta = \varepsilon^{1/2}(x - \lambda t), \quad \tau = \varepsilon^{3/2} t \quad (9) \]

where \( \varepsilon \) is the small parameter and \( \lambda \) is the solitary wave velocity; normalized by \( C_d \). The variable \( n_d, u_d, n_m, u_m, Z_d \) and \( \varphi \) are expanded as.

\[ n_d = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 + \cdots \quad (10a) \]

\[ u_d = \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \cdots \quad (10b) \]

\[ Z_d = 1 + \varepsilon Z_{d1} + \varepsilon^2 Z_{d2} + \varepsilon^3 Z_{d3} + \cdots \quad (10c) \]
\[ \phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \ldots \ldots \ldots \ldots \ldots, \] (10d)

Substituting this expansion (10) into equations (3), using \( \psi = Z \psi_o \), we could find the lost order of \( \varepsilon \) as

\[ n_{d1} = -R \phi_1, u_{d1} = -\lambda R \phi_1, Z_{d1} = \gamma \phi_1, \] (11)

where

\[ R = \frac{1}{(\lambda^2 - 3\sigma)}. \]

The linear dispersion relation is given by

\[ 1 + k = \frac{1}{\lambda^2 - 3\sigma} \] (12)

Thus and the solitary wave velocity \( \lambda \) is given by

\[ \lambda = [3\sigma + (1 + k)^{-1}]^{1/2}. \] (13)

where

\[ k = \frac{\left[1 - L \beta - s \psi_o + \alpha_1 \gamma (F_1 + F_2 s \alpha_1 \gamma \psi_o)\right]}{(-1 + L \beta - F_2 \alpha_1 \gamma)}, \]

\[ L = (-1 + \alpha_1 F_1 + s \psi_o + F_2 s \alpha_1 \gamma \psi_o), \quad \alpha_1 = \alpha \sqrt{1/\gamma}. \]

From the second terms in \( \varepsilon \), \( O(\varepsilon^2) \), we can obtain the desired K-dV equation,

\[ \frac{\partial \phi}{\partial \tau} + AB \phi \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} = 0, \] (14)

where

\[ A = 1/(2 \lambda R^2), \quad B = [3R(k - \lambda^2 (\lambda^2 + \sigma)) + s^2 (\mu (1 - \alpha k^3) - \beta \gamma) - 2k_1], \]

with

\[ k = (-1 + L \beta - F_2 \alpha_1 \gamma)^{-1} s[1 - L \beta^2 + F_2 \alpha_1 \gamma^2 - k[2 + (2 + k) L \beta^2 - 2F_2 \alpha_1 \gamma^2] \]
\[ + s(1 + F_2 \alpha_1 \gamma^3) \psi_o]. \]
On the other hand, the second order perturbed quantities \( n_{d2}, u_{d2} \) and \( Z_{d2} \) are given in terms of \( \phi_1 \) and \( \phi_2 \) and their derivatives as

\[
\begin{align*}
n_{d2} &= -R \phi_2 + \Omega_0 \phi_1^2 + 2AR^2 \Lambda \frac{\partial \phi_1}{\partial \xi^2}, \\
u_{d2} &= -R \Lambda \phi_2 + \Omega_1 \phi_1^2 + AR^2 (\Lambda^2 + 3\sigma) \frac{\partial^3 \phi_1}{\partial \xi^3}, \\
Z_{d2} &= k \phi_2 + k_1 \phi_1
\end{align*}
\] (15a) (15b) (15c)

where \( \Omega_0 \) and \( \Omega_1 \) are given in the Appendix.

From the equation of \( O(\varepsilon^3) \), we obtain a linear inhomogeneous equation of K-dV-type for the second-order perturbed potential \( \phi_2 \)

\[
\begin{align*}
\frac{\partial \phi_2}{\partial \tau} + AB \frac{\partial}{\partial \xi} (\phi_1 \phi_2) + B \frac{\partial^3 \phi_1}{\partial \xi^3} = \\
-H_1 \phi_1^2 \frac{\partial \phi_1}{\partial \xi} - H_2 \frac{\partial \phi_1}{\partial \xi} \frac{\partial^2 \phi_1}{\partial \xi^2} - H_3 \phi_1 \frac{\partial^3 \phi_1}{\partial \xi^3} - H_4 \phi_1 \frac{\partial^3 \phi_1}{\partial \xi^3},
\end{align*}
\] (16)

where \( H_q (q = 1, 2, 3, 4) \) is given in the Appendix.

