Physics of the Interaction of Charged Particles with Nuclei

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Abstract

The fundamentals of low energy nuclear reaction physics are consid-
ered. The main discussion concerns the physical substance of particle-
nuclear interaction phenomena. The corresponding mathematical re-
lations are introduced with detailed explanation. All the necessary in-
formation to understand the subject is immediately given in the text.
Nuclear physics models used for the cross section calculations are de-
scribed and how the model parameters are adjusted is discussed. The
aim of the lecture is to give a material scientist an insight into the
nuclear physics theory in the limits, which are necessary in order to
understand the principals of its application in the problem of nuclear
data.
1 Introduction

Nuclear physics studying the structure and properties of atomic nuclei dates back to the discovery of the atomic nucleus in 1911. A tremendous progress has been achieved since then due to efforts of experimentalists and theoreticians both in understanding the nature of the matter and in application of the acquired knowledge in different areas. However, until now there is no comprehensive theory which could describe all the experimentally observed properties of particle-nucleus interaction from the first principles. This is a consequence of the complexity of the nucleus as a physical object. Even in case the nucleon-nucleon interaction force would be known, and it would be possible to assume that this interaction does not depend on the presence of other nucleons, the many body problem of nuclear structure could hardly be resolved. So different models were developed in order to describe nuclear structure and mechanisms of the nuclear processes. These models are based on some simplifications and usually each model is suitable only for the description of a restricted number of nuclear phenomena. Comparison of the model calculations with experimental data followed by successive improvements of the model is a typical approach in nuclear physics.

Ion Beam Analysis (IBA) acquires information about composition and structure of the sample employing spectroscopy of products of the interaction of accelerated ions with the nuclei containing in the sample. There are a number of different IBA methods based on the registration of elastically scattered particles or the products of nuclear reactions and a reliable source of nuclear cross section data is needed for all of them except for Rutherford backscattering for which the cross section can be calculated according to known formula and for PIXE which is based on atomic rather than on nuclear physics. When some 30 years ago the first steps were made in IBA this work was carried out by nuclear physicists. Step by step IBA became more and more routine and a new generation of scientists came in the field. Now the present generation belongs mainly to a community of material science and has, as a rule, no nuclear physics background. So far as a projectile-nucleus interaction underlies the IBA methods some knowledge in the field is necessary for material scientists. In addition it appeared that nuclear physics theory is a powerful tool in the evaluation of the differential cross sections for IBA [1]. A vice versa process to that made when nuclear models were developed is now applied to evaluate measured cross sections on the base of their consistency with nuclear models. The aim of this lecture is to give a
material scientist an insight into the low energy nuclear physics theory in
the limits, which are necessary in order to understand the principals of its
application in the problem of nuclear data.

2 Nuclear Forces

An atomic nucleus is a strongly bound system of nucleons located in a small
domain with a typical size of

\[ R \approx (1.1 \div 1.5) \cdot A^{1/3} fm \quad (1 fm = 10^{-13} cm) \quad (1) \]

Nucleons are held together inside nuclei due to nuclear forces. These
forces are strong attractive forces acting only at short distances. They pos-
sess property of saturation, due to which nuclear forces are attributed ex-
change character (exchange forces). Nuclear forces depend on spin, not on
electric charge, and are not central forces. The nature of the nuclear forces
has not yet fully been clarified.

Nuclear forces are said to be strong forces, in the sense that they are
at least 100 times greater than very strong Coulomb forces taken at short
nuclear distances of about 1 fm. The short range of nuclear forces leads to
a strict demarcation of the regions where only long-range Coulomb forces,
or only nuclear forces show up as the latter suppress the Coulomb forces at
short distances.

The dependence of the force on the space coordinates is described by
means of the potential. The presence of one of the interacting bodies is
expressed through the potential as a function of the distance from the body
center and the force at the point \( r \), directed from the first body to the second,
is found as a potential derivative with respect to the space coordinates at
this point.

Assuming nucleus is a uniformly charged sphere the electrostatic poten-
tial energy for the projectile-nucleus system can be written as

\[ V_C (r) = \begin{cases} \ \frac{Zze^2}{r} & \text{for } r \geq R \\ \frac{Zze^2}{2R} \left( 3 - \frac{r^2}{R^2} \right) & \text{for } r \leq R \end{cases} \quad (2) \]

where \( Z \) and \( z \) are charge numbers of the nucleus and the projectile respectively.

