Neutrinos as Astrophysical Probes

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Abstract

These informal notes are an introduction to the topic of neutrino astronomy, and in particular to neutrinos from core collapse supernovae. They are prepared having in mind a curious reader, beginning to work in this multidisciplinary topic—that involves particle physics 'phenomenology', nuclear physics, experimental neutrino physics and astrophysics. After an introduction to the methods and goals of neutrinos astronomy, we concentrate on core collapse supernovae, as (one of) the most promising astrophysical source of neutrinos. The introduction is organized almost as a tale, the last part is a bit more technical. We discuss the impact of flavor oscillations on the supernova neutrino signal (=the change of perspective due to recent achievements) and consider one specific example of signal in detail. Three appendices corroborate the text with further details and some basics on flavor oscillations; but no attempt of a complete bibliographical survey is done (in practice, we include only those references that we know better, those that we believe can be useful for a ‘modern’ introduction to the subject, and suggest to the reader to use public databases for papers [1] and for experiments [2] to fill the gaps we leave).
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1 Neutrino astronomy, methods and goals

1.1 Main neutrino features

The electron, muon and tau neutrinos are by definition those neutral fermions coupled by weak CC interactions to the corresponding charged leptons. (A few important acronyms that we use: CC=charged currents, NC=neutral currents, CR=cosmic rays, SN=supernova). Thus, one knows that in the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ a muon neutrino is produced, and in the possibly subsequent decay $\mu^+ \rightarrow \bar{\nu}_\mu \nu_e e^+$ an electron neutrino and a muon antineutrino are formed.\footnote{These examples are important. Charged pions and other mesons are frequently formed by collisions of CR with Earth atmosphere and in several other cosmic sites. $\pi^0$ are also formed, and since $\pi^0 \rightarrow \gamma \gamma$, neutrino and gamma radiation are related at production. Another important case when neutrinos and photons are produced together is a nuclear reactions networks where beta and gamma radiation occur at the same time. $\gamma$'s are much more easily absorbed than neutrinos; they can be observed more easily, but for the same reason, their propagation can be more easily affected. In certain cases, neutrinos will be the most important signal (think for instance to neutrinos from big-bang nucleosynthesis, from the sun, or from a core collapse supernova). One can summarize saying that photons and neutrinos are often related, but they carry a different message. }\footnote{By contrast, protons and nuclei of CR radiation are deflected by galactic of $\sim$ few $\mu$G and extragalactic magnetic fields, at or below nG, and are not expected to point to their sources except perhaps at the very highest energies. Fast galactic neutrons instead are another interesting neutral probe.} The fact that these particles are neutral and stable makes them important astrophysical probes; they are expected to point in the direction of the astrophysical site of production, as in the more standard case of astronomy with photons.\footnote{Here we have in mind the case of ‘point astrophysical sources'; but, of course, ‘diffuse sources’ are also of importance.}

Due to their weak interaction cross section, neutrinos are almost invisible. However, they can sometime interact and carry away or deposit energy in terrestrial detectors. In other words, neutrinos can sometimes manifest themselves as missing energy phenomena; even better, one can use as neutrino tracker the visible particles originated by neutrinos. Most usual neutrino trackers are $e^\pm$, $\mu^\pm$, and $\gamma$ (a nucleus excited by a neutrinos can emit a $\gamma$ or a neutron; in certain conditions, the latter is absorbed by a nucleus, releasing visible $\gamma$'s later).

Some neutrino interactions are of special interest for the following discussion. First,

$$\nu_e \rightarrow \nu_e, \quad \nu_\mu \rightarrow \nu_\mu \quad [\text{CC and NC elastic scattering}]$$

In these reactions an $e$ at rest—say, from an atomic shell—is hit by the neutrino and acquires kinetic energy. An important feature is that the hit $e$ maintains the direction of the neutrino when $E_\nu \gg m_e$ (“directionality”). The cross section is low, $\sigma \sim G_F^2 m_e E_\nu$ ($G_F$ is the Fermi coupling). The (lowest energy) reactions with
neutrons are:
\[ \bar{\nu}_e p \rightarrow e^+ n, \quad \nu_e n \rightarrow ep \text{ [neutrino] absorption on nucleon – or inverse beta decay] } \]
\[ \bar{\nu}_e (A,Z) \rightarrow e^+ (A,Z-1), \nu_e (A,Z) \rightarrow e (A,Z+1) \text{ [neutrino] absorption on nucleus] } \]
(2)

these reactions are only slightly directional (more quantitative statements requires

care to details, see e.g. App. A). Their cross section is larger, \( \sigma \sim G_F^2 A^2 E^2_{\nu} \), when a

nucleus composed by \( A \) nucleons reacts as a whole (coherent scattering). At

higher energies this becomes \( \sim G_F^2 A \nu E_{\nu} \) (incoherent scattering). In the latter

case, the nucleus is broken, and/or hadronic resonances are excited. Usually, the

most interesting product is the charged lepton.

1.2 Concepts of neutrino telescopes

Let us describe some concepts of neutrino detector, to illustrate what people mean

by a ‘neutrino telescope’.3 (Supernova neutrino detectors fall in the first concept,

normally.)

* One can instrument a large volume, possibly vetoing for external particle, and wait

for a charged particle coming apparently from nowhere—in activity, created by a

neutrino interaction. (Better to be underground for low counting rates, like those

related to natural neutrino radiation.) Active volume can be a bubble chamber, a

scintillator, a Cherenkov radiator, a layered target. This method works from sub-

MeV to several GeV energies, because it is subject to the condition that the (main

part of the) event is contained in the detector. The number of events scales as

\[ N = \text{Number of targets} \times \sigma_\nu \times F_\nu \times \text{Time} \text{ (} F \text{ denotes generically a flux).} \]

In particular, the signal scales as the volume of the detector.

* One can set a muon counter and timing system underground (underwater or

under-ice), for muons that originate from neutrinos—as those coming from below.

Detectors are located underground to avoid CR \( \mu \). This is the oldest method, and

works since muons suffer of mild energy losses till \( \sim 500 \text{ GeV} \) (that corresponds

roughly to \( \text{Range}_\mu \sim 1 \text{ km in water} \). It applies from energies around a GeV till

several hundred TeV; then the Earth becomes opaque even to neutrinos (see e.g.[3]).

The number of events and the \( \nu \)-induced muon flux scale respectively as:

\[ N = \text{Surface} \times F_\mu \times \text{Time and } F_\mu = F_{\nu\mu} \times \sigma_{\nu\mu} \times N_A \times \text{Range}_{\nu}. \]

In particular, the signal scales as the area of the detector (actual target being the

Earth, the sea or the ice where the detector is located).

* By an extension of previous concept, one could use the Earth atmosphere as a

target for high energy neutrinos to produce muons (inclined air showers); or, use

mountains to convert almost horizontal \( \nu_\tau \) of very high energy into visible taus. In

this way, we could observe neutrinos of highest energies. The search of inclined air

showers is just a spin-off of extensive air shower arrays research activity. Till now,

however, no positive detection has been claimed.

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3Warnings: As it is common in physics, different concepts are blurred and useful at
best for first orientation; in present case, they depend on the type of particle, on the size
of the detector ...
In principle we would like to measure a lot of quantities: a) direction of the charged lepton; b) its energy; c) its charge; d) tag the flavor; e) tag the time of arrival; f) check occurrence of secondaries (n, γ, charged hadrons). In practice, one has to find a compromise between the various and contrasting needs of an experiment, e.g. between the wish to have a very 'granular' detector to see all the details of the reaction, and the need to monitor a big amount of matter.

1.3 Chances for neutrino astronomy

In short, the goal is to use neutrinos to probe astrophysical sources; the information from ν can be complementary to the one from γ. Some important possibilities in this connection are:

1) Solar neutrinos [0.1-20 MeV] There is little doubt that this is 'ν-astronomy'. Among the results of a very successful program of observations pioneered by Homestake we quote: a) low energy ν experiments Gallex/GNO and SAGE prove that the pp-chain (initiated by pp → D^1 + ν_e) is the main energy source; b) the physics of the center of the sun (ρ_e ~ 150 g/cc) is probed. There is consistency with the theory of solar oscillation eigen-modes (helioseismology). c) Neutrino oscillations of a type predicted in MSW [12] theory are indicated. Future observations will aim at the Beryllium line (Borexino, KAMLAND) and at real time pp-neutrino detection.

2) Atmospheric neutrinos [0.05-1000 GeV] CR come isotropically on Earth and they are not completely understood; they are not thought as astronomy, but they belong to astrophysics as much as to particle physics. The study of CR secondaries, like gamma’s, muons or atmospheric neutrinos, permits us to investigate CR spectra and their interactions with Earth atmosphere, which is not that different from possible sites of production of CR. Atmospheric neutrinos give a very significant indication of oscillations, especially thanks to Super-Kamiokande results. In the present context, atmospheric neutrinos will be thought just as an important background.

3) Neutrinos from cosmic sources [unknown energies] This is a vast field, and includes a large variety of approaches of observation and of objects; presumably, also unknown objects [4]. For instance, one can search for an excess of neutrino events over the expected background by selecting a solid angle—observation window—around a cosmic source (say, an active galactic nucleus) or an appropriate time window around a cosmic event (say, a gamma ray burst). Other possibilities are to search for self-trigger (excess of multiple ‘neutrino’ events), or coincidence with

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4We should recall the important role of some theorists: J.N. Bahcall, tightly connected with this experiment, and G.T. Zatsepin and V.A. Kuzmin, who advocated solar neutrino astronomy.

5This reconciles SNO observations (1/3 of expected ν_e) with those of low energy ν experiments (where the deficit is less than 1/2). KAMLAND experiment supports strongly this picture; more discussion later.

6MACRO, Soudan2 and K2K support these results. Again K2K, Minos, and CNGS long-baseline experiments will further test these results with man-made neutrino beams.
other neutrino- or with gravitational wave-detectors. The observation of point (or diffuse) sources is a very important goal: e.g., $\nu$ (and $\gamma$) astronomy above TeV can shed light on the problem of the origin of CR. Till now, several experiments like LSD, MACRO, LVD, Super-Kamiokande, Baksan, AMANDA, EAS-TOP, HiRES and other ones produced upper limits on the fluxes. In future, this type of search will be conducted by AUGER, ICECUBE, ANTARES. One of the main hopes is that the neutrino energy spectrum remains very hard till $\sim$100 TeV, as suggested by observed gamma spectra at 1-10 TeV (another one is that the prompt neutrino background—from charm—is not huge.)

(4) Supernova neutrinos [few-100 MeV] (this ‘cosmic’ source is singled out, since it is the topic of the rest of the talk). As recalled in next section, most of core collapse supernova energy is carried off by neutrinos of all flavors. About 20 events were detected in 1987 from such a supernova [5] located in the Large Magellanic Cloud (SN1987A); usually, all these events are attributed to inverse beta decay reaction, the one with the largest cross section (see App. A). The observations of Kamiokande II, IMB, Baksan (and perhaps Mont Blanc detector) begun extragalactic neutrino astronomy. The agreement with the expectations is reasonable. Many operating neutrino detectors like Super-Kamiokande, SNO, LVD, KamLAND, Baksan, AMANDA could be blessed by the next galactic supernova. Other detectors like ICARUS and Borexino will also be able to contribute to galactic supernovae monitoring in the future. This activity promises to have a big payoff in astro/physics currency: core collapse SN are a possible source of infrared, visible, X, and $\gamma$ radiation, and possibly of gravitational waves; they are of key importance for origin of galactic CR, galactic reprocessing of elements; etc. In the following, we focus only on supernova neutrinos.

1.4 Galactic, extragalactic, and relic supernovae

We close this introduction by classifying and discussing the possible observations of SN neutrinos. (Note that, unless said otherwise, the term supernova means always core collapse supernova in these notes, even though this is an abuse of notation—supernovae of type IA are very important in cosmology and astrophysics, and are not core collapse events).

The hope of existing neutrino telescopes is the explosion of a galactic supernova, for the simple fact that the $1/D^2$ scaling of the flux is severe. For reference, in water or scintillator detector one expects roughly

$$300 \, \nu\text{-events}/\text{kton for a distance } D = 10 \, \text{kpc},$$

when the galaxy has a radius of some 15 kpc, and we are located at 8.5 kpc from its center.\footnote{One could expect that the chances of getting a supernova where matter is more abundant are higher (the galactic center), but one can also object that younger matter, conducive to SN formation, lies elsewhere (in the spiral arms). However, we are unaware of the existence of a ‘catalog of explosive stars of our galaxy’, or of calculations of weighted matter distributions of our galaxy.} (As everybody should know, 1 pc is $3.1 \times 10^{18}$ cm). Various authors
estimated the rate of occurrence of core collapse supernovae; for our galaxy, this ranges from \( \sim 1/10 \) to \( \sim 1/(100 \text{ y}) \). A recent study [6] of the correlations of \( \sim 200 \) observed supernovae at cosmological distances with the blue luminosity of their host galaxy yields \( 1/(50-100 \text{ y}) \). A \( \sim 1/(10 \text{ y}) \) lower limit can be already established, since existing \( \nu \)-telescopes did not observe any event yet. Often, one recalls the possibility that SN events can take place in optically obscured regions of our galaxy; however, one should also remind that, beside \( \nu \), there are other manners to investigate the occurrence of such a phenomenon, e.g. from the released infrared radiation.

Curiously enough, galactic neutrino astronomy is still to begin, but extragalactic neutrino astronomy began several years ago with SN1987A. In principle, one should profit of the wealth of galaxies around us (say, those in the 'local group') to get events at human-scale pace. In practice, this is difficult, because core collapse SN takes place only in spiral or irregular galaxies, and not in elliptical ones.\(^9\) The only other large spiral galaxy of the local group is Andromeda (M31) but (1) its mass is presumably half of our galaxy, (2) its distance is about 700 kpc. A half-megaton detector (as the one suggested as a followup of Super-Kamiokande to continue proton decay search) should get 30 events if efficiency is unit. Perhaps, the best chance would be another SN from Large Magellanic cloud (an irregular galaxy) but the odds for such an event are not high.

Another interesting possibility is the search for relic supernovae, namely the neutrino radiation emitted from past supernovae. The practical method is to select an energy window around 20 – 40 MeV, where atmospheric or other neutrino background is small, searching for an accumulation of neutrino events there with more-or-less known distribution. The best limit has been obtained by the Super-Kamiokande [7], and the sensitivity is approaching the one requested to probe interesting theoretical models. In principle, one can suppress the main background (muons produced below the Cherenkov threshold) by identifying the neutron from inverse beta decay—see also App. A. This could be perhaps possible by loading the water with an appropriate nucleus, that should absorb the neutron and yield visible \( \gamma \) eventually.\(^{10}\)

### 2 Supernova neutrinos

In sects. 2.1 and 2.2 we present theoretical expectations on supernova neutrinos. More precisely, we describe the expected sequence of events of the `delayed scenario'

\(^8\)The main unknown comes from the fact that we ignore which is the type of the galaxy that hosts us; this implies a factor 2 of uncertainty.

\(^9\)Their stellar population is older, and star forming regions are absent or very rare; in a sense, the stars of 10-40 solar masses are a problem of youth.

\(^{10}\)Neutron identification by \( p + n \rightarrow D + \gamma (2.2 \text{ MeV}) \) was proved in scintillators (furthermore, no Cherenkov threshold impedes); however no existing or planned scintillator has a mass above 1 kton.
[8], see also [9]. In sect.2.3 we discuss generalities of SN neutrino oscillations. We provide the basic concepts and formulae, discuss the effects on the fluxes at a rather general level, and recall some important number. (The basic terminology and result are recalled in App. C, but a reader who does not know oscillations could conveniently consult review articles or texts before reading this section; more detailed, specific accounts are in [13].) Finally, we complete the discussion and show an application of the formalism in sect.2A, by considering the reaction $\nu_e$ Ar$\rightarrow$K $e^-$ as a signal of supernova neutrinos.

However, the reader should be warned; it turned out to be difficult (perhaps impossible) to simulate a SN explosion. This could be due to a very complex dynamics; or, it could indicate that some ingredient is missing (such as an essential role of rotation, of magnetic fields, etc); or that there is nothing like a 'standard explosion'; or, worse, a combination of previous possibilities. In short, we have not a "standard SN model" yet, and this makes supernova neutrinos even more interesting.

2.1 Gravitational collapse and the 'delayed scenario'

Usually, the life of a star is characterized by a quasi-equilibrium state between gravity and nuclear forces. However, the dramatic conclusion the brief-some million years-life of a very massive star of $\sim 10 - 40 ~M_\odot$ is something very different, a core collapse supernova.

Stellar evolution forms an iron-core, inert to nuclear reactions. This is supported by degeneracy pressure of (quasi)free electrons, but when it reaches the Chandrasekhar mass of $\sim 1.4M_\odot$ (radius $\sim 8 \times 10^9$ km) it collapses under its weight.\footnote{The gravitational pressure is $P_g \sim G_N M^2/R^4$. The $e^-$ pressure is $P_e \sim u/v$ where $v$ is the specific volume, and the internal energy $u$ is $u_{\text{Fermi}}$ or $p^2_e/(2m_e)$ depending on whether electrons are relativistic or not ($p_{\text{Fermi}}$=Fermi momentum); thus, $P_e \sim \hbar c n_e^{5/3}$ or $P_e \sim \hbar^2/(2m_e n_e)^{5/3}$. Since $n_e \sim M/(R^3 m_n)$, non-relativistic $e^-$ lead to the scaling $P_e \sim 1/R^5$ and an equilibrium can be reached; for relativistic ones $P_e \sim 1/R^3$, and equilibrium is impossible after the core reaches the Chandrasekhar mass of $M \sim (\hbar c G_N)^{3/2}/m_n^2$.} When the so called inner-core reaches nuclear densities, a shock-wave forms. Then, this enters a phase of stall trying to make its way through the outer core of $\sim 0.6M_\odot$—"accretion". During this phase, convective motions and neutrinos should revive the shock (that subsequently will eject outer star's layers). The inner core settles in a new quasi-equilibrium state called protoneutron star, that cools and contracts radiating neutrinos of all types—"cooling". Eventually this leads to the formation of a neutron star, occasionally seen as a pulsar. Its mass is $M_{\text{ns}} = 1 - 2M_\odot$, and its radius scales roughly as $R_{\text{ns}} \approx 17$ km $(1.85M_\odot/M_{\text{ns}})^{1/3}$, due to degenerate character of the equation of state. The most important feature to note is that the gravitational binding energy released is huge, roughly (3/5 is for a uniform density distribution)

$$\mathcal{E}_B \simeq G_N \frac{3M_{\text{ns}}^2}{5R_{\text{ns}}} \approx (1 - 5) \times 10^{53} \text{ erg}$$
<table>
<thead>
<tr>
<th>Conventional name</th>
<th>Description</th>
<th>Duration</th>
<th>% of $\mathcal{E}_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>infall (early neutronization) [only $\nu_e$]</td>
<td>Collapse begins, $ep \to \nu_e n$, $\nu$-trapping increases</td>
<td>$\sim 100$ millisecond</td>
<td>less than 1%</td>
</tr>
<tr>
<td>flash [only $\nu_e$]</td>
<td>Bounce. Flash obtains when the $\nu$-sphere is reached</td>
<td>few msec $[t = 0]$</td>
<td>$\sim 1%$</td>
</tr>
<tr>
<td>accretion [$\nu_e, \bar{\nu}<em>e, \nu</em>\tau, \bar{\nu}_\tau$?]</td>
<td>Shock stalls, then resumes (how?) $e^+ e^- \to \nu_\tau \bar{\nu}_\tau$ begins</td>
<td>fraction of a second</td>
<td>$\sim 10 - 20%$</td>
</tr>
<tr>
<td>cooling [all $\nu$ types]</td>
<td>Proto NS cools and contracts emitting $\nu$'s</td>
<td>till 10-100 seconds</td>
<td>$\sim 80 - 90%$</td>
</tr>
</tbody>
</table>

that is about $\sim 10\%$ of the rest mass energy $M_{ns} c^2$. This is much bigger than the kinetic energy of the ejecta $E_{\text{kin}} \sim 10^{51}$ ergs $\approx 1\% \, \mathcal{E}_B$ (a typical velocity of the shock wave is 4-5000 km/s). Also much bigger than what is needed to dissociate the outer iron core $0.6M_\odot / m_n \times (2.2 \; \text{MeV}) = 2 \times 10^{51}$ erg since the mass of $^{56}$Fe is 123 MeV smaller than $13\, m_n + 4\, m_n$—but this could be optimistic, and the energy losses suffered by the shock wave even larger. The energy that goes in photons is very small, $E_{\text{phm}} \approx 10^{49}$ erg $\approx 0.01\% \, \mathcal{E}_B$ (sufficient to outshine host galaxy though!), and the gravitational wave part is unknown (and depends on the detailed dynamics of the collapse) but it is probably even less.\footnote{A naive guess is $G_\odot (Mv^2/2)^2/R \sim \mathcal{E}_B \beta^4$; it means some billionth of $\mathcal{E}_B$ with $v \sim 4000$ km/s.} The overwhelming part of this huge energy is carried away by neutrinos. The neutrino luminosity can be roughly estimated noting that $\sim 10^{53}$ erg are emitted in a few seconds in the cooling timescale, and thus $L_\nu \approx 3 \times 10^{19} L_\odot$: the supernova neutrino burst outshines the entire visible universe. (Incidentally, we feel there is something poetic in these quasi-spherical SN neutrino shells that propagate freely in the Universe).

### 2.2 Neutrino fluxes

Here, we describe in some detail the neutrino fluxes. First we discuss the general characteristics and present a phenomenological survey, and then we discuss how their luminosity, energy spectrum, and possible non-thermal effects can be parameterized. We ought to recall the 3 relevant types of neutrino fluxes:

\[ \nu_e, \bar{\nu}_e \text{ and } \nu_x \]

where $x$ is anyone among muon and tau (anti)neutrinos ($\nu_\mu$ and $\bar{\nu}_\mu$ have similar properties, $\nu_\mu$ and $\nu_\tau$ are produced by NC in the same manner, and probably, muons are present only in the innermost core; thus $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$ and $\bar{\nu}_\tau$ should have a very similar distribution).

Let us begin by describing the general properties of the neutrino fluxes. In delayed scenario the collapse has four main phases. Correspondingly, we distinguish between two early neutrino emissions, named “infall” (or early neutronization) and “flash” and the two late phases of “accretion” and “cooling” (or thermal phase): see the table given above. The most uncertain phase is certainly the one of “accre-
tion”, that, together with “cooling”, accounts for most of the energetics. Perhaps, one could argue that a fair estimation of errors should be just 100%. In support of this (apparently too conservative) statement, we recall that we have not \textit{ab initio} calculations of these fluxes, and alternative scenarios have been proposed. Furthermore, the calculations that tried to estimate the effect of rotation (by Imshennik and collaborators, and more recently by Fryer and Heger [10]) found very different fluxes, and in particular a severe suppression of muon and tau neutrinos.

Reference ranges on average neutrino energies found in a number of numerical calculations are:

\begin{align*}
\langle E_{\nu_e} \rangle &= 10 - 12 \text{ MeV}, \\
\langle E_{\nu_x} \rangle &= 12 - 17 \text{ MeV}, \\
\langle E_{\nu_{\mu}} \rangle &= 18 - 27 \text{ MeV},
\end{align*}

(The reason of this hierarchy is that neutrinos that interact more decouple in more external regions of the star.) The approximate amount of the total energy $E_B$ carried away by the specific flavor is

$\begin{align*}
E_{\nu_x} &= 17 - 22 \% E_B, \\
E_{\nu_e} &= 17 - 28 \% E_B, \\
E_{\nu_{\mu}} &= 16 - 12 \% E_B,
\end{align*}$

The approximate equality found in numerical calculations has been called ‘equipartition’, but in our understanding, there is no profound reason behind this result. We recall again that these numbers should be regarded with caution.

Next, we would like to introduce a general formalism to describe \textit{parameterized neutrino fluxes}. Such a description requires (a) to know the distance of production $D$, (b) the distribution over the solid angle (expected to be isotropic, up to $\sim 1 - 10$ \% corrections at most) (c) to assign a luminosity function $d\mathcal{E}_i/dt$ (neutrinos per unit time), (d) to describe the energy spectrum, presumably of Fermi-Dirac type:

\begin{equation}
\frac{n(x, \eta)}{[1 + \exp(x - \eta)]} = \frac{x^2}{1 + \exp(x - \eta)}, \quad \text{with } x = E/T_i
\end{equation}

where $T_i$ is the temperature. Possible non-thermal effects are often described by introducing an effective chemical potential parameter $\eta_i$ (not subject to the condition $\eta = -\eta$)\textsuperscript{12}--the pinching factor\textsuperscript{13}--that modifies the shape of the distribution. (However, it is not excluded that the true non-thermal effects are even more dramatic, and the high energy tail of the spectrum is cutoff as $e^{-x(E/E_0)^2}$.) In formulae, the neutrino flux $F^0_i$ is:

\begin{equation}
F^0_i \equiv \frac{dN^0_i}{d\Omega dt dE} = \frac{1}{4\pi D^2} \frac{d\mathcal{E}_i}{dt} \frac{n(E/T_i, \eta_i)}{F^0_3(\eta_i) T_i^2} \quad i = e, \bar{\nu}_e \text{ or } x
\end{equation}

\textsuperscript{12}This name arises since, for fixed average energy $\langle E \rangle$, a value $\eta > 0$ leads to a distribution suppressed at low and high energies. The reason [9] why this happens at high energy is simply that hotter neutrinos are in contact with cooler regions than average neutrinos. A typical cross section that increases fast with energy and has a large threshold is $\nu^\nu_\mu^\nu + ^{12}\text{C} \rightarrow ^{12}\text{N}e^-$: changing $\eta$ from zero to 2 decreases by 20 \% the event number, if $\langle E \rangle = 23$ MeV.
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... (the index 0 recalls us that we do not take into account oscillations during propagation, \( a \) is the area transverse to the flux, and the meaning of \( F_3 \) is recalled in App. B). The condition of normalization is that \( \int dE \ E \ dN^0/\ dtdE = d\xi_i/\ dt \). Therefore, one has to calculate (or to reconstruct experimentally) 3 functions of the time for each type of neutrino, which is a difficult task.

