Gravitational Radiation from Gamma-Ray Bursts

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Abstract

Gamma Ray Bursts (GRBs) are the most relativistic objects known so far, involving, on one hand an ultra-relativistic motion with a Lorentz factor $\Gamma > 100$ and on the other hand an accreting newborn black hole. The two main routes leading to this scenario: binary neutron star mergers and Collapsar - the collapse of a rotating star to a black hole, are classical sources for gravitational radiation. Additionally one expects a specific gravitational radiation pulse associated with the acceleration of the relativistic ejecta. I consider here the implication of the observed rates of GRBs to the possibility of detection of a gravitational radiation signal associated with a GRB. Unfortunately I find that, with currently planned detectors it is impossible to detect the direct gravitational radiation associated with the GRB. It is also quite unlikely to detect gravitational radiation associated with Collapsars. However, the detection of gravitational radiation from a neutron star merger associated with a GRB is likely.
1 Introduction

Gamma Ray Bursts (GRBs) are the most relativistic objects known so far. According to the fireball model [1, 2] the observed gamma-rays are produced within relativistic shocks arising when an ultra-relativistic ejecta (with a Lorentz factor $\Gamma > 100$) is slowed down. The GRB arises due to internal shocks within the ultra-relativistic flow. The afterglow is produced when the ejecta is slowed down by the surrounding medium. The “inner engine” that accelerates the relativistic flow and powers the GRB involves in most models a newly formed black hole surrounded by a thick accretion disk around it. The ultra-relativistic ejecta is, most likely, in the form of two jets along the rotation axis of this system.

It is only natural to expect that with these processes GRBs will be accompanied by “strong” gravitational radiation signals. We identify two different stages that can produce gravitational radiation. First, gravitational radiation can arise when the black hole forms. Currently there are two competing models for the inner engine: A binary neutron star merger, in which case we should expect a “classical” chirping signal of a merger [3], or a Collapsar, which involves an asymmetric collapse of a massive star, whose gravitational radiation signal resembles the one from a rapidly rotating core collapse [4]. In both models we find a black hole surrounded by a massive accretion disk. The accretion of this disk onto the black hole powers the GRB. Some of the accreting material is accelerated, most likely along the symmetry axis, to ultra-relativistic velocities. This ejecta produces the GRB. In addition to the gravitational radiation produced during the formation of the “central engine” we also expect gravitational radiation from the acceleration stage in which the relativistic ejecta is accelerated to an ultra relativistic velocity. This emission is unique to GRBs. It should appear in association with GRBs regardless of the nature of the model leading to the GRB.

I examine here the possibility that this radiation signal will be detected in coincidence with a GRB by future gravitational radiation detectors. I discuss in section 2 some general features of GRBs which serve as a background for the following discussion. In sections 3 and 4 I discuss gravitational radiation from GRBs associated with mergers (section 3) and Collapsars (section 4). In section 5 I review the basic features of the fireball model and I calculate the gravitational radiation emitted during the acceleration phase of relativistic ejecta. The results are summarized in section 6.

2 Gamma-Ray Bursts

Three related features of GRBs are most relevant to our discussion: rate, beaming and energy. I refer the reader to a recent review [1, 2] for additional details on GRBs.
2.1 Isotropic Rates

The rate of GRBs is calculated by fitting the observed BATSE $\log N/\log S$ (actually the $C/C_{\text{min}}$) distribution [5, 6] to theoretical models. In this work I use the recent results of Schmidt [7, 8] who confirmed previous indications that GRