LOW-FREQUENCY WAVES IN MAGNETIZED
DUSTY PLASMAS REVISITED

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Abstract

The general dispersion relation of any wave is examined for low-frequency waves in a homogeneous dusty plasma in the presence of an external magnetic field. The low-frequency parallel electromagnetic wave propagates as a dust cyclotron wave or a whistler in the frequency range below the ion cyclotron frequency. In the same frequency regime, the transverse electromagnetic magnetosonic wave is modified with a cutoff frequency at the dust-ion lower-hybrid frequency, which reduces to the usual magnetosonic wave in absence of the dust. Electrostatic dust-lower-hybrid mode is also recovered propagating nearly perpendicular to the magnetic field with finite ion temperature and cold dust particles which for strong ion-Larmor radius effect reduces to the usual dust-acoustic wave driven by the ion pressure.
Dusty plasmas differ from multi-component plasmas because of the presence of relatively massive and highly charged dust grains which introduce new time and space scales causing new waves and instabilities. Fluctuation of the dust charge and dust size distribution, etc. are other unique features of dusty plasmas. Considering a simple model of constant point particles for the highly charged and massive dust grains, a number of low-frequency short-wavelength electrostatic waves have been found in dusty plasmas with or without the presence of external magnetic field [1]. Obviously, the low-frequency electromagnetic waves will also be modified or new modes may appear in magnetized dusty plasmas, which have not been explored rigorously.

Kotsarenko et al. [2] have studied low-frequency electromagnetic waves in a cold dusty plasma using fluid equations without paying attention to their damping or growth. Das et al. [3] have studied the kinetic Alfvén waves below the dust cyclotron frequency considering unmagnetized electrons and ions, but magnetized massive dust grains. In real physical situations, dust can be rather taken cold and unmagnetized with finite temperature for electrons and ions. Neglecting thermal-kinetic effects and using fluid model, Shukla and Rahman [4] previously have investigated low-frequency electromagnetic waves in nonuniform dusty magnetoplasmas. In recent years, the electrostatic dust-lower-hybrid instability with a background of neutrals and streaming electrons and ions similar to the laboratory conditions has been investigated [5,6]. Kinetic Alfvén waves have also been studied involving the dynamics of the magnetized or unmagnetized dust grains [7]. Hence, a rigorous kinetic theory is necessary to study the waves and instabilities, particularly, the long-wavelength low-frequency electromagnetic waves in a dusty plasma. These long wavelength electromagnetic waves are important for space plasma environments.

In this Brief Communication, we first study the low-frequency electrostatic/electromagnetic waves with particular emphasis on cyclotron, whistler, and magnetosonic waves in a uniformly magnetized dusty plasma. The long wavelength low-frequency electromagnetic waves are important for space plasma environments. In the electrostatic limit, we find the dust-lower-hybrid mode propagating nearly perpendicular to the external magnetic field.

The motion of electrons, ions and the dust particles in the presence of any mode \((\omega, k)\) either electrostatic or electromagnetic in a homogeneous magnetized dusty plasma is described by the linear Vlasov equation

\[
\frac{\partial f_\alpha}{\partial t} + (v \cdot \nabla) f_\alpha + \frac{q_\alpha}{m_\alpha} \left[ E(r, t) + \frac{v \times (B_0 + B(r, t))}{c} \right] \cdot \nabla_v f_{ao} = 0, \tag{1}
\]

where \(\alpha = e, i, d\) and \(E(r, t)\) and \(B(r, t)\) are the self-consistent electric and magnetic fields of the propagating mode; \(m_\alpha, q_\alpha, B_0\), and \(c\) are the mass, charge, external magnetic field \(\parallel \hat{z}\), and the velocity of light in a vacuum, respectively. The equilibrium distribution function, \(f_{ao}\) is taken as Maxwellian

\[
f_{ao} = (m_\alpha/2\pi T_\alpha)^{3/2} \exp \left(-v^2/2v^2_{ta}\right), \tag{2}
\]
where \( v_\alpha = (T_\alpha/m_\alpha)^{1/2} \) is the thermal velocity, \( T_\alpha \) being the temperature in energy units.