For this reason, or just to get a 'synthetic' description, it is common use to introduce the time integrated fluxes, that are parameterized in a very similar manner, namely by (1) an effective temperature \( T_i \), (2) an energy fraction parameter \( f_i \)

\[
\int dt \frac{d\xi_i}{dt} = (1 - \varepsilon) f_i \xi_B
\]

(here, we singled out the energy fraction \( \varepsilon \xi_B \) that goes in the \( \nu_e \) ‘flash’, and fractioned the rest by \( f \) and finally by (3) a parameter \( \eta \), for non-thermal effects. We would like to argue that a minimal set of parameters, beside to \( \xi_B \) and \( \varepsilon \), should be the following:

\[
T_\varepsilon, \kappa = T_\varepsilon/T_B, f \text{ and } \eta
\]

where

(i) \( T_\varepsilon \) = the temperature of antineutrinos (presumably, easy to observe);
(ii) \( \kappa \) = increase in temperature of \( \mu/\tau \) (anti)\( \nu \) (oscillations, and NC events imply \( \kappa \));
(iii) \( f = f_\varepsilon = f_\bar{\nu} \) = the fraction of electron (anti)\( \nu \) (presumably \( f = 1/4 - 1/6 \), which constrains \( f_\varepsilon = (1 - 2f)/4 \) (the case \( f = 1/6 \) represents 'equipartition'); and finally
(iv) an effective pinching parameter \( \eta \geq 0 \), equal for all types of neutrinos (this is not expected to be an accurate description, but could be adequate in practice).

Usually, \( T_\varepsilon \) is not a very important parameter to describe the neutrino signal, simply because this is the lowest temperature, however this can be estimated by a 'reasonable' condition on the emitted lepton number \( N_\ell^0 - N_\ell^0 \), and the parameters of eq.7:

\[
T_\varepsilon = T_\varepsilon/\left[ 1 + (N_\ell^0 - N_\ell^0)(T_\varepsilon F_3(\eta)/F_\ell(\eta)) / (f \xi_B) \right]
\]

At a further level of refinement, we introduce time dependent features, and distinguish between 'cooling' and 'accretion' neutrinos. E.g., we have a cooling component whose luminosity \( d\xi_i/\ dt \) scales as \( T_i^3 \), and whose temperature obeys a time law as:

\[
T_i(t) = T_i(0)/(1 + t/\tau)
\]

(the constant \( \tau \sim 10 - 100 \) sec has to be extracted from the data or computed). On top of that, we add for \( t < \tau_a \) another rather luminous phase, presumably with a marked non-thermal behavior (\( \eta \neq 0 \)) and with its own effective temperature. Since the efficiency of energy transfer to matter is not large, (anti)\( \nu_e \) should carry a sizable fraction of energy (\( \nu_a \) are of little use to revive the shock, but perhaps, only few of them are produced in this phase).

It is possible to introduce a rough model (see e.g. [11]) for these phases, us-
ing effective “neutrino radiation” spheres\textsuperscript{14} to describe black-body emission in the cooling phase. Similarly, one can model the accretion phase by suggesting that the non-thermal neutrino production is from $e^\pm$ interactions with the accreting matter. This suggests that the fluxes are proportional to the cross sections: Thus, their scaling should be more similar to $E^4_{\nu}$ than to $E^2_{\nu}$. It is rather interesting that there is some hint of such a luminous phase already from SN1987A neutrino signal, see again [11]; in our view, this indication is encouraging for theory and for future observations, even if this is not supposed to convince skeptics.

2.3 Effects of neutrino oscillations

Let us begin by describing the effect of neutrino oscillations in the star. Oscillations do not affect neutral current events, if we postulate to have only 3 types of neutrinos. In fact, the flux $F^0_\nu + F^0_\mu + F^0_\tau$ is not changed by reshuffling the fluxes (‘NC are flavor blind’). Oscillations modify only CC events. To describe this phenomenon, we need just 2 functions $P_{\nu e}$ and $P_{\nu \mu}$, the electron neutrino/antineutrino survival probabilities (since the $\mu$ and $\tau$ flux are supposed to be identical\textsuperscript{15}). In order to calculate $P_{\nu e}$, one has to solve the evolution equation described by the effective hamiltonian (similarly for $\overline{\nu}_e$):

$$H_{\text{eff}} = 2.534 \cdot U \frac{\text{diag}(m^2)}{E^\nu} U^\dagger + 3.868 \cdot 10^{-7} \rho Y_e \cdot \text{diag}(1,0,0)$$

(8)

where $\nu$ masses $m_\nu$ are in eV and energy is in MeV (see App. C). The supernova density $\rho$ (in g/cc) and the fraction of electrons $Y_e = N_e/(N_\mu + N_\tau)$ must be taken from some pre-supernova model, including the modifications due to the shock wave when needed. For orientation, a pre-supernova mantle density $\rho \sim 4 \cdot 10^3 (r_0/r)^3$ g/cc with $r_0 = 10^4$ km and $Y_e \sim 1/2$ can be used. At the center of the star, the second term of eq.8 (‘matter’ term) dominates: Thus, $\nu_e$ coincides with the local mass eigenstates when $\nu_e$ are produced. When they exit from the star, instead, the first term (‘vacuum’ term) dominates. What happens in between? Depending on the size of the unknown mixing angle $\theta_{13}$ ($U_{e3} = \sin \theta_{13}$) and on $N_e$ distribution, it can happen that neutrinos remain always local mass eigenstates $\nu_e = \nu_3^m(t)$ exiting the star as $\nu_3$ (see fig. 1). In formulae:

$$P_{\nu e} = |\langle \nu_e | \nu_e(t) \rangle|^2 = \begin{cases} U_{e3}^2 \sim 0 & \text{if } \theta_{13} > 1^\circ \text{ adiabatic conversion of } \nu_e \rightarrow \nu_3 \\ U_{e2}^2 \sim 0.3 & \text{if } \theta_{13} < 0.1^\circ \text{ adiabatic conversion of } \nu_e \rightarrow \nu_2 \end{cases}$$

(9)

Solar neutrino mixing makes almost certainly adiabatic the second conversion, if the first should fail. Similarly, $P_{\overline{\nu}e} = |\langle \overline{\nu}_e | \overline{\nu}_e(t) \rangle|^2 = U_{e1}^2 \sim 0.7$. Thus, the flux of $\nu_e$

\textsuperscript{14}Even if, one could believe that expected deviation from spherical symmetry are large, especially for early phases, and for deep layers of the collapsing star.