Nuclear forces are also introduced through the potential energy of the
nucleon interaction. The positive potential creates repulsive forces, and the
negative potential creates attractive forces. Therefore, the potential energy is positive if it corresponds to repulsive forces, and it is negative for attractive forces. As a result, the potential energy of the point proton interaction with the nucleus may be presented as is shown in Fig.1

\[ V(r) \]

Figure 1: The nuclear and Coulomb potentials of the nucleus.

The Coulomb repulsion changes abruptly to attraction at the distance of the radius of action of nuclear forces, i.e. at the boundary of the nucleus \( R \). The transition from repulsion to attractions proceeds, though rapidly but continuously, in the region of the space coordinate \( R \). So, to a certain degree of accuracy the nuclear potential is pictured in the form of a square potential well which is about 40÷50 MeV deep.

For a charged projectile to reach the range of action of nuclear forces, it should possess some kinetic energy \( T \) sufficient to overcome the Coulomb potential barrier of height

\[ B_C = \frac{Ze^2}{R} \]  

which is of the order of 1 MeV even in the interaction of singly charged particles with the lightest nuclei.

According to quantum mechanics the transparency of the Coulomb barrier is given by the formulae

\[ D \approx e^{-\frac{2}{\hbar} \int_{r_1}^{r_2} \sqrt{2\mu(V_C-T)} \, dr} \]

(4)

where \( \mu = \frac{Mm}{M+m} \) is reduced mass, \( r_1 = R \) and \( r_2 \) is derived from the relation

\[ T = \frac{Ze^2}{r_2} \].
Thus though electric charge of atomic nuclei hinders the initiation of nuclear reactions with low energy charged particles the reactions are still feasible at energies below the potential barrier. These are so-called under-barrier reactions. The penetrability of Coulomb barrier increases very rapidly as $T$ approaches $B_C$ (Eq. (3)). Therefore, if $T$ does not greatly differ from $B_C$, the under-barrier reactions take place with remarkable probability.

3 Rutherford Scattering

If the interaction is solely due to electric forces the differential cross section of elastic scattering is derived from energy and angular momentum conservation laws using a concept of impact force. As far as the law of the interaction (i.e. dependence of the force on the distance) is known it is possible to find a dependence of the scattering angle $\theta$ on the impact parameter $b$ which is expressed in a non-relativistic case by the relation

$$
tan\theta = \frac{2Ze^2}{mv^2b}.
$$

Figure 2: Scattering of a charged particle by the electric field of the atomic nucleus.

For a single unmoveable nucleus placed on the path of the ion beam of intensity equal to $N$ particles per square cm in 1 sec the number of the ions
scattered in the angle interval from $\theta$ to $\theta + d\theta$ is $dN = 2\pi b db N$ where $b$ and $db$ are derived from eq.(5). The value

$$d\sigma = \frac{dN}{N} = 2\pi b db$$

(6)

is differential cross section which is expressed for the target containing $n$ nuclei per unit area by the well-known Rutherford formulae

$$d\sigma = n \left( \frac{Zze^2}{mv^2} \right) \frac{d\Omega}{4\sin^2\theta}$$

(7)

Distinct of pure Coulomb scattering the cross section cannot be calculated from an algebraic formulae in case of the nuclear interaction. As far as nuclear forces are acting only at very short distances a classical approach to the consideration of the scattering process is no longer applicable. The de Broglie postulates combining the corpuscular and wave properties of microparticles served as the foundation for the theory of the motion of projectiles and their interactions with nuclei.

4 Potential Scattering Formalism

In quantum mechanics, the state of a particle is described by the wave function $\psi(x, y, z)$ which, in the stationary case, depends only on the space coordinates. The specific form of the wave function is determined by solving the Schroedinger equation including the term expressing the particle interaction law. The square of the wave function modulus is the distribution of the probability for the particle to have any space coordinates $(x, y, z)$. The wave function does not indicate the sequence in which the space coordinates are occupied with time, as is required when describing the motion of a classical particle, because this has no meaning for microobjects. For microparticles, the conception of moving along a trajectory analogous to the trajectory of the classical particle does not exist. This circumstance is most clearly indicated by one fundamental corollary of the de Broglie postulates known as the Heisenberg uncertainty principle.

