COSMOLOGICAL CONSTRAINTS ON NEUTRINO OSCILLATIONS

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Abstract

Solar, atmospheric and terrestrial neutrino experiments have provided evidence for neutrino oscillations. These neutrino anomalies were successfully explained in terms of neutrino oscillations, the dominant channels being flavour neutrino oscillations. The role of sterile neutrinos and the active-sterile subdominant channels are being explored presently. Therefore, we discuss all cosmological effects of active-sterile neutrino oscillations on the early Universe evolution, and particularly the effects on the nucleosynthesis epoch.

Numerical analysis of the cosmological production of He-4, $Y_p$ in the presence of $\nu_e \leftrightarrow \nu_s$, effective after $\nu_e$ decoupling from the equilibrium, was provided for the full neutrino oscillations parameter range. These neutrino oscillations lead always to an overproduction of He-4. We have obtained isohelium contours corresponding to different levels of He-4 overproduction, $\delta Y_p/Y_p$, for initial population of the sterile state in the range $0 \leq \delta N_s \leq 0.5$.

Cosmological constraints on oscillation parameters, obtained on the base of the calculated isohelium contours and $Y_p$ observational data, are discussed. We present the constraints corresponding to $\delta N_s = 0.0$ and $0.5$, and helium overproduction $\delta Y_p/Y_p = 3\%$. These cosmological constraints, being more stringent than the ones provided from the neutrino experimental data, provide valuable information for the impact of sterile neutrino in the neutrino anomalies and for the neutrino physics in general.

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1 Introduction

In the last 20 years physical cosmology has made rapid development, marked by the precision measurements of the light elements abundances and the discovery of the cosmic microwave background (CMB) anisotropy by the balloon and spacecraft experiments DASI, Boomerang, Maxima and WMAP, pointing to the early tiny inhomogeneities, from which recent structures have developed, and providing information for many Universe characteristics.

Primordial abundances data and the cosmological theory of their formation, the so-called Big Bang Nucleosynthesis (BBN) theory, as well as CMB data and theory are often used as powerful probes of the physics beyond the standard electroweak model. Particularly, BBN provides stringent constraints on neutrino physics and especially neutrino oscillations.

In this paper we will discuss the present experimental status of neutrino oscillations (sec.2), neutrino oscillation effects in the early Universe (sec.3), BBN with neutrino oscillations (sec.4) and the cosmological constraints on oscillation parameters (sec.5). We will present the update of BBN constraints accounting for two new effects of oscillations on the early Universe physics and also a generalization of the constraints for the case of non-zero population of the sterile neutrino state at the onset of oscillations.

2 Evidence for neutrino oscillations

The hypothesis of neutrino oscillations was proposed more than 40 years ago by B. Pontecorvo [1] as a possible explanation of the solar neutrino deficit found at the Davis solar neutrino experiment.

Neutrino oscillations occur if mass eigenstates $\nu_i$ are distinct from the flavour eigenstates $\nu_f$: $\nu_i = U_{if} \nu_f$ ($f = e, \mu, \tau$), i.e. if neutrino has mass and non-zero mixing. In the simple two-neutrino oscillation case in vacuum the flavour composition changes with time - the probability of a given neutrino type in an initially homogeneous neutrino beam of the same type is:

$$P_{ff}(t) = 1 - \sin^2 2\theta \sin^2(\delta m^2 t/4E),$$

where $\delta m^2$, the neutrino squared mass difference, and $\theta$, the oscillations mixing angle, are the oscillation parameters and $E$ is the neutrino energy. The electron neutrino, produced in the solar core, undergoes transformations into another type and hence, the registered electron neutrino flux by the solar neutrino experiment must be reduced in comparison with the flux expected from the Sun.

Since first Davis measurements, three types of neutrino experiments, solar, atmospheric and terrestrial ones have provided indication and/or evidence that neutrino oscillations really exist in Nature.

