ROLE OF SELECTIVE INTERACTION IN WEALTH DISTRIBUTION

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Abstract

In our simplified description 'money' is wealth. A kinetic theory model of money is investigated where two agents interact (trade) selectively and exchange random amount of money between them while keeping total money of all the agents constant. The probability distributions of individual money ($P(m)$ vs. $m$) is seen to be influenced by certain modes of selective interactions. The distributions shift away from Boltzmann-Gibbs like exponential distribution and in some cases distributions emerge with power law tails known as Pareto's law ($P(m) \propto m^{-(1+\alpha)}$).
Introduction

Money is important to everyone. However, this is unequally shared among individuals in a society. The flow of money from one hand to the other is a continuous and a complicated process; there is much to understand. Out of Economists’ domain, recent attempts are being made by Statistical Physicists to exploit apparent similarities between statistical physical descriptions of system of particles and economic agents in a society. The subject of ‘Econophysics’ is thus born. Econophysics of Wealth distributions is an active area which involves in interpreting and analysing real economic data of money, wealth or income distributions of all kinds of people pertaining to different societies and Nations \[3\]. A number of statistical physical models are already in the literature \[1\] in connection with the above. Understanding the emergence of Pareto’s law \( P(m) \propto m^{-(1+\alpha)} \), now more than a century old, is one of the most important agenda. Some early attempts \[4\] have been made to understand the wealth distributions, especially the Pareto’s law where the index \( \alpha \) is found to be between 1 to 2.5 more or less universally.

Some recent works assume economic activities to be analogous to elastic collisions and have Kinetic theory like models \[5, 6\]. Analogy is drawn between Money \( m \) and Energy \( E \) where temperature \( T \) is average money \( \langle m \rangle \) of any individual at equilibrium. In such a model any two agents chosen randomly from a total number of agents \( N \) are allowed to interact (trade) stochastically and thus money is exchanged. The stochasticity is introduced through a parameter \( 0 < \epsilon < 1 \) into the interaction. The interaction (trade) is such that one agent wins and the other looses the same amount so that the sum of their money remains constant before and after interaction (trading). This way it ensures the total amount of money of all the agents \( \sum m_i = M \) constant throughout. Therefore, this is a conservative system. The model is described below in a few steps.

The Model:

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\begin{align*}
    m_i(t + 1) &= \epsilon(m_i(t) + m_j(t)) \\
    m_j(t + 1) &= (1 - \epsilon)(m_i(t) + m_j(t)), \\
    m_i(t + 1) + m_j(t + 1) &= m_i(t) + m_j(t),
\end{align*}
\]

where \( m_i \) and \( m_j \) are money of the \( i \)-th and \( j \)-th agents respectively. Here we have \( t \) being discrete ‘time’ which is referred to as a single interaction between two agents.

Computer Simulation Results

The computer simulation results reported here, are done with system sizes (=total number of agents) \( N = 1000 \). In all cases we allowed the system to equilibrate up to \( t = 10^5 \) time steps. Averaging is done over 5000 realizations in each case and data is collected after the system assumes equilibrium. The final distribution of course does not depend on the initial
configuration (initial distribution of money with the agents). We calculate average over many
different realizations; which means over a number of ways of the random selection of a pair of
agents and also the stochastic term $\epsilon$.

If we allow the agents to interact for a sufficient time we arrive at an equilibrium distribution
of money among individual agents. This is quite the same way we arrive at the equilibrium energy
distribution of a system of gas particles elastically colliding and exchanging energy with each
other. The equilibrium temperature here is average money $< m >$ per agent. We arrive at a
Boltzmann-Gibbs type distribution ($P(m) \propto \exp(-m/ < m >)$) of individual money. This is
verified by numerical simulation.

If we intend to take average of money of a single agent over a long time then we are supposed
to get same value for all the agents. Therefore, the distribution of individual time averaged
money appears to be a delta function as checked by numerical simulation. However, when
average is taken over short time period, the delta function broadens to appear as a modified
exponential distribution. Finally, the distribution of individual money at a particular time
is pure exponential as mentioned earlier. This is all in the randomly interacting agents with
stochastic gain or loss.

Two agents can interact with each other in a number of ways. Distribution of money is
dependent on how the two randomly selected agents decide to interact when they meet: whether
it will be random sharing of their aggregate money or with some rules [5]. It is shown in our
earlier work [2] that the kind of $2 \times 2$ transition matrix dictates the emergence of the distribution
whether it is purely exponential or a modified exponential distribution ($P(m) \propto m' \exp(-m/ < m >)$). In this work we would like to show that the selection of partners does influence the
distribution of individual money.

Before discussing the effect of selection on individual wealth distribution, let us examine
family wealth distribution. The agents in the society may not be individualistic. They belong to
certain families. This is also quite reasonable to assume that an agent does not trade or interact
with another member of the same family. In such cases interaction is definitely restricted and a
way of selective interaction. While doing computer simulation we colour all the agents belonging
to the same family. It is taken care that two members of a same family do not have trade
relationship. Now if we ask what is the money distribution of individual families, we simply add
up the contributions of the members and then find out the distributions. In Fig.1 we plot the
family wealth distribution of a society for three cases: (i) all the families consist of 2 members
each, (ii) all the families consist of 4 members each, and (iii) families of various sizes between
single member to one with member of four. The distributions are clearly not purely exponential,
but modified exponential distributions with different peaks and different widths which is quite
expected. The probability of zero income of a family is zero when there are families of size more
than one.