Nuclear scattering is considered below, first for the simplest case of the projectile with no charge. According to quantum mechanics a particle state is described by the wave function $\psi$, which is obtained as a solution of the wave equation. For the case of elastic scattering of spinless non-identical
particles the wave equation has the form of a Schroedinger equation with a spherically symmetric potential \( V(r) \)

\[
\Delta \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad ,
\]

where

\[
\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z^2} \quad .
\]

Prior to scattering, the wave function \( \psi \) for the particle with a given momentum \( p \) has the form of a plane wave:

\[
\psi = e^{ikz} ,
\]

where \( k \) is a propagation vector

\[
k = \frac{p}{\hbar} = \frac{1}{\lambda} .
\]

Here \( \lambda = \lambda/2\pi \), where \( \lambda \) is the de Broglie wavelength.

This function is a solution of eq.(8) in case of \( V(r) = 0 \), i.e. the equation of the form

\[
\Delta \psi + \frac{2m}{\hbar^2} E \psi = 0
\]

and is normalized to correspond to the flux density equal to the projectiles velocity.

In the course of scattering the plane wave interacts with the field of nucleus \( V(r) \), that gives rise to a spherical wave divergent from the center of the interaction. This wave has the form of

\[
f(\theta) \frac{e^{ikr}}{r} .
\]

Thus the last stage of the scattering process (after scattering) is depicted by a superposition of the two waves - plane and spherical ones:

\[
e^{ikz} + \frac{e^{ikr} f(\theta)}{r} .
\]

Here \( \theta \) is a scattering angle (see Fig.3); \( f(\theta) \) is an amplitude of the divergent wave; the \( 1/r \) factor stands for decreasing of the flux in reverse proportionality to the square of the distance.
The square of the modulus of the scattered wave amplitude is equal to the differential cross section

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2.$$  \hspace{1cm} (15)

This is easy to prove. By definition the differential cross section $d\sigma$ is equal to the fraction $dN/N$ of the initial particles flux $N$ scattered into the given solid angle $d\Omega$. Assuming the density of particles in the primary beam being equal to unity one obtains $N = v$, where $v$ is the particles velocity. For $dN$ one obtains (see Fig.3)

$$dN = f(\theta) \frac{e^{ikr}}{r} v r^2 \sin\theta d\theta d\varphi$$ \hspace{1cm} (16)

Taking into account that velocity does not change in the elastic scattering and that $\sin\theta d\theta d\varphi = d\Omega$ one finally obtains that

$$d\sigma = \frac{dN}{N} = \frac{|f(\theta)|^2 v^2 d\Omega}{r^2 v} = |f(\theta)|^2 d\Omega.$$ \hspace{1cm} (17)

or

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2.$$ \hspace{1cm} (18)

The angular distribution of the scattered particles is defined by the $f(\theta)$ function. For the quantitative analysis of the elastic scattering eq. (8) and
(12) are considered in spherical coordinates. The general solution of these equations has the form of
\[ \psi = \sum_{l=0}^{\infty} A_l P_l (\cos \theta) R_{kl}(r), \]
where \( R_l(r) \) is a radial wave function; \( P_l(\cos \theta) \) is Legendre polynomial \((P_0=1, P_1=\cos \theta, P_2=(3\cos \theta-1)/2,...)\).

Far from the center of scattering (at large distances \( r \)) the radial function for each of \( l \) can be represented in the form of two partial spherical waves one of which is converging \( e^{-i(kr-l\pi/2)} \) and the other is divergent \( e^{i(kr-l\pi/2)} \).

For the initial stage depicted by a plane wave both waves have equal amplitudes and
\[ R_{kl}(r) = e^{i(kr-l\pi/2)} - e^{-i(kr-l\pi/2)}. \]

So the plane wave expressed through an expansion over Legendre polynomials has the form of
\[ e^{ikz} = \sum_{l=0}^{\infty} \frac{(2l+1)}{2ikr} P_l (\cos \theta) \left[ e^{i(kr-l\pi/2)} - e^{-i(kr-l\pi/2)} \right]. \]

Here each of the spherical waves corresponds to the particle's moving with given orbital momentum \( l \) and is characterized by the angular distribution \( P_l(\cos \theta) \) (see Fig.4).

Figure 4: Legendre polynomials angular dependence.