(i) Evidence for solar neutrino oscillations was obtained at the solar neutrino experiments Homestake, Kamiokande, SuperKamioka, Gallex, SAGE, SNO and the terrestrial Kamland experiment. All solar neutrino experiments detected considerably lower neutrino flux than the predicted one by the standard Solar Model. Furthermore, the suppression dependance on the
neutrino energy was found. Depending on the energy range, the measured neutrino fluxes consist 0.3 to 0.7 of the predicted value. This problem was called solar neutrino anomaly.

Recent results of SNO and Kamland experiments are considered to confirm definitely neutrino oscillations as the solution to the solar neutrino anomaly. The measurement of neutral currents and charged currents fluxes at SNO experiment provided $\sim 5\sigma$ signal for neutrino flavour transitions that is not strongly dependent on the Solar Model. Kamland experiment chose among the different solar neutrino oscillation solutions. Solar neutrino data prefers flavour oscillation solutions to active-sterile ones, as was pointed first from cosmology considerations almost 15 years ago, and in the light of the recent results of the terrestrial experiment KamLAND, the Large Mixing Angle solution, corresponding to $\delta m^2 \sim 8.10^{-5}$ eV$^2$ and $\tan^2\theta \sim 0.45$, is the chosen one [2].

(ii) Evidence for atmospheric neutrino oscillations was obtained at Super-KamioKa, Macro, Soudan 2, IMB atmospheric neutrino experiments and recently by the terrestrial K2K experiment. These underground neutrino experiments as well as the earlier experiments IMB and Kamiokande, have measured ratio of the atmospheric muon to the electron neutrino flux $r$ considerably lower than the expected one $r_{MC} \sim \nu_\mu/\nu_e = 2$ for energies less than 1 GeV. Besides, a dependence of the muon neutrino deficit on the neutrino energy and zenith angle dependence was observed, instead of the expected isotropical flux. This so-called atmospheric neutrino anomaly has been known already for more than 10 years.

It was explained in terms of neutrino oscillations of muon to tau neutrino $\nu_\mu \leftrightarrow \nu_\tau$ as a dominant channel and nearly maximal mixing and $\delta m^2 \sim 2.5 \times 10^{-3}$ eV$^2$. The oscillations into sterile neutrino were disfavoured, because of the absence of suppression of oscillations by the medium, expected in the sterile case at high energies [3].

(iii) There exist also laboratory experiments, the so-called terrestrial experiments LSND [4], K2K [5] and KamLAND [6], which data have given an indication for oscillations, too.

$K2K$ has measured muon neutrino deficit in a beam coming from KEK to Kamiokande. A hint of spectrum distortion was also indicated by the analysis. The results are consistent with SuperKamioka atmospheric data and confirmed the atmospheric neutrino oscillation solution.

$KamLAND$ (Kamioka Liquid Scintillator Anti-Neutrino Detector) terrestrial experiment has measured $\bar{\nu}_e$ deficit in the flux of antineutrinos coming from reactors. KamLAND results chose the LMA solution, as the oscillation solution to the solar neutrino problem and further reduced the previously allowed LMA region.
The short baseline *LSND* (Los Alamos Liquid Scintillation Neutrino Detector) experiment has registered the appearance of electron antineutrino in a flux of muon antineutrino. This anomaly might be interpreted as $\nu_\mu \leftrightarrow \nu_e$ oscillations with $\delta m^2 = O(1 \text{ eV}^2)$ and $\sin^2 2\theta = O(0.003)$. \(^2\)

Thus, neutrino experiments results confirmed the existence of non-zero neutrino mass and mixing. Each of these neutrino anomalies were resolved by flavour neutrino oscillations. Active to sterile neutrino oscillations were proven to have a subleading role and their impact on the anomalies is now under consideration. Recently some indications were found from the analysis of the data from solar neutrino experiments that some subdominant admixture of sterile neutrino is not only allowed but desirable [7]. Hence, it is interesting to discuss active-sterile oscillations effects and the existing constraints on them in more detail.

Active-sterile oscillations were strongly restricted by cosmological considerations long before the experimental neutrino data could put constraints on them (for reviews see [8] and [9]). Non-zero neutrino mass and mixing affect Universe evolution expansion rate, neutrino densities and neutrino energy spectrum, thus influencing the neutrino involved processes, as in particular cosmological nucleosynthesis. In the following sections I will discuss in more detail the cosmological effects of neutrino oscillations and the cosmological constraints on oscillation parameters.