Now we again return to individual wealth (or money) distribution where a more interesting
part is around the tail. In most of the developed Countries and capitalist kind of Economies there is disparity. Probability distribution of money of the majority is different from that of a handful of the minority (rich people). Tails of the distributions (of real data) are fitted well with a power law (Pareto’s law). The emergence of power law is supported by a number of models though the proper reason behind its emergence is yet to be understood well. Pareto’s law is observed quite universally across many nations and societies. Let us examine what the role of selection of pairs of agents may be into the distribution of individual wealth. Transaction of money between agents is linked with so many factors where selection or personal choice could be one of those. At the first step, we perform a computer experiment where we allow any two agents to interact randomly as before. Money is exchanged continually and as a result distribution keeps changing: some people become poor, some become rich and so on. Now we imagine the interaction may be such that the resulting 'disparity' (among all the agents) increases. This means numerically we calculate the dispersion (variance, $\sigma = <m^2> - <m>^2$) after each interaction (between two agents) and if $\sigma$ is seen to be greater than the previously calculated value, the transaction is allowed, otherwise not. In this way we keep on increasing $\sigma$ ('disparity' in a way). However, it does not increase forever as the system itself is conserved (total money $M =$ constant). When $\sigma$ attains maximum limit we stop the simulation. The idea is to check the distribution (of individual money), how it behaves in the limit of maximum $\sigma$. The resulting distribution is plotted in Fig.2 where it is seen that the tail clearly obeys power law with a power close to 3. 

Figure 1: Family wealth distribution: two curves are for families of all equal sizes and one is for families of various sizes between 1 and 4. Unit is arbitrary.
Therefore, the exponent $\alpha$ in Pareto’s law is approximately 2. We do not immediately claim this to compare with real data where the value of $\alpha$ comes around 1.5. Rather we claim that some ways of selection may invoke power law. In a real society a large amount of economic disparity is often observed among people. The detail mechanism leading to disparity is not always clear but it may be connected to power law in wealth distribution.

Figure 2: Wealth distribution in the limit of maximum ‘disparity’. To have a comparison an exponential function $100.\exp(-x)$ and a power law function $2000.x^{-3}$ are plotted alongwith.

We observe that some different ways of incorporating selective interactions (or preferences) can also give rise to power law in the wealth distribution. Let us define a ‘class’ of an agent by some index. The class may be understood in terms of their efficiency of earning money, or by some other related property. Now when two agents interact they can convert the appropriate amount of money proportional to their efficiency factor in their favour or against. Now we rewrite equations (2) and (3) as the following:

\begin{align}
    m_i(t + 1) &= \epsilon_im_i(t) + \epsilon_jm_j(t) \\
    m_j(t + 1) &= (1 - \epsilon_i)m_i(t) + (1 - \epsilon_j)m_j(t),
\end{align}

The above equations can be viewed as more general within the framework of kinetic theory kind model. A detailed analysis in terms of $2 \times 2$ Transition matrices can be found in ref.[2]. Here the $\epsilon_i$ factors are randomly assigned to agents at the beginning and they are kept fixed (quenched). Now suppose we have a situation that all the agents do not interact with each other.
as it often happens in a real society. Very rich people may not interact with very poor people, so to say. Agents go by some norm or decision. Suppose the decision is taken in favour of trade (interaction) when the ratio of two class factors does not exceed a certain value. Let us say, $\epsilon_i > \epsilon_j$ and $\epsilon_i/\epsilon_j = \eta$. In our simulation we examine two cases with $\eta < 4$ and $\eta < 8$. The results are plotted in Fig.3. A power law is clearly observed with a power close to 4 which means the Pareto index $\alpha$ is close to 3 (the slopes for two curves are close to each other).

![Figure 3: Wealth distribution due to selective interaction restricted by 'class': Two curves for two limiting class ratios 4 and 8. A power law function (solid line) is plotted alongwith for comparison.](image)

**Discussions and Conclusions**

We may conclude that selective interaction of some kind within this conserved model can be able to produce power law in the distribution. The question of decision making (and of selection) in some form or other is of course important in a real society. In the framework of Kinetic theory type model, random interactions between agents lead to exponential distribution of individual money. Depending on the interaction and how the money is shared during interaction, the distribution changes away from exponential one and in some cases power law is also observed.

We have shown here that if the interacting pair of agents are not chosen randomly but with some selection procedure, the distribution definitely alters. In some cases we arrive at power law distributions. The reason is not very clear though. However, more meaningful and realistic models may be constructed based on this information. In the real society people most probably do not interact arbitrarily and randomly, they do so with purpose and thinking. Some kind of
personal selection or preference is always there which may be incorporated in some way or other. The effect of incorporation of Minority Game kind of strategy and selection into this kind of model would be interesting to see.

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