Suppose the projectile possesses kinetic momentum \( p \) and angular momentum \( l \). Then from a comparison between classical and quantum mechanical relations for the modulus of the angular momentum
\[ |\vec{l}| = pp = \hbar l \sqrt{(l + 1)} \]
it follows that
\[ \rho = \frac{\hbar}{p} \sqrt{l(l+1)} = \lambda \sqrt{l(l+1)} , \]
\[ (23) \]
i.e. the initial beam behaves as if it were subdivided into cylindrical zones with radii defined by eq.(20), as shown in Fig.5. A significant difference between classical and quantum mechanical predictions for the scattering process is evident: in a classical approach the particle having zero impact parameter scatters straight in the back direction whereas angular distribution for the corresponding \((l = 0)\) partial wave is isotropic.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{beam_splitting}
\caption{The illustration of the initial beam splitting into the partial waves corresponding to the angular momenta.}
\end{figure}

In the process of scattering an additional divergent spherical wave arises. So the ratio between convergent and divergent waves changes. The change of the ratio can be formally taken into account by a coefficient at the divergent wave
\[ R_{kl}(r) = S_l e^{i(kr-\frac{l\pi}{2})} - e^{-i(kr-\frac{l\pi}{2})} . \]
\[ (24) \]

In the case of the elastic scattering the fluxes for the convergent and divergent waves should be equal to each other for each of \(l\). This means that \(|S_l|^2 = 1\). So the factor \(S_l\) can be written as
\[ S_l = e^{2i\delta_l} \]
\[ (25) \]
where \(\delta_l\) is called a phase shift.

Physically the phase shift is explained by the difference of the wave velocity in the presence of the nuclear forces field and outside the nucleus as is illustrated in Fig.6.

The partial wave after scattering has then the form of
\[ R_{kl}(r) = e^{i(kr-\frac{l\pi}{2}+2\delta_l)} - e^{-i(kr-\frac{l\pi}{2})} . \]
\[ (26) \]
The solution of eq.(8) for the final stage of scattering is
\[ e^{ikz} + f(\theta) \frac{e^{ikr}}{r} = \sum_{l=0}^{\infty} \frac{(2l + 1) i^l}{2ikr} P_l(\cos\theta) \left[ S_l e^{i(kr - l\frac{\pi}{2})} - e^{-i(kr - l\frac{\pi}{2})} \right]. \]
(27)

The following relation between the scattering amplitude and phases can be derived
\[ f(\theta) = \frac{1}{2ik} \sum_l (2l + 1) \left( e^{2i\delta_l} - 1 \right) P_l(\cos\theta). \]
(28)

Summing up, the differential cross section for elastic scattering is calculated from eq.(15), the scattering amplitude being expressed through phase shifts \( \delta_l \) according to eq.(28). The phase shifts for partial waves are calculated by resolving Schroedinger equation (8) with assumed potential \( V(r) \). This equation is split into angular and radial ones. The asymptotic general solution for the radial equation is
\[ R_{kl} \approx \sqrt{\frac{2}{\pi r}} \sin \left( kr - l\frac{\pi}{2} + \delta_l \right). \]
(29)

The phase shifts \( \delta_l \) are defined by the edge conditions. The phase shifts are functions of \( k \) and \( l \) but do not depend on the scattering angle.

If the projectile is charged it interacts with combined Coulomb and nuclear fields of the target nucleus. The relation for the scattering amplitude is then
\[ f(\theta) = f_C(\theta) + \frac{1}{2ik} \sum_{l=0}^{\infty} (2l + 1) (S_l - 1) e^{2i\phi_l} P_l(\cos\theta), \]
(30)
where $f_C(\theta)$ and $\sigma_l$ are amplitude and phase shift of the Coulomb scattering respectively.

The $S_l$ values defined by eq.(25) can be considered as elements of some diagonal matrix which is called a scattering matrix. In case of pure elastic scattering phase shifts $\delta_l$ are real numbers. However they become complex if inelastic scattering is also present in the scattering process. This corresponds to decreasing of the amplitude of the divergent waves i.e. $|S_l| < 1$.

In case a projectile possesses non zero spin all the ideology described above is retained valid. However, the equations become more complicated since radial wave equation splits into $(2s+1)$ equation. Suppose projectiles are protons which spin is $1/2$. Then the spin of bombarding particles may be combined with angular momentum $l$ in two ways to produce the total angular momentum $j=l\pm 1/2$.

The proton elastic scattering differential cross section is obtained in this case through resolving Schroedinger equations for partial waves as $d\sigma/d\Omega = |A(\theta)|^2 + |B(\theta)|^2$, the scattering amplitudes $A(\theta)$ and $B(\theta)$ being defined by the following relations

$$A(\theta) = f_C(\theta) + \frac{1}{2k} \sum_{l=1}^{\infty} \left[(l+1)S_l^+ + lS_l^- - (2l+1)\right] \exp(2i\sigma_l) P_l(cos \theta);$$

$$B(\theta) = \frac{1}{2k} \sum_{l=0}^{\infty} \left(S_l^+ - S_l^-\right) \exp(2i\sigma_l) P_l^1 (cos \theta),$$

where $f_C(\theta)$ is an amplitude of Coulomb scattering, $\sigma_l$ are Coulomb phase shifts, $P_l^1 (cos \theta)$ are Legendre polynomials, $P_l^1 (cos \theta)$ are associated Legendre polynomials, $S_l^+$ and $S_l^-$ are scattering matrix elements for different spin orientation, $k$ is a wave number.