3 Neutrino Oscillations Effects in the Early Universe

3.1 Big Bang Nucleosynthesis

The idea for the production of elements through nuclear reactions in the hot plasma during the early stage of the Universe evolution belongs to George Gamov and was proposed and developed in the 1930s and 1940s [10]. In the following 70 years this idea has grown to an elegant and famous theory - theory of Big Bang Nucleosynthesis (BBN), explaining successfully the data on the abundances of the light elements D, He-3, He-4 and Li-7. The only parameter of the standard BBN - the baryon-to-photon ratio $\eta$ was recently determined by CMB anisotropy measurements and was used as an input in BBN[11, 12]. Today the physical processes typical for the BBN epoch are well known and BBN is a most powerful probe for new physics, like the physics predicting neutrino oscillations and non-zero neutrino mass.

From BBN considerations most stringent constraints on neutrino active-sterile oscillations parameters were obtained. In particular, active-sterile solar oscillation and atmospheric solutions were excluded many years before the global analysis of experimental data pointed to the preference of flavour oscillations.

For a precise analysis of the oscillations effect on BBN, helium-4 is used traditionally, as far as the most reliable and abundant data now available are for that element. \(^4\)He is a result of a

\(^2\)In case the LSND result is confirmed by the ongoing MiniBoo experiment at Fermilab, an addition of at least one light singlet neutrino (sterile neutrino $\nu_s$) is required, because three different mass differences, needed for the explanation of the solar, atmospheric and LSND anomaly require minimum 4 different neutrino masses.
complex network of nuclear reactions, which proceed after the freezing of the neutron-to-proton ratio $n/p$, however its abundance depends mostly on two compelling processes, determining the $n/p$-freezing, namely the Universe’s cooling rate, $H(t) \sim \sqrt{g_{\text{eff}}} T^2$ and the interaction rates of the weak processes, interchanging neutrons and protons: $\Gamma_w \sim G_F^2 E^2 \nu T^3$. Hence, the produced helium is a strong function of the effective number of relativistic degrees of freedom at the BBN epoch, $g_{\text{eff}}$, and the neutron mean lifetime $\tau_n$, which parametrizes the weak interactions strength. It is also sensitive to the neutrino number densities and spectrum. SBBN assumes three neutrino flavours, zero lepton asymmetry and equilibrium neutrino number densities and spectrum.

### 3.2 Neutrino oscillations effects

The oscillations effect on BBN was first considered for vacuum flavour oscillations [13]. Since there is a slight deviation from equilibrium in that case (temperatures of different flavour neutrinos are nearly the same), flavour oscillations have a negligible effect on nucleosynthesis.

*Active–sterile vacuum neutrino oscillations* provide better opportunities for BBN influence, because they can increase the effective number of light degrees of freedom during nucleosynthesis [13] and lead to a strong distortion of the neutrino spectrum [14].

The thermal background in the prenucleosynthesis epoch may strongly affect the propagation of neutrinos and, hence it is appropriate to consider oscillations in medium. The medium distinguishes between different neutrino types due to different interactions [15, 16, 17, 18]. This leads to different average potentials $V_f$ for different neutrino types: $V_f = Q \pm L$, where $f = e, \mu, \tau$, $Q = -b E T^4 / (\delta m^2 M_W^2)$, $L = -a E T^3 L^\alpha / (\delta m^2)$, $L^\alpha$ is given through the fermion asymmetries of the plasma, $a$ and $b$ are positive constants different for the different neutrino types, $-L$ corresponds to the neutrino and $+L$ to the antineutrino case. The effects of the medium can be hidden in $\delta m^2$ and $\theta$. In the adiabatic case the matter mixing angle is expressed through the vacuum oscillation parameters and the characteristics of the medium:

$$\sin^2 \theta_m = \sin^2 \theta / [\sin^2 \theta + (Q \mp L - \cos 2\theta)^2],$$

Although, in general, the medium suppresses oscillations by decreasing their amplitude, there also exists a possibility of enhanced oscillation transfer in case a resonant condition between the parameters of the medium and the oscillation parameters holds: $Q \mp L = \cos 2\theta$. Then the mixing in matter becomes maximal, independently of the value of the vacuum mixing angle, i.e. resonant transfer takes place.