Without loss of any generality, we assume the wavenumber vector \( \mathbf{k} \) to lie in the \( xz \)-plane \( (\mathbf{k} = \hat{x} k_\perp + \hat{z} k_\parallel) \) and following refs.[8-10] we solve the Vlasov-Maxwell set of equations with standard procedures and obtain the dispersion relation of any wave from

\[
| \varepsilon - \frac{c^2 k^2}{\omega^2} I + \frac{c^2 k^2}{\omega^2} kk | = 0,
\]

where

\[
\varepsilon = I + 4\pi i\sigma/\omega,
\]

\( I \) is the unit dyadic and the components of the conductivity tensor can be obtained from \( J(\omega, k) = \mathbf{\sigma} \cdot \mathbf{E}(\omega, k) \) as

\[
\sigma_{xx} = \sum_\alpha \sum_l \frac{\left( -\pi i n_\alpha v_\perp^2 \omega_\alpha \right)}{T_\alpha k_\perp} \int \int \frac{dv_\perp dv_\perp}{\omega_\alpha + k_\parallel v_\parallel - \omega} a_v l f_{\alpha o},
\]

\[
\sigma_{xy} = \sigma_{yx} = \sum_\alpha \sum_l \frac{\left( -\pi i n_\alpha v_\perp^2 \omega_\alpha \right)}{T_\alpha} \int \int \frac{dv_\perp dv_\perp^2}{\omega_\alpha + k_\parallel v_\parallel - \omega} a_v l f_{\alpha o},
\]

\[
\sigma_{zz} = \sum_\alpha \sum_l \frac{\left( -\pi i n_\alpha v_\perp^2 \omega_\alpha \right)}{T_\alpha} \int \int \frac{dv_\perp dv_\perp}{\omega_\alpha + k_\parallel v_\parallel - \omega} a_v l f_{\alpha o},
\]

\[a_x = 2l \omega_\alpha J_l / k_\perp, \quad a_y = 2i v_\perp J_l', \quad a_z = 2v_\parallel J_l.\]

Here, \( \nu = x, y, z \), the prime on the Bessel function denotes derivative with respect to its argument \( (k_\perp v_\perp/\omega_\alpha) \), and \( n_\alpha \) is the average number density of the species \( \alpha \).

We consider a three-component dusty plasma embedded in a uniform magnetic field \( B_o \parallel \hat{z} \).

Using Eqs.(3-8), we obtain the components of the dielectric response function for the presence of electrostatic and electromagnetic waves in uniform magnetooactive cold dusty plasmas including collisions as

\[
| k^2 I - kk - \omega^2 \varepsilon / c^2 | = 0,
\]

where

\[
\varepsilon_{xx} = \varepsilon_{yy} = 1 + \sum_\alpha \frac{\omega_{pa}^2}{\omega_{ca}^2 - f_\alpha^2} \cdot \frac{f_\alpha}{\omega} \varepsilon_{xy} = -\varepsilon_{yx} = -i \sum_\alpha \frac{\omega_{pa}^2}{\omega_{ca}^2 - f_\alpha^2} \cdot \frac{\omega_{ca}}{\omega},
\]

\[
\varepsilon_{zz} = 1 - \sum_\alpha \frac{\omega_{pa}^2}{\omega f_\alpha}, \quad \varepsilon_{xz} = \varepsilon_{yx} = \varepsilon_{zy} = 0.
\]

Here, \( \alpha = e, i, d, f \), \( f_\alpha = \omega + iv_\alpha - k_\parallel u_\alpha \), \( \omega_{pa}^2 = 4\pi e^2 Z_d^2 n_\alpha / m_\alpha \), \( \omega_{ca} = Z_\alpha e B_o / m_\alpha c \), \( Z_\alpha = 1 \) for electrons and ions, \( = Z_d \), the number of electronic charge on a dust grain; \( m_\alpha, n_\alpha, c, \) and \( u_\alpha \) are the mass, number density of species \( \alpha \), the velocity of light in vacuum, and the plasma flow velocity, respectively.
Using the dielectric functions, Eqs.(10) in the cold plasma limit and Eqs.(5-8) in the hot plasma limit, we examine the low-frequency waves, electrostatic and electromagnetic, propagating in two standard geometries.