The above representation of the elastic scattering process produces the cross section with a smooth dependence on energy. Some rather broad resonances called “shape (or size) resonances” are observed only at energies when conditions for standing waves to form in the nucleus potential well are fulfilled (Fig.7). These resonances correspond to the single particle states in the potential well.

5 Compound Nucleus Model

The mechanism of scattering, considered so far, is called direct or potential scattering since it proceeds through direct interaction of a single bombarding
particle with a potential well representing a nucleus. Nuclear interaction at low energies can proceed also in two stages through the mechanism of a compound nucleus (Fig.8). The first stage of the interaction is the absorption of the bombarding particle by the target nucleus and the production of an intermediate, or compound, nucleus. The compound nucleus is always highly excited because the absorbed particle brings both its kinetic energy and the binding energy of the absorbed nucleons into the produced nucleus. The second stage is the decay of the compound nucleus with the emission of this or that particle. The original particle may always be such a particle, and here again the original nucleus is formed. A typical lifetime for a compound nucleus is \( \sim 10^{-14} \) sec that is very long as compared with the time of direct interaction defined as a time \( (10^{-23} \div 10^{-21} \text{ sec}) \) needed for the bombarding particle passes through the region occupied by the nucleus potential well.

For the case of light nuclei the compound nucleus has discrete energy levels as shown in Fig.9 and so the cross section of the elastic scattering through this mechanism has a resonance structure. Because of the relatively long lifetime and due to the uncertainty relation (written in energy-time coordinates it is \( \Delta E \cdot \Delta t \geq \hbar \)) the widths of the compound nucleus levels are rather small. So are the widths of the resonances observed in the cross section.

One of the ways to take resonance scattering into account is to add Breit-
Wigner resonance terms to the diagonal elements of the scattering matrix:

\[
S^\pm_l = \exp \left(2i\lambda^\pm_l\right) \left[ \exp \left(-2\mu^\pm_l\right) + \exp(2i\phi_p) \frac{i\Gamma_p}{E_0 - E - \frac{1}{2}i\Gamma} \right],
\]

where \(\lambda^\pm_l + i\mu^\pm_l\) is the off-resonance nuclear phase shift describing the elastic scattering of particles of energy \(E\) from spin zero nuclei. The quantities \(E_0\), \(\Gamma\), and \(\Gamma_p\) are the energy, total width and partial elastic width, respectively. The subscript \(l\) is the relative angular momentum of the proton and the target in units of \(\hbar\). The plus and minus signs in superscripts refer to summing of orbital and spin momenta with different mutual orientation. The quantity \(\phi_p\) is a resonance phase shift.

Because of the interference between potential and resonance scattering the excitation function has a typical structure with resonances pictured as dips and bumps rather than as Breit-Wigner functions (Fig.10).

In case of the nuclei of middle and heavy mass the energy level density is high and the width \(\Gamma\) of resonances exceeds the distance \(D\) between them, \(\Gamma \gg D\) at a relatively low excitation energy (see Fig.9). Then a continuous background produced by the scattering via compound nucleus with overlapped levels is observed in the scattering yield. This background can be evaluated in the framework of a statistical model. It is assumed in this model that the compound nucleus decay is independent from the way the compound nucleus was created (yet all the conservation laws - energy, momentum, parity etc. - naturally are fulfilled). If the number of the overlapped levels is great enough it becomes possible to depict the properties of
the compound nucleus by averaging over excited states. Due to the averaging the quantum mechanical effects disappear and the semiclassical approach using statistical physics methods becomes possible. Computer codes based on the Hauser-Feshbach formalism [2] are widely in use for such calculations (see e.g. [3]). The input data needed are level-density parameters and transmission coefficients. The level-density parameters can be found e.g. in a Reference Input Parameter Library (RIPL) [4] produced in the result of the recent IAEA coordinated research project. The transmission coefficients are calculated using the optical model discussed below.