The oscillations effect of *matter active–sterile oscillations* on BBN, efficient before BBN epoch, was considered in refs. [19, 20, 21]. Active–sterile oscillations that are efficient before the active neutrinos decoupling, are capable of exciting additional degrees of freedom into the plasma and lead to depletion of electron neutrinos, due to $\nu_e \leftrightarrow \nu_s$ [19, 20, 21].

In the last years the spectrum distortion due to $\nu_e \leftrightarrow \nu_s$ was found to have considerable
Figure 1: The figure gives a snapshot of the spectrum distortion caused by oscillations with mass difference $|\delta m^2| = 10^{-7}$ eV$^2$ and mixing $\sin^2 2\theta = 0.1$ at $T = 0.7$ MeV for different degrees of population of the steriles, namely $\delta N_s = 0$ (lower curve), $\delta N_s = 0.5$ and $\delta N_s = 0.8$ (upper curve). The dashed curve gives the equilibrium spectrum for comparison.

effect on BBN [22], both for the nonresonant [23] and resonant oscillations case [24]. In the resonant case, a growth of neutrino–antineutrino asymmetry generated by such oscillations was found possible [26, 22], and its effect on helium-4 production is noticeable [22] as well.

So today we know that neutrino oscillations considerably influence the neutrino involved processes in the Universe by

(a) **Excitation of additional degrees of freedom**, leading to faster Universe expansion $H(t) \sim g_{eff}^{1/2}$ and $^4\text{He}$ overproduction up to 5%, $\delta Y_p \sim 0.013 \delta N_s$.

(b) **Distortion of the neutrino spectrum**, in case of oscillations between partially populated sterile neutrino state $0 \leq \delta N_s \leq 1$ and electron neutrino. This distortion leads both to a *depletion of the active neutrino number densities* $N_\nu$: $N_\nu \sim \int dE E^2 n_\nu(E)$ and a decrease of the $\Gamma_w$, causing an earlier $n/p$-freezing and an overproduction of $^4\text{He}$ yield. The spectrum distortion is the greatest in the case the sterile state is empty at the start of oscillations, $\delta N_s = N_{\nu e}/N_{\nu s} = 0$.

It decreases with the increase of the degree of population of the sterile state at the onset of oscillations[25] as illustrated in the following figure (Fig.1).

(c) **Production of the neutrino-antineutrino asymmetry**

Neutrino-antineutrino asymmetry may be generated during the resonant transfer of neutrinos [22, 26] and it exerts back effect on oscillating neutrino and changes its oscillation pattern. Even when its value is small to have a direct kinetic effect on the synthesis of light elements, i.e. $L << 0.01$, it effects indirectly BBN suppressing oscillations at small mixing angles, leading to less overproduction of He-4 compared to the case without the account of asymmetry growth [24].

Spectrum distortion effect is the dominant one and for a wide range of oscillation parameters
considerably effects BBN. We denote further on the kinetic effects (b) and (c) by \( \delta \nu_{\mathrm{kin}} \).

4 Production of He-4 in the Presence of Neutrino Oscillations

The production of He-4 in case of oscillations effective before electron neutrino freezing, was considered both analytically [19, 20, 27, 28] and numerically [21] accounting precisely for a) and partially for b) effects (namely, estimating the depletion of the neutrino number densities, assuming equilibrium neutrino energy spectrum). Analytical description was found in the case of small mixing angles and 'large' mass differences \( \delta m^2 > 10^{-6} \text{ eV}^2 \), and for the case without spectrum distortion effects [27, 28].