**Low-frequency electrostatic waves**

Let us now consider the electrostatic waves having frequency range below the ion cyclotron frequency, \( \omega \ll \omega_{ci} \) with unmagnetized but mobile dust component. In this case, the dust dynamics is also expected to play an important role. In the high frequency limit, \( \omega_{ci} \ll \omega \), the dust particles do not have any significant role on the electrostatic waves.

In the electrostatic limit (\( k \parallel \mathbf{E} \)) and in the presence of cold and unmagnetized dust grains, the dispersion relation of the longitudinal waves with \( k = \hat{x}k_\perp + \hat{z}k_\parallel \) in the above frequency limit, is given by

\[
k_\perp^2 \epsilon_{xx} + k_\parallel^2 \epsilon_{zz} = \frac{\omega^2}{c^2} \epsilon_{xx} \epsilon_{zz},
\]

that is,

\[
\omega^2 = \omega_{dlh}^2 \left( 1 + \frac{k_\parallel^2 \omega_{pe}^2}{k_\perp^2 \omega_{pd}^2} \right), \quad k_\perp^2 \gg k_\parallel^2,
\]

where \( \omega_{dlh}^2 = \frac{\omega_{pd}^2 \omega_{ci}^2}{\omega_{pe}^2} = \omega_{ci} \omega_{cd} \left( Z_m n_{do}/n_{io} \right) \). This electrostatic dust-lower-hybrid mode having a cutoff at \( \omega = \omega_{dlh} \) and propagates as a dust-mode for nearly perpendicular direction to the external magnetic field (\( k_\parallel \ll k_\perp \)). It may be mentioned here that the existence of the dust-lower-hybrid mode was first pointed out in the literature [5] with a view to finding the magnetic field generalization of the analogous dust-acoustic wave involving the dust dynamics in the unmagnetized plasmas. The effect of ion-neutral collisions and their instability conditions in a dusty plasma with streaming ions and electrons were studied later [6].

To take into account the effect of temperature of electrons and ions with cold unmagnetized dust motion on the low-frequency electrostatic waves, viz., the dust-lower-hybrid waves, we employ Eqs.(5-8) to obtain the components of the dielectric tensor in a magnetized dusty plasma. In the electrostatic limit (\( k \parallel \mathbf{E} \)), for nearly perpendicular propagation (\( k_\parallel \ll k_\perp \)) with \( k_\parallel v_{ti} \ll \omega \) and in the presence of cold and unmagnetized dust grains, we obtain the dispersion relation from \( \epsilon_{xx} \simeq 0 \) where

\[
\epsilon_{xx} = 1 + \sum \sum_n \frac{\omega_{\parallel \alpha}^2}{\sqrt{2k_\parallel v_{\parallel \alpha}} Z_n \left( \sqrt{2k_\parallel v_{\parallel \alpha}} \right)} \frac{n^2 I_n(b_{\alpha}) \exp(-b_{\alpha})}{b_{\alpha}},
\]

\[
 \simeq 1 + \frac{\omega_{\parallel \alpha}^2}{\omega_{ci}^2} \left( 1 - \frac{3}{4} b_i \right) - \frac{\omega_{\parallel \alpha}^2}{\omega_{ci}^2} b_i, \quad b_i \ll 1.
\]

Here, \( b_{\alpha} = k_\parallel^2 v_{\parallel \alpha}^2 / \omega_{ci}^2, \quad v_{\parallel \alpha}^2 = T_\alpha / m_{\alpha}, \quad Z \) is the plasma dispersion function of its argument, \( I_n \) is the modified Bessel function; \( T_\alpha \) being the temperature of the species \( \alpha \). Thus, for high density plasma \( \omega_{\parallel \alpha}^2 / \omega_{ci}^2 \gg \omega_{pe}^2 / \omega_{ce}^2 \gg 1 \), we obtain the dispersion relation of the electrostatic
dust-lower-hybrid mode propagating transverse to the direction of the external magnetic field as