### 4. Stationary solution

We investigate a stationary solution by applying the renormalization method introduced by Kodama & Taniuti [28] to the reductive perturbation method. According to this method, equations (24) and (18) are modified as

\[
\begin{align*}
\frac{\partial \phi_1}{\partial \tau} + AB \frac{\partial}{\partial \xi} \phi_1 + B \frac{\partial^3 \phi_1}{\partial \xi^3} + \delta \nu \frac{\partial}{\partial \xi} \phi_1 &= 0, \\
\frac{\partial \phi_2}{\partial \tau} + AB \frac{\partial}{\partial \xi} (\phi_1 \phi_2) + B \frac{\partial^3 \phi_1}{\partial \xi^3} + \delta \nu \frac{\partial}{\partial \xi} \phi_2 &= s_2(\phi_1) + \delta \nu \frac{\partial}{\partial \xi} \phi_1,
\end{align*}
\] (17a) (17b)

where \( s(\phi_1) \) represents the source terms in equation (17), the parameter \( \partial \nu \) is introduced so that the secular terms in \( s(\phi_1) \) are cancelled out by the term \( \partial \nu (\partial \phi_1 / \partial \xi) \). Let us find the stationary solution by defining the new independent variable \( \eta \) as

\[
\eta = \xi - (\nu + \partial \nu) \tau
\] (18)
\[
\frac{\partial^2 \phi_1}{\partial \eta^2} + (2B)^{-1}(AB \phi_1 - 2\nu)\phi_1 = 0,
\]

(19a)

\[
\frac{\partial^2 \phi_2}{\partial \eta^2} + 2(2B)^{-1}(AB \phi_1 - \nu)\phi_2 = 2(2B)^{-1}\int_{-\infty}^{\eta} \left[ s(\phi_1) + \nu_1 \frac{\partial \phi_1}{\partial \eta} \right] d\eta
\]

(19b)

Under this transformation and important imposing vanishing boundary conditions on \(\phi_1\), \(\phi_2\) and their derivatives up to the second order as \(|\eta| \rightarrow \infty\), we obtain the

\[
\phi_1 = \frac{\partial^a}{\partial \eta^a} \phi_1 = \frac{\partial^a}{\partial \eta^a} \phi_2 = 0, \quad (n = 1, 2)
\]

(20a)

we obtain from equation (19a) the stationary renormalization solitary wave solution

\[
\phi_1 = \phi_0 \sec h^2 d\eta \omega
\]

(20b)

Figure 8 shows the relation between the soliton \(\phi_1\) and \(\eta\) for different value of \(a\), where \(a = \{0.1, 0.4, 0.8\}, \nu = \{2\}\)

Figure 9 shows the relation between the soliton \(\phi_1\) and \(\eta\) for different value of \(a\), where \(a = \{0.1, 0.4, 0.8\}, \nu = \{3.5\}\)

Figure 10 shows the relation between the soliton \(\phi_1\) and \(\eta\) for different value of \(a\), where \(a = \{0.1, 0.4, 0.8\}, \nu = \{5\}\)

Corresponding to the solution for \(\phi_1(\eta)\) in (17a), the source terms of (19b) is given by

\[
2(2B)^{-1}\int_{-\infty}^{\eta} \left[ s_2(\phi_1) + \nu_1 \frac{\partial \phi_1}{\partial \eta} \right] d\eta = [(\partial \nu - 16\omega^4 C_4) \sec h^2 \omega \omega \eta + 2\omega^2 \phi_1 (60\omega^4 C_4 +

(C_2 + C_3) \phi_1) \sec h^2 \omega \omega \eta + 1/3 \phi_1 (-360 \omega^4 H_4 - 6\omega^2 (C_2 + 2 C_3) \phi_1 + C_4 \phi_1^2 \sec h^2 \omega \omega \eta)
\]

(21)

Now to drop out the secular terms in \(s_2(\phi_1)\), we put

\[
\partial \nu = 16\omega^4 C_4
\]

By introducing the new variable.

\[
\mu = \tanh \omega \eta
\]

(22)
Equation (19a), with the source term (21), is transformed into the associated inhomogeneous Legendre equation

\[
\frac{\partial}{\partial \eta} [(1 - \mu^2) \frac{\partial}{\partial \eta} \phi_2] + (12 - \frac{4}{1 - \mu^2}) \phi_2 = J_1(1 - \mu^2) + J_2(1 - \mu^2)^2
\]  

(23)

where

\[
J_1 = (8B/\nu u)2\omega^2 \phi_0 [−60\omega^2 C_4 + (C_2 + C_3)],
J_2 = (8B/3\nu u) \phi_0 [−360\omega^2 C_4 − 6\omega^2 (C_2 + 2C_3) \phi_0 + C_1 \phi_0^2],
\]

Two independent complementary solutions are given by the associated Legendre function of first and second kind as.

\[
P_3^2 = 15\mu(1 - \mu^2),
\]  

(24a)

\[
Q_3^2 = (15/2)\mu(1 - \mu^2)\ln\frac{(1 + \mu)}{1 - \mu} + (1 - \mu^2)^{-1} + 5 - 15(1 - \mu^2)
\]  

(24b)

and thus the complementary solution is given by

\[
\phi_{2c} = D_115\mu(1 - \mu^2) + D_2[(15/2)\mu(1 - \mu^2)\ln\frac{(1 + \mu)}{(1 - \mu)} + (1 - \mu^2)^{-1} + 5 - 15(1 - \mu^2)],
\]  

(25)

The particular solution of (25) can be obtained, using the method of variation of parameters, as

\[
\phi_2 = L_1(\mu)P_3^2(\mu) + L_2(\mu)Q_3^2(\mu),
\]  

(26)

where

\[
L_1(\mu) = \int \frac{Q_3^2(\mu)T(\mu) \, d\mu}{(1 - \mu^2)W(P_3^2, Q_3^2)},
\]  

(27a)

\[
L_2(\mu) = \int \frac{P_3^2(\mu)T(\mu) \, d\mu}{(1 - \mu^2)W(P_3^2, Q_3^2)},
\]  

(27b)

\[
T(\mu) = N(1 - \mu^2) + M(1 - \mu^2)^2,
\]

\[
W(P_3^2, Q_3^2) = P_3^2 \frac{\partial Q_3^2}{\partial \mu} - Q_3^2 \frac{\partial P_3^2}{\partial \mu} = 120(1 - \mu^2)^{-1}
\]

Substituting for \(L_1(\mu)\) and \(L_2(\mu)\), the particular solution (23) is given by:
\[ \phi_{zp} = \frac{1}{24} \{ -4 A_i [-1 + \text{Tanh}^2 (\alpha \eta)] - 3 B_i [-1 + \text{Tanh}^4 (\alpha \eta)] \} \]  

(28)

where

\[ A_i = (4/3u)2\omega^2 \phi_o [-60\omega^2 C_4 + (C_2 + C_3)], \]

(29a)

\[ B_i = (4/3u)\phi_o [-360\omega^4 C_4 - 6\omega^2 (C_2 + 2C_3)\phi_o + C_3 \phi_o^2] \]

(29b)

The final solution of \( \phi \) is a combination of the solutions for \( \phi_1 \) and \( \phi_2 \)

\[ \phi_{zp} = \frac{1}{24} \{ -4 A_i [-1 + \text{Tanh}^2 (\alpha \eta)] - 3 B_i [-1 + \text{Tanh}^4 (\alpha \eta)] \} + \phi_o \text{sech}^2 (\alpha \eta) \]  

(30)

Figure 11 shows the relation between \( \phi \) and \( \eta \) where, \( \alpha = \{0.1, 0.4, 0.6, 0.8\} \) and \( v = 2 \).

Figure 12 shows the relation between \( \phi \) and \( \eta \) where, \( \alpha = \{0.1, 0.4, 0.6, 0.8\} \) and \( v = 3.5 \).

Figure 13 shows the relation between \( \phi \) and \( \eta \) where, \( \alpha = \{0.1, 0.4, 0.6, 0.8\} \) and \( v = 5 \).

Figure 14 shows the relation between \( \phi \) and \( \eta \) where, \( \gamma = \{0.1, 0.5, 0.9\} \) and \( v = 2 \).

Figure 15 shows the relation between \( \phi \) and \( \eta \) where, \( \gamma = \{0.1, 0.5, 0.9\} \) and \( v = 3.5 \).