For nonequilibrium neutrino oscillations, effective after active neutrino decoupling, i.e. for \( (\delta m^2/eV^2) \sin^4 \theta < 10^{-7} \), the spectrum distortion effect was shown to play a considerable role. For that case a complete selfconsistent numerical analysis of the kinetics of the oscillating neutrinos, the nucleons freeze-out and the asymmetry evolution was provided [22, 23, 24, 31]. The exact kinetic equations for the neutrino density matrix in momentum space were selfconsistently solved with the kinetic equations of the nucleons during the \( n/p \)-freezing. The kinetic equations accounted simultaneously for expansion, neutrino oscillations and neutrino forward scattering.

The first equation describes the kinetics of the neutrino ensembles in terms of the density matrix of neutrino \( \rho \) and anti-neutrino \( \bar{\rho} \). The second equation describes the kinetic evolution of the neutrons.

\[
\frac{\partial \rho(t)}{\partial t} = H_{\nu} \frac{\partial \rho(t)}{\partial p_{\nu}} + \text{amp; } + i \left[ \mathcal{H}_{\nu}, \rho(t) \right] + \text{amp; } \frac{\partial n_n}{\partial t} = H_{n} \frac{\partial n_n}{\partial p_n} + \text{amp; } + \int d\Omega(e^-p, \nu) |A(e^-p \rightarrow \nu n)|^2 \left[ n_e - n_p(1 - \rho_{LL}) - n_n \rho_{LL}(1 - n_e^-) \right] + \text{amp; } - \int d\Omega(\bar{e}^+p, \bar{\nu}) |A(e^+n \rightarrow \bar{\nu} \bar{p})|^2 \left[ n_{e^+} + n_n (1 - \bar{\rho}_{LL}) - n_p \bar{\rho}_{LL}(1 - n_{e^+}) \right].
\]

where \( \alpha_{ij} = U_{il}^* U_{j\ell} \), \( p_{\nu} \) is the momentum of electron neutrino, \( n \) stands for the number density of the interacting particles, \( d\Omega(i,j,k) \) is a phase-space factor, and \( A \) is the amplitude of the corresponding process. The plus sign in front of \( \mathcal{L} \) corresponds to the neutrino ensemble, the minus sign - to the anti-neutrino ensemble. Mixing only in the electron sector was assumed: \( \nu_i = U_{il} \nu_l \) (\( l = e, s \)). The initial condition for the neutrino ensembles in the interaction basis was assumed to be of the form:

\[
\rho = n_{\nu}^{eq} \left( \begin{array}{c} 1 \\ \text{amp;} 0 \\ \text{amp;} S \end{array} \right),
\]

where \( n_{\nu}^{eq} = \exp(-E_{\nu}/T)/(1 + \exp(-E_{\nu}/T)) \), while \( S \) measures the degree of population of the sterile state and \( \mathcal{H}_{\nu} \) is the free neutrino Hamiltonian. The 'non-local' term \( Q \) arises as a
$W/Z$ propagator effect, $Q \sim E_{\nu} T$. $\mathcal{L}$ is proportional to the fermion asymmetry of the plasma and is essentially expressed through the neutrino asymmetries $\mathcal{L} \sim 2L_{\nu_e} + L_{\nu_{\mu}} + L_{\nu_{\tau}}$, where $L_{\mu,\tau} \sim (N_{\mu,\tau} - N_{\bar{\mu},\bar{\tau}})/N_\gamma$ and $L_{\nu_e} \sim \int d^3p (\rho_{LL} - \bar{\rho}_{LL})/N_\gamma$.

The analysis was performed for the full set of oscillations parameters of the model and the temperature interval $[2.0, 0.3]$ MeV.

An empirical approximation formula for the interplay of the kinetic and density increase effect on the overproduction of He-4 was found[25]:

$$\delta Y_p = 0.013[\delta N^\text{max}_{\text{kin}}(1 - \delta N_s) + \delta N_s],$$

where $\delta N^\text{max}_{\text{kin}}$ is the value calculated for an initially empty sterile state, i.e. $\delta N_{\text{tot}} = \delta N^\text{max}_{\text{kin}}(1 - \delta N_s) + \delta N_s$.