\[ \omega^2 = \omega_{dh}^2 \left( 1 + \frac{3}{4} b_i \right), \quad b_i \ll 1 \]

\[ \omega^2 = \omega_{dh}^2 + \frac{3}{4} k^2 \perp \epsilon_d^2, \quad (14) \]

where \( \epsilon_d^2 = Z_d^2 T_i n_{do} / m_d n_{io} \). This is the dispersion relation of the dust-lower-hybrid wave in a finite ion temperature dusty plasma. Electrons do not have a significant contribution for the perpendicular dielectric response function. Thus, in absence of the magnetic field, we recover the usual dust-acoustic wave \([1]\). The damping rate of the dust-lower-hybrid mode for small but finite \( k_\parallel \) is given by

\[ \gamma_L = -\sqrt{\frac{\pi}{8} \frac{\omega_d^2}{k_\parallel v_{ti}}} \left\{ \exp \left[ -\left( \frac{\omega + \omega_{ci}}{\sqrt{2} k_\parallel v_{ti}} \right)^2 \right] \right\} \right. + \exp \left[ -\left( \frac{\omega - \omega_{ci}}{\sqrt{2} k_\parallel v_{ti}} \right)^2 \right] \right\} \right. \]  

\[ (15) \]

Since \( \exp \left( -1/x^2 \right) \) falls sharply than rise of \( 1/x \) for small \( x \), \( \gamma_L \) is negligible for nearly perpendicular propagation.

For the large ion finite Larmor radius effect, \( b_i \gg 1 \), \( \epsilon_{xx} \simeq \omega_d^2 / \omega_{ci}^2 b_i - \omega_{pd}^2 / \omega^2 \), and we obtain

\[ \omega^2 = \omega_{dh}^2 b_i = k^2 \perp \epsilon_d^2, \quad k_\perp \gg k_\parallel. \quad (16) \]

For the large finite-Larmor radius effect, ions also follow straight line orbits like unmagnetized dust grains. Electrons play an insignificant role in the perpendicular motion, but they play an important role for motion parallel to the magnetic field. Thus, we obtain the dust-acoustic mode of the unmagnetized plasmas.

Electrostatic wave propagating parallel to the magnetic field with phase velocity between the thermal velocities of electrons and ions is the usual dust-acoustic wave. Electrostatic waves having frequencies below or nearly equal to the dust-cyclotron frequency may propagate as dust-cyclotron, dust-Bernstein modes, or Alfvén waves where it will be extremely difficult to magnetize the massive dust grains. Thus, waves with \( \omega_{cd} \ll \omega \ll \omega_{ci} \) will involve the dust characteristics and may be detected in laboratory conditions. The dust-lower-hybrid wave may be regarded as the analogue of dust-acoustic wave in unmagnetized dusty plasmas.

**Electromagnetic waves**

i) EM waves propagating parallel to \( \hat{B}_o \parallel \hat{z} \parallel k \)

High-frequency waves above the ion-cyclotron range of frequencies \( (\omega \gg \omega_{ci}) \) will not be affected much except for a small correction due to dust dynamics. In the frequency range between the cyclotron frequencies of ions and dust particles, \( \omega_{cd} \ll \omega \ll \omega_{ci} \), the wave dispersion relation
will involve the dust parameters. For transverse electromagnetic waves propagating parallel to
the magnetic field within this frequency range, the dispersion relation is given by

\[ k^2 - \frac{\omega^2}{c^2} \epsilon_{xx} = i \frac{\omega^2}{c^2} \epsilon_{xy}. \]  

(17)

Thus, for the right-hand-circularly polarized electromagnetic waves \((E_x = iE_y)\) for \(\omega_{cd} \ll \omega \ll \omega_{ci}\),

\[ k^2 = \frac{\omega_{pi}^2}{c^2} \left( 1 + \frac{1 - \omega/\omega_{ci}}{\delta} \right) \frac{\omega}{\omega_{ci}} - \omega \left( 1 - \frac{\omega_{pd}^2/\omega_{pi}}{\omega_{ci}} \right), \]

(18)

where the non-neutrality parameter, \(\delta = n_{io}/n_{eo}\). The left-hand-circularly polarized waves will
not propagate as they will be evanescent within this frequency range.