Even at high excitation energy when compound nucleus levels are overlapped, there are still some sharp resonances superimposed on a continuous background. These resonances are caused by the population in the compound nucleus \((Z,N)\) of so-called isobaric analogue states which have a rather simple structure, because they look alike low-lying states in the nucleus having \(Z-1\) proton and \(N+1\) neutron; a proton takes on the role of a neutron and vice versa. Because of the independence of nuclear forces on electric charge these nuclei are similar, with states being displaced due to the difference in the Coulomb energy between \((Z,N)\) and \((Z-1,N+1)\) nuclei.

In the intermediate case when \(\Gamma \sim D\) the so-called Ericson fluctuations of the cross sections are observed [5]. These fluctuations are uncorrelated over the energy, angle and reaction channel. Thus only statistical properties of the fluctuations not detailed structure of the cross section can be calculated.

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**Figure 9:** A scheme of nuclear levels of light (left) and heavy (right) nuclei.
A comprehensive review of compound nuclear reactions can be found elsewhere [6].

6 Optical Model

If nuclear reactions contribute to the total cross section along with elastic scattering this should be taken into account. Though some progress has been achieved in microscopic theory of nuclear reactions it is practical to apply a phenomenological approach consisting in consideration of the projectile interaction with the nucleus as a whole, the nucleus being represented by an appropriate potential. The potential parameters are found through fitting theoretical calculations to the available experimental data. To make this approach more physical the potential shape is derived from the known features of the nucleon-nucleon interaction and from distributions of matter and charge in the nucleus.

In the so-called optical model [7] nucleus is represented by means of a complex potential. The interaction of the projectile with the nucleus is then reduced to de-Broglie’s wave refraction and absorption by an opaque sphere. The name of the model originates from the formal analogy with the light plane wave passing through a semitransparent sphere.

Also refraction and absorption of the light is described by a complex
the complex potential of the form

\[ U = V + iW \]  

(33)

is used to take into account scattering and absorption of the projectile by the nucleus. The real part of the potential is responsible for scattering due to the direct mechanism whereas the imaginary part stands for the absorption. It is implied in the model that the compound nucleus formed in the result of the absorption of the incident particle is excited to such an extent that its energy levels are completely overlapped.

The standard form of the optical potential is as follows:

\[ U (r) = U_C (r) + U_R (r) + iU_1 (r) + U_{so} (r), \]

(34)

where \( U_C \) is the Coulomb potential defined by eq.(2),

\[ U_R (r) = -V_R f_R (r) \]

(35)

\[ U_1 (r) = 4a_3 W_D \frac{df_1 (r)}{dr} \]

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\[ U_{so} = \left( \frac{\hbar}{m_\pi c} \right)^2 V_{so} \frac{1}{r} \frac{df_{so}}{dr} l \cdot s \]  

(37)

\[ f_R(r) = \left[ 1 + \exp \left( \frac{r - R_x}{a_x} \right) \right]^{-1} \]  

(38)

\[ R_x = r_x A^{1/3} \]  

(39)

The potential terms represent, in sequence, the real central volume potential of the depth \( V_R \), the imaginary central surface potential of the depth \( W_D \) (volume absorption is negligible at low energies), and the surface spin-orbit potential of the depth \( V_{so} \), while \( f_x(r) \) is a Saxon-Woods formfactor, \( R_x \) is a half value radius, \( a_x \) is a diffusivity parameter, \( A \) is a target mass number, \( m_\pi \) is a \( \pi \)-meson mass, \( c \) is light velocity, \( l \) and \( s \) denote angular momentum and spin operators respectively.

![Graph](image_url)

Figure 12: The dependence of the calculated cross sections on the imaginary potential in case of low energy proton scattering from silicon.

The optical model does not take into account specific features of a particular nucleus. Thus resonances (except for single particle ones) are not reproduced in the framework of this model. Resonances are specific features of a particular compound nucleus whereas the optical model describes nuclear matter as a whole and so the optical potential parameters have only
a general trend on mass number and energy. Due to more than 30 years of application of the optical model the general features of phenomenological optical potential parameters are well established. So-called global sets of parameters obtained through the optimization procedure that was based on a wide collection of experimental data were developed [8] - [10]. A Reference Input Parameter Library [4] also contains the recommended optical model parameters. Generally the results obtained with optical model for scattering of nucleons for nuclei with mass number $A > 30$ are quite reliable in the energy range of $10 < E < 100$ MeV. Several computer codes were developed for optical model calculations. The SCAT2 code written by O. Bersillon [11] is one of the most popular.