Figure 16 shows the relation between \( \phi \) and \( \eta \) where, \( \gamma = \{0.1, 0.5, 0.9\} \) and \( v = 5 \).

5. Conclusion

By employing a reductive perturbation technique, many different effects such as fluctuated charge, dust temperature, dust potential and streaming negative ion has been investigated. For every additional physical parameter, we can summarize the results as follows.

1- For higher values of the ratio \( v \): \( \{v = 3.5, 5\} \) the fluctuated charge increases and reaches a stable value at \( \{\varphi = 7.5\} \), but at low value of ratio \( v: \{v = 2\} \) the fluctuated charge increases by the increase of the potential; the value of the fluctuated charge then decreases at \( \{\varphi = 7.5\} \), where it behaves much like the effect of non-streaming velocity of negative ion.

2- By increasing the ratio of negative ions' density (\( a \)), the fluctuated charge reduces; also, the entire curve decreases by increasing values of the ratio \( v \).

3- The fluctuated charge increases by increasing the temperature of the streaming negative ions,

Finally, we could deduce an important effect from the curves of the solitons solution, when the density of negative ions increases (\( a \)) the amplitude of the soliton solution increases the ratio to \( v = \{2\} \), which mean the energy of the system of the dust grain increases. On the other hand, the amplitude of the soliton solution reduces with the increasing of the streaming negative ion density (\( a \)) at the ratios of \( v \):

\( \{v = 3.5, 5\} \) which means that energy of the dust grain reduces.
In this study, the K-dV-type is obtained, and changes with the density \((\alpha)\) and the temperature \((\gamma)\) of the streaming velocity of negative ions. The humping of the curve of the total soliton \(\varphi\) decreases by increasing the effect of the temperature \((\gamma)\), but by increasing the ratio of \(\nu\), the humping increases of the whole curve.

6. Appendix

\begin{align}
\Omega_o &= (R/2)[-k + R(2AB\lambda + 3R(\lambda^2 + \sigma))], \\
\Omega_i &= (R/2)[-k\lambda + R(AB(\lambda^2 + \sigma) + R\lambda(\lambda^2 + 9\sigma))], \\
C_1 &= (2\lambda R^2)^{-1} \\
[6k_2 - 2k_3 R - 6k_i R + s^3(\mu + \alpha \gamma^3 \mu + \beta^3 \nu) + 2k(ABR^2 \lambda + 3R^3(\lambda^2 + \sigma) + \Omega_o + 2\Omega_2] \\
C_2 &= [-f_1 + 2\lambda R^2 \lambda]/(2\lambda R^2), \\
C_3 &= [-2k_1 - 2k R - f_2 + f_3] / (2\lambda R^2), \\
C_4 &= [-3\lambda^2 R(\lambda^2 + \sigma)] / (2\lambda R^2),
\end{align}

with

\begin{align}
\Omega_3 &= R[-k + 3k R^2(\lambda^2 + \sigma) - 2AB[2R^3(\lambda^2 + 3\lambda \sigma) + \lambda \Omega_o + \Omega_i] - R \\
&[R^3(5\lambda^4 + 30\lambda^2 \sigma + 9 \sigma^2) + 3\sigma \Omega_o + 2\lambda \Omega_i],
\end{align}

\begin{align}
f_1 &= AR^3[9AB(\lambda^2 + \sigma) + 4R\lambda(\lambda^2 + 3\sigma)], \\
f_2 &= R[-k + 3R(\lambda^2 + \sigma)], \\
f_3 &= s^2[\alpha \gamma^2 - 1] \mu + \beta^2 \nu],
\end{align}
References

Figure 1

Figure 2

Figure 3
fluctuate charge $Z_d$ and the ratio of the density of negative charge to positive ion at $v=2,3.5,5$

Figure 4

Relation between fluctuated charge $Z_d$ and the inverse ratio of Gama

Figure 5

Figure 6
Figure 10

\( \psi = 5 \)

\( \alpha = 0.1, 0.4, 0.6, 0.8 \)

\( \psi_1, \psi = \psi_1 + \psi_2 \)

Figure 11
Figure 12

Figure 13
Figure 14

Figure 15
\{u=5\}, \gamma = 0.1, 0.5, 0.9

\begin{align*}
\phi_1, \ (\phi = \phi_1 + \phi_2) \\
\phi_1, \ (\phi = \phi_1 + \phi_2)
\end{align*}

Figure 16