Primordial $^4\text{He}$, $Y_p$ in the presence of $\nu_e \leftrightarrow \nu_s$ oscillations, effective after $\nu_e$ decoupling, may be considerably overproduced. The overproduction is maximal for the case of initially empty $\nu_s$ state $\delta N_s = 0$ - up to 32% in the resonant oscillation case (i.e. equivalent to 6 additional neutrinos in equilibrium, $\delta N_{\text{kin}} = 6$) and up to 14% in the non-resonant one [31]. For $\delta N_s = 0$ case the kinetic effect of oscillations $\delta N_{\text{kin}}$ due to spectrum distortion plays the dominant role in the overproduction of $^4\text{He}$.

5 BBN constraints on oscillation parameters

BBN constraints corresponding to $\delta Y_p/Y_p = 3\%$ overproduction of $^4\text{He}$ and initial population of the sterile neutrino $0.0 < \delta N_s < 0.6$ were calculated [30, 25]. BBN constraints for the nonresonant and the resonant cases of electron–sterile oscillation parameters are shown in fig. 2. The plots correspond to 3% overproduction of He-4. On the right-hand side the nonresonant case is presented and on the left the resonant oscillation case is given. The dashed curve corresponds to the case of initially empty sterile state $\delta N_s = N_{\nu} - 3 = 0$, the solid - to $\delta N_s = 0.5$.

The cosmological constraints exclude almost completely LOW solution to the solar neutrino problem, besides the LMA solution and sterile atmospheric solution, excluded in previous works. This result is consistent with the global analysis of the neutrino experiments data, which do not favour $\nu_e \leftrightarrow \nu_s$ solutions as dominant solutions. However, the cosmological constraints are more restrictive by several orders of magnitude concerning the neutrino squared mass differences.

The $\delta N_s > 0$ constraints become more stringent with the increase of the sterile state population. They slightly increased the BBN exclusion region of oscillation parameters.

Having in mind the still-existing observational uncertainty of $^4\text{He}$ measurements and especially the existence of a large systematic error indicated by the existence of two different $^4\text{He}$

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3In general, however, the population of the sterile neutrinos $\nu_s$ $\delta N_s$ may be non-zero in the pre-BBN epoch. $\nu_s$ may be present at the onset of the nucleosynthesis epoch. There may be different reasons for their production — they are naturally produced in GUT models, in models with large extra dimensions and Manyfold Universe models, in mirror matter models, in $\nu_{\mu,\tau} \leftrightarrow \nu_s$ oscillations in the preceding epoch.
measurements, we have provided detail numerical calculations of iso-helium contours corresponding to $\delta Y_p/Y_p = 5\%$ as well [32]. Relaxation of the constraints with the increase of $\delta N_s$ value was observed. Our numerical analysis has shown that up to $\delta N_s = 0.5$ the cosmological constraints corresponding to 5% He overproduction are slightly relaxed and remain almost stringent, as before. However, the constraints can be considerably relaxed for higher $\delta N_s$ values.

6 Conclusions

As early as 1990 cosmological constraints on active-sterile neutrino oscillations were obtained, based on BBN production of helium-4, pointing to the preference of the flavour oscillation solutions, while solar sterile solutions were disfavoured experimentally in 1998 and more definitely excluded only recently. This is a good illustration of the predictive power of cosmology and its use as an independent tool for revealing secrets of Nature.

We discovered two additional effects of oscillations in the early Universe, namely distortion of the neutrino energy spectrum and oscillations generated neutrino-antineutrino asymmetry, which may have strong influence on early Universe processes, and in particular on BBN. We analysed numerically the influence of those oscillations effects on BBN and updated the cosmological constraints on electron-sterile oscillation parameters. Due to the account of the kinetic effect of the spectrum distortion stronger cosmological constraints were derived, excluding solar LOW active-sterile solution as well.

Also the influence of non-zero initial population of the sterile neutrino state on oscillations
effects on BBN was studied. Generalized cosmological constraints corresponding to 3% He-overproduction and different level of initial population of the sterile neutrino were obtained. They are in agreement with the analysis of the experimental neutrino oscillations data and provide the stringest constraints on neutrino mass differences.

The results are important for determining neutrino properties, and may be useful for clarifying the role of the sterile neutrino in the solar and atmospheric neutrino anomalies, as well as for constraining non-standard physics models.

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