At energies in the vicinity of the Coulomb barrier and in the sub-Coulomb region some anomalies were found in systematic dependence of the optical potential parameters on mass number and energy. An intensive study of the low energy anomalies in the optical potential behaviour was made in the early 80s. The peculiarities, that were found, are as follows. The strength parameters often have strong energy dependence in the vicinity of the Coulomb barrier. The real potential radial dependence is more complicated than the Saxon-Woods form. The imaginary part of potential reveals non-systematic dependence on nucleus mass number. Absorption is peaked at the nucleus surface. The radius of the imaginary potential diminishes with decreasing energy while its diffusivities increases. Calculations made at low energies in the framework of the optical model are very sensitive to the parameters used. So the results obtained with global sets appear to be unrealistic. Several attempts have been made to develop a global set for low energy region (see e.g. [12]). However for close reproduction of the measured angular distribution or excitation function the parameters of the model should be fitted to the particular set of experimental data. Authors of the RIPL [4] claim to produce recommended optical model parameters including low energy proton scattering on light nuclei. However the optical model is not applicable at all in this case because a basic assumption of the averaging over compound nucleus levels is not fulfilled. It is evident from Fig.11 that the cross sections calculated using optical models with RIPL input parameters are absolutely inconsistent with experimental data in this case. Another theory rather than optical model should be employed in this situation.

In most cases of low energy charged particle scattering the contribution of the reaction channels is negligible and so the imaginary potential is close to zero. The calculated cross section is extremely sensitive to this parameter,
as illustrated in Fig.(12). As far as the imaginary part of the potential is equal to zero the cross section is represented by $S$-matrix formalism rather than by the optical model.

\[
\begin{align*}
1p & \quad 1p_{1/2} \quad 1p_{3/2} \\
1s & \quad 1s_{1/2}
\end{align*}
\]

Figure 13: Splitting of the levels due to spin-orbital interaction.

It is interesting to note that the differential cross section at higher energies is insensitive to the spin-orbit potential (38) and it influences only polarization data. In the region of separated resonances such is not the case. Because of the spin-orbit interaction the energy levels split with respect to the total angular momentum, as shown in Fig.13.

![Cross section vs. energy graph](image)

Figure 14: Dependence of the distance between split resonances on the spin-orbital potential.

As a result the distance between split resonances strongly depends on the spin-orbit potential (see Fig.14).
7 R-matrix Theory

The R-matrix theory describes a nuclear reaction proceeding via formation of intermediate states of the compound nucleus in terms of some set of states that can be associated with those of the compound nucleus. If the wave-function and its derivative are known at the boundary of the nucleus it can be found everywhere outside the nucleus. The idea of the R-matrix approach is that the scattering matrix is expressed through R-matrix which is defined to connect values $\psi_l$ with its derivative at the nucleus boundary

$$\psi_l(a) = R_l a \left( \frac{d\psi_l}{dr} \right)_{r=a}.$$  \hfill (40)

Matrix elements $R_l$ are expressed as

$$R_l = \sum_{\lambda} \frac{\gamma_{l,\lambda}^2}{E_{\lambda} - E}.$$  \hfill (41)

where

$$\gamma_{l,\lambda} = \left( \frac{\hbar}{2ma} \right)^{1/2} \psi_{l,\lambda}(a).$$  \hfill (42)

Functions $\psi_{l,\lambda}$ correspond to actual states $E_{\lambda}$ of the nucleus. Quantities $\gamma_{l,\lambda}$, called reduced width amplitudes, are connected with the energy width $\Gamma$ of real states ($\gamma_{l,\lambda}^2 \sim \Gamma_{l,\lambda}$). It is shown in the theory that the cross-section can be written in terms of the R-matrix. The differential cross section for the scattering of charged particles is a sum of three terms which correspond to pure Coulomb scattering, resonance and reaction scattering and interference between Coulomb and resonance scattering. Potential scattering is taken into account through the so-called hard sphere scattering phase shift. The R-matrix theory is a formal one in the sense that it uses expansion of the wave function over eigenvalues regardless of the nature of the states.

Application of the R-matrix theory is quite simple in the case of a zero spin target nuclei, especially if the distances between compound nucleus levels are great enough and level interference effects are so negligible. The parameters of nucleus levels needed for the calculations can be taken from Ajzenberg-Selove’s compilation for A<20 (available online at www.tunl.duke.edu/nucldata/fas/fas_list.shtml) and from Endt and Van der Leun’s compilation for A=21-44 nuclei.