\textsuperscript{15}Indeed, we see a $\nu_e$ if it stays the same or if $\nu_\mu$ or $\nu_\tau$ oscillate into $\nu_e$: $F_e = P_{\nu e} F^0_\nu + P_{\nu e} F^0_\mu + P_{\nu e} F^0_\tau$. Rewriting $F_{\nu e} = P_{\nu e} F^0_e + (P_{\nu e} + P_{\nu e}) F^0_\nu$ and recalling that $1 = P_{\nu e} + P_{\nu e} + P_{\nu e}$, we conclude the proof.
Figure 1: These plots are the energy levels of neutrinos and antineutrinos inside the star; innermost regions to the right. As visible from the leftmost regions, we assume that the neutrino spectrum (the levels of the vacuum hamiltonian) obey a 'normal' mass hierarchy (we do not emphasize another possibility compatible with the data, 'inverse' mass hierarchy; common mass scale is immaterial for oscillation however).

becomes:

\[ F_\nu = \begin{cases} 
F_\nu^0 & \text{if } \theta_{13} > 1^\circ \\
0.3F_\nu^0 + 0.7F_\nu^0 & \text{if } \theta_{13} < 0.1^\circ 
\end{cases} \quad (10) \]

(intermediate cases are also possible) while \( F_\bar{\nu} = 0.7F_\nu^0 + 0.3F_\nu^0 \). Since we expect that \( F_\nu^0 \neq F_\nu^0 \) and \( F_\nu^0 \neq F_\nu^0 \), oscillations should modify the expected supernova neutrinos fluxes. These modifications are large (e.g. the flash yields little in CC; NC events range from 70 to 100 %) and can be observable, but the message that we want to stress here is simply that these effects should be taken into account in order to interpret the SN neutrino signal correctly.

Now, we consider the 'earth matter effect' and show that with current parameter, it is not expected to be very large. As we saw, in a possible scenario (=normal mass hierarchy, very small \( U_{e3} \)) neutrinos exit from the star as \( |\nu_e\rangle \rightarrow |\nu_2\rangle \) and \( |\bar{\nu}_e\rangle \rightarrow |\bar{\nu}_1\rangle \) due to the MSW effect [12]; in other words,

\[ P_{ee} = \sin^2 \theta \sim 0.3 \text{ and } P_{\bar{e}e} = \cos^2 \theta \sim 0.7 \]

If (anti)neutrinos cross the Earth before hitting the detector, new oscillations will occur (since vacuum eigenstates are not eigenstates in the Earth matter) and previous expressions will be modified. For constant density (say, Earth mantle) the solution of a two-flavor version of eq.8 gives:

\[ P_{ee} = \sin^2 \theta_{12} \left[ 1 + \frac{4 \varepsilon \cos^2 \theta_{12}}{(1+\varepsilon)^2 - 4 \varepsilon \cos^2 \theta_{12}} \cdot \sin^2 \left( \frac{\Delta m^2 L}{4E} \sqrt{(1+\varepsilon)^2 - 4 \varepsilon \cos^2 \theta_{12}} \right) \right] \quad (11) \]
with $\theta_{12} \approx 34^\circ$ and $\Delta m^2 \approx 7 \cdot 10^{-5}$ eV$^2$, where

$$\varepsilon = \frac{\sqrt{2} \; G_F \; N_{e\beta}}{\Delta m^2 / 2E} \approx 9 \cdot 10^{-3} \frac{N_{e\beta}}{E/(20 \text{ MeV})} \frac{(2 \text{ gr/cc}) \cdot E/(7 \cdot 10^{-5} \text{ eV}^2)}{\Delta m^2}
$$

For antineutrinos, just replace $\theta_{12} \rightarrow 90^\circ - \theta_{12}$. Earth matter effect is larger than for solar neutrinos, simply because supernova energies are larger (see eq.8). This can give rise to spectacular wiggles, especially if large energies events are seen. If it can be proved experimentally that Earth matter effect does (not) occur to neutrinos, this will result into an upper (lower) bound on $U_{e3}$; however, numerical considerations based on previous formulae suggest that this investigation will be demanding. If (or when) the position of the supernova will be known, it will be possible to include such an effect, reducing ambiguities in the interpretation of the signal.

### 2.4 An application: $\nu_e$ absorption on Argon

In order to complete the discussion and to show an application of the formalism, we will consider in detail the specific supernova neutrino signal provided by the reaction of absorption

$$\nu_e + \text{Ar} \rightarrow K + e^-,$$

that has a large cross section. This signal could be seen by the forthcoming detector ICARUS, or a similar detector based on the liquid argon technology.\(^\text{16}\) (We are not going to discuss the more difficult and important question of ‘what we can learn from supernova neutrinos’; whose answer will of course depend on which neutrino detectors will be working when next galactic supernova will explode, and what will be the distance of this supernova; but it is almost from granted that we will learn a lot from the $\nu_e + p \rightarrow n^+e^+$ reaction for the reasons recalled in App.A)

The number of events in a detector of 3 ktton is about 400, for a supernova exploding at $D = 10$ kpc. To calculate this number, one simply multiplies the flux (including oscillations) by the number of target nuclei and by the cross section of

\(^{16}\)Other detectors can see the $\nu_e$ signal with other reactions, even if usually this is not the main signal. For instance: (1) $\nu_e + ^{16}\text{O} \rightarrow e + ^{16}\text{F}$ (with $Q = 15.4$ MeV) can be exploited at water Čerenkov detectors as SuperKamiokande or SNO due to the angular distribution ($^\text{16}\text{F}$ rapidly decays by proton emission); (2) $\nu_e + ^{12}\text{C} \rightarrow e + ^{12}\text{B}$ (with $Q = 17.4$ MeV) can be seen in scintillators detectors (LVD, Borexino, KamLAND, BAKSAN), with the great advantage of offering a double tag, due to the $\beta^+$ decay of Boron; (3) $\nu_e + D \rightarrow e + p + p$ (with $Q = 1.4$ MeV, and a large cross section) can be used at the inner part of SNO (the signal is given by a lone electron, in contrast with neutral current, or electron antineutrino, reactions on deuterium that are tagged by additional neutrons). In a sense, the detection of $\nu_e$ requires to stay closer to the philosophy of solar neutrinos, and to employ literally solar detectors. Note that a big $Q$ value amplifies the difference between the case with and without oscillations.
Neutrinos as Astrophysical Probes

the reaction [15], and then integrates over the possible neutrino energies. (We use the formalism of ‘time integrated’ fluxes here).

The inputs are: (a) a neutrino temperature of \( T_e = 5.1 \text{ MeV} \), (b) a spectrum without pinching (\( \eta = 0 \)), (c) an exact equipartition of the fluxes (\( f = 1/6 \)), (d) a normal hierarchy of neutrino masses (e) \( U_{e3} \) large enough to produce \( P_{ee} \sim 0 \)—that is, \( \nu_e \rightarrow \nu_3 \). Following the indications of the most recent calculations [9], we assume that in absence of oscillations the temperature of electron neutrinos is \( T_e = 4.1 \text{ MeV} \), that is closer to \( T_e \) than though in the past.

How this number changes, with reasonable changes of the input parameters? To answer this question, we can calculate the percentage variation \( 100 \times \delta N/N \) of the number of absorbed \( \nu_e \) under a number of alternative hypotheses:

\[
\begin{array}{|c|c|c|c|c|}
\hline
& T + \Delta T & T - \Delta T & f \rightarrow 1/8 & \eta \rightarrow 2 & P_{ee} \rightarrow 0.3 \\
\hline
\text{+51\%} & \text{-45\%} & \text{-25\%} & \text{+15\%} & \text{-16\%} \\
\hline
\end{array}
\]

The first two columns show the effect of changing the temperature by \( \Delta T = \pm 1.3 \text{ MeV} \); the third column is the effect of having a pinched (‘non-thermal’) spectrum; the fourth column, describes the effect of non-equipartitioned fluxes; the last column, assumes that \( \nu_e \rightarrow \nu_2 \) due to very small \( U_{e3} \). This shows that the present uncertainty in the temperature has a big impact on the expected signal, about 50\%. It shows also that a mixture of various phenomena can affect the flux at the \( \sim 20\% \) level. To separate these effects clearly, it will be important to study several properties of the neutrino signal, like distributions in time and energy, and use several reactions.

3 Summary and discussion

In this introduction to \( \nu \)-astronomy, we focused mostly on supernova neutrinos. Our aim was to help the orientation of a reader in this field, so we did not give a comprehensive study (i.e. we did not consider all theoretical possibilities or scenarios, or reactions to detect neutrinos. Rather, we offered a selection of the background information, provided some basic formula, and showed illustrative calculations. Let us conclude by recalling the main points discussed:

\( \star \) Neutrino astronomy is theoretically appealing and rich of promises. Supernova neutrinos are a very well defined and interesting possibility.

\( \star \) Neutrino observations from SN1987A are not in contradiction with the general theoretical picture. However, supernova explosions are still mysterious, and this warrants more discussion and stimulates more efforts.

\( \star \) Next galactic supernova will permit us much more precise observations, and this will be certainly very helpful to progress. In particular, we would like to have detectors as stable as possible, as big as possible, and sensitive to various types of neutrinos.

\( \star \) The effects of oscillations are important. One could combine experiments and information in order to make cleaner inferences, but astrophysical uncertainties
should be thought as an essential systematics for this purpose. (In other words, there are chances to learn something on neutrinos, but, in our view, the primary aim of these observations is just supernova astrophysics.)

* All this is fine; the most important task left is an exercise of patience, 0–100 years for next galactic supernova.

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A An example of cross section

The ‘inverse beta decay’ reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ is particularly important for actual neutrino detection. Indeed, it has a large cross section, and water Cherenkov and scintillator based detectors have a large amount of free protons. For illustration, we recall here a simple approximation of this reaction, and make reference to [14] for a more complete discussion.

The tree level cross section in terms of Mandelstam invariants $s, t, u$ is:

$$\frac{d\sigma}{dt} = \frac{G_F^2 \cos^2 \theta_W}{2\pi(s - m_P^2)}[A(t) - B(t)(s - u) + C(t)(s - u)^2]$$  \hspace{1cm} (A.1)

$A, B, C$ are well approximated as $c_1 + t \cdot c_2$ at the energy of supernova neutrino detection:

$$A \approx M^2(1 - g^2)(t - m_P^2) - M^2\Delta^2(1 + g^2) - 2m_P^2M\Delta g(1 + \xi)$$

$$B \approx g(1 + \xi)t, \hspace{1cm} C \approx (1 + g^2)/4$$  \hspace{1cm} (A.2)

where $M = (m_n + m_p)/2, \Delta = m_n - m_p, \xi = 3.706$ and $g = -1.270 \pm 0.003$. Eq. A.1 is related by Jacobians to the cross sections differential in the lepton energy $E_\nu$, or in the angle $\theta = 0 - \pi$ between the incoming neutrino and the charged positron:

$$\frac{d\sigma}{dE_\nu} = 2m_P \frac{d\sigma}{dt} \quad \text{and} \quad \frac{d\sigma}{d\cos \theta} = \frac{eP_\nu}{1 + \epsilon(1 - E_\nu/E_\nu \cos \theta)} \frac{d\sigma}{dE_\nu}$$  \hspace{1cm} (A.3)

Of course, for the first formula one has to express the Mandelstam variables in terms of $E_\nu$ and $E_\nu$, e.g. $t = m_n^2 - m_P^2 - 2m_P(E_\nu - E_\nu)$. To evaluate the second formula, one first defines $\epsilon = E_\nu/m_P$ and calculates the positron energy $E_\nu$ (and momentum $p_\nu$) from $E_\nu = [(E_\nu - \delta)(1 + \epsilon) + \epsilon \cos \theta((E_\nu - \delta)^2 - m_P^2k^2)^{1/2}]/k$, where $\delta = (m_n^2 - m_P^2 - m_P^2)/(2m_P)$, and $k = (1 + \epsilon)^2 - (\epsilon \cos \theta)^2$. Note that $E_\nu$ is one-to-one with $E_\nu$ at zeroth order in $\epsilon$. The threshold of the reaction is at $E_\nu > 1.806$ MeV.

B Fermi integrals and polylog

The Fermi integral is the function:

$$F_n(\eta) = \int_0^\infty \frac{x^n}{1 + e^x-\eta} \, dx = -P(n + 1, -e^\eta) \cdot n!$$  \hspace{1cm} (B.1)

where $P$ is the polylogarithm function. This is useful to express energy momenta:

$$\langle E \rangle = T \cdot \frac{F_3(\eta)}{F_2(\eta)}, \hspace{1cm} \langle E^2 \rangle = T^2 \cdot \frac{F_4(\eta)}{F_2(\eta)}, \hspace{1cm} \langle \delta E \rangle = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$$  \hspace{1cm} (B.2)

The series expansion $P(n, z) = \sum_{m=1}^\infty z^m/m^n$ leads to some identities:

$$P(0, z) = z/(1 - z); \hspace{1cm} P(1, z) = -\log(1 - z); \hspace{1cm} P(n, 1, z) = \int_0^z P(n, t) \frac{dt}{t}$$  \hspace{1cm} (B.3)
At $z = \pm 1$, the polylogarithm can be expressed by the $Z$-function:

$$P(n, 1) = Z(n), \quad P(n, -1) = -(1 - 2^{-n})Z(n)$$

$(Z(2, 3, 4, 5) = 1.645, 1.202, 1.082, 1.037)$ (connection of Bose and Fermi integrals) (B.4)

## C A reminder on neutrino masses and oscillations

(1) The mixing matrix $U$ (introduced by Sakata and collaborators–Nagoya group–in 1962) connects neutrino fields of given flavor and of given mass:

$$\nu_i(x) = U_{\ell i} \nu_\ell(x), \quad \text{where } \ell = e, \mu, \tau \text{ and } i = 1, 2, 3$$

(C.1)

This implies a relation between states of ultrarelativistic neutrinos. In fact, the decomposition in oscillators $\nu(x) = \sum_{\rho\lambda}(b_{\rho\lambda}^\dagger \nu_\lambda e^{i\rho t} + b_{\rho\lambda} \nu_\lambda e^{-i\rho t})$ implies $b_{\ell i}^\dagger = U_{\ell i} b_i^\dagger$ and $a_{\ell i}^\dagger = U_{\ell i}^* a_i^\dagger$, so:

$$|\nu_{\ell i}| = U_{\ell i}^* |\nu_i|$$

$$|\bar{\nu}_{\ell i}| = U_{\ell i} |\bar{\nu}_i|$$

(C.2)

No changes if the type of mass is Dirac instead than Majorana (last one being theorist’s favorite).

(2) From previous considerations, it follows that if we produce a state of flavor $\ell$ at $t = 0$ it will acquire overlap with other states at later time (“appearance” of a new flavor) and at the same time it will loose overlap with itself (“disappearance”). This was shown by B. Pontecorvo in 1967, though, the first idea dates back to 1957. Thus, a state with momentum $p$ becomes

$$|\nu_{\ell i}(t)| = U_{\ell i}^* |\nu_i(0)|$$

(C.3)

The energy of neutrinos with different masses cannot remain the same in the course of the propagation, since $E_i \approx p + m_i^2/(2p)$ (ultrarelativistic approximation always applies to the cases of interest). When the distance between production and detection satisfies $L \approx t \gg E/\Delta m_{ij}^2$, $|\nu_{\ell i}(t)|$ becomes different from $|\nu_{\ell i}(0)|$, if the mixings $U_{\ell i}$ are large enough. As usual, $\Delta m_{ij}^2 = m_j^2 - m_i^2$.

(3) The effective hamiltonian of propagation in matter contains $\pm \sqrt{2}G_F N_e \text{diag}(1, 0, 0)$ ($N_e = \rho Y_e/m_n = e^-$ number density; $+$ is for $\nu_e$, $-$ for $\bar{\nu}_e$). This term is linear in the Fermi coupling, and describes a coherent interaction of neutrinos with the matter where it propagates. This can drive neutrinos to be “local” mass eigenstates during the propagation, thus exiting from a star as vacuum eigenstates, e.g.: $|\nu_e \rangle \rightarrow |\nu_2 \rangle$.

In the sun, this effect (named after MSW after Wolfenstein, Mikheyev and Smirnov [12]) is partial, and it is pronounced for highest energy neutrino events, e.g. those at SNO; for the supernova, it can be complete.

(4) Putting aside the indications of LSND indication (that will be tested at MiniBooNE) we know from a number of experiments that the usual 3 neutrino flavors
most likely oscillate among them, and this points to the following (roughly 1 sigma) ranges of the parameters:

\[
\begin{align*}
\Delta m_{12}^2 &= 7.1 \pm 0.7 \cdot 10^{-5} \text{ eV}^2 & \theta_{12} &= 34^\circ \pm 3^\circ \\
\Delta m_{23}^2 &= 2.2 \pm 0.5 \cdot 10^{-3} \text{ eV}^2 & \theta_{23} &= 45^\circ \pm 7^\circ & \theta_{13} < 10^\circ
\end{align*}
\]  

(C.4)

The 3 mixing angles given above parameterize the unitary mixing matrix \( U_{\alpha i} \):

\[
|U_{e3}| = \sin \theta_{13}, \quad |U_{e2}/U_{\alpha 1}| = \tan \theta_{12} \text{ (solar mix.)}, \quad |U_{\mu 3}/U_{\tau 3}| = \tan \theta_{23} \text{ (atmospheric mix.)}.
\]

(5) We have bounds on neutrino masses (in this sense, we do not know only the oscillation parameters \( \Delta m_{ij}^2 \)) from other sources: from \( \beta \)-decay (Mainz, Troitsk),

\[
\sqrt{\sum_i |U_{e i}^2| m_i^2} \leq 2.2 \text{ eV}; \quad \text{from neutrinoless double beta decay (Heidelberg-Moscow at Gran Sasso, IGEX)}, \quad |\sum_i U_{e i}^2 m_i| \leq 0.3 - 1 \text{ eV} \quad \text{[a claim was made that the transition has been observed, but in our opinion, with a weak significance]; from galaxy surveys (2dF)}\]

\[
\sum_i m_i < 1.8 \text{ eV} \quad \text{or combined cosmological results (including WMAP)} \quad \sum_i m_i < 0.7 \text{ eV}. \quad \text{We do not discuss them further, but we note that they suggest that the kinematic search of effects of neutrino masses with SN neutrinos is difficult or impossible.}\
\]
References