When the direction of propagation makes an angle \(\theta\) with the direction of the external
magnetic field, we find \(\epsilon_{xy} \gg \epsilon_{xx}\) for \(\omega_{cd} \ll \omega \ll \omega_{ci}\) and consequently, a whistler mode will propagate

\[ k^2 = \frac{\omega_{pi}^2 (1 + 1/\delta)}{c^2} \frac{\omega}{\omega_{ci} \cos \theta}. \]

(19)

The electromagnetic waves below the dust-cyclotron frequency \((\omega \ll \omega_{cd})\) will propagate as a
compressional Alfvén wave, a shear Alfvén wave, or a kinetic Alfvén wave in a finite temperature
plasma. These waves with \(\omega \ll \omega_{cd}\) have been studied extensively in ref.[7].

ii) **EM waves propagating perpendicular to \(B_0 \parallel \hat{z} \perp k\)**

Let us now consider the transverse electromagnetic waves \((k \perp E \perp \hat{z})\) propagating perpendicular
to the direction of the magnetic field \((k_{||} = 0)\). Since the massive dust grains are considered
cold and unmagnetized and the electrons and ions have opposite gyrations around magnetic field
lines, and \(\delta \simeq 1\), we have \(\epsilon_{xx} \gg \epsilon_{xy}\) within \(\omega_{cd} \ll \omega \ll \omega_{ci}\). Then, the dispersion relation is obtained from Eq.(3) as

\[ k_{\perp}^2 = \frac{\omega^2}{c^2} \left( 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega^2} \right). \]

(20)

Thus, for the high density limit, \(\omega_{pi}^2/\omega_{ci}^2 \gg \omega_{pe}^2/\omega_{ce}^2 \gg 1\),

\[ \omega^2 = \omega_{dlh}^2 + k_{\perp}^2 v_A^2, \]

(21)

where \(v_A = c\omega_{ci}/\omega_{pi}\), and the dust-lower-hybrid frequency \(\omega_{dlh}^2 = \omega_{pd}^2\omega_{ci}^2/\omega_{pi}^2\). Obviously, this
low-frequency electromagnetic mode reduces to the usual magnetosonic wave \((\omega^2 = k_{\perp}^2 v_A^2)\) in
a usual electron-ion plasma in absence of the dust. Thus, Eq.(21) describes the dust-modified
magnetosonic wave in a dusty plasma. There now appears a new cutoff frequency for the
magnetosonic wave due to the dust dynamics within this frequency regime.

To summarize, we have made a detailed theory of low-frequency waves using the fluid and
kinetic models of plasmas in a magnetized dusty plasma. The dust magnetosonic wave \((\omega_{cd} \ll \omega \ll \omega_{ci})\) is found to be drastically modified by the dust dynamics with the appearance of a
cutoff at the dust-lower-hybrid frequency. The transverse electrostatic dust-lower-hybrid wave propagates due to small but finite ion temperature of the plasma [cf. Eq.(21)], which for strong finite-Larmor radius effect reduces to the dust-acoustic wave driven by the ion pressure. The lightest component of the plasma, viz., electrons do not contribute to this mode. The damping rate of this dust-lower-hybrid mode is small and vanishes for exact transverse propagation. The dust-lower-hybrid waves studied here may explain the electrostatic/electromagnetic noise at the dust-lower-hybrid frequencies in the spectral observation from the astrophysical dusty plasma systems. The short wavelength electrostatic dust-lower-hybrid mode may be detected using typical parameters in a laboratory experiment in the presence of an external magnetic field.

It may be mentioned here that the instabilities of these dust-lower-hybrid modes, due to various free-energy sources in a magnetized dusty plasma, are being investigated in detail. The mode-coupling nonlinear interactions of other low-frequency electromagnetic and electrostatic waves with the dust-modes studied here are also of much importance in a magnetized dusty plasma, and the work in these lines is in progress.

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REFERENCES