Though formulas appropriate for the analysis of proton scattering from non-zero spin targets have been presented in many papers the complexity
of the analysis in this case significantly increases. For a target of spin $I_t$ and a projectile of spin $I_p$ the two spins are coupled to form a channel spin $s$. This channel spin is then combined with the relative orbital momentum $l$ to form the spin of the compound nucleus state. For proton scattering there are two values of the entrance channel spin $s = I_t \pm 1/2$ and for elastic scattering both $s$ and $l$ mixing are possible. With such mixing the measured elastic scattering cross section includes reaction terms with $s \neq s'$ and $l \neq l'$ where primed values correspond to the exit channel. The spin of the channel remarkably influences the shape of a resonance curve, as shown in Fig.15.

A comprehensive review of $R$-matrix theory is presented by Lane and Thomas [13]. A practical description of the analysis of resonance excitation functions for proton elastic scattering is given in Ref.[14].

8 Deuteron Induced Reactions

Reactions due to deuterons possess some specific features. The deuteron consists of one proton and one neutron, its binding energy is 2.22 MeV, or about 1 MeV per nucleon, which is much less than 8 MeV, the mean binding energy of the nucleon in most nuclei. In addition, the mean distance at
which nucleons are spaced from each other in the deuteron composition is relatively long ($\sim 4$ fm). The particle possessing such properties proves to be able to interact with nuclei not only with the production of a compound nucleus but by direct interaction. If in the deuteron nucleus collision, a compound nucleus is produced with the capture of both nucleons, then its excitation energy appears to be very high due to the great difference in the binding energies of two nucleons in the nucleus and in the deuteron, i.e., about 14 MeV. Therefore, all the reactions due to deuterons ($d, p$), ($d,n$), ($d,\alpha$), are exoenergetic and have high yields. Apart from the production of the compound nucleus, the reactions ($d,p$) and ($d,n$) can proceed in some other way. Because of the weak binding of nucleons in the deuteron and of the relatively great distance between them, the deuteron-nucleus interaction may result in the absorption of only one nucleon, while the other nucleon will remain beyond the boundaries of the nucleus and continue its motion predominantly in the direction of the initial flight. If the kinetic energy of the deuteron is lower than the height of the potential barrier of the nucleus, then the yield of the ($d,p$)-reaction turns out to be comparable with the yield of the ($d,n$)-reaction for light and intermediate nuclei, and for heavy nuclei it is even several times higher than the latter. Such a behaviour of the yields of the ($d,p$)- ($d,n$)-reactions contradicts the compound nucleus mechanism because in the decay of the compound nucleus the emission of protons is always more difficult than that of neutrons, and especially in the case of heavy nuclei.

For the reactions induced by low energy deuterons at light nuclei it is assumed that the main contribution to the cross section of the process is given by the following three mechanisms: direct stripping when incident deuteron leaves one of its nucleons in the target nucleus, resonant mechanism and in some cases a compound nucleus statistical mechanism. It is accepted, that the complete amplitude $T$ of process is $T=D+R$, where $D$ is the amplitude of the direct process of stripping, which is calculated within the framework of a method of deformed waves, and $R$ is the amplitude of resonant process. The statistical compound mechanism contribution if any is incoherent and it may be simply added. Complete and partial width of formation and desintegration of resonances in the system, which are necessary in order to calculate the amplitude of $R$, are defined by fitting the model predictions to the available experimental cross sections of elastic deuteron scattering and ($d,p$)-reaction. The satisfactory description of the experimental data for $^{12}\text{C}(d,p_{0})^{13}\text{C}$ reaction is feasible (see Fig.16). However, for a reliable
description of a whole set of (d,p)-reaction data a development of the model in several directions is required.

Figure 16: Theoretical description of the \(^{12}\text{C}(d,p)^{13}\text{C}\) reaction cross section.

9 Conclusion

Nowadays low energy nuclear physics is a sufficiently studied field. Reaction mechanisms are known and appropriate models have been developed. However, satisfactory agreement between measured data and theoretical calculations, which is sufficient as a rule in order to support a model, does not provide a reliable base for cross section \textit{a priori} prediction. In addition nuclear reaction models use many adjustable parameters. Though some systematics and “global” sets of these parameters exist, fitting is always needed in order to represent a particular cross section. Moreover, in some important IBA cases reaction mechanisms are in general known but there is no code which provides necessary calculations. Although nuclear physics theory cannot provide sufficiently accurate cross section data when the calculations are based simply on first principals, theory does provide a powerful tool for data evaluation.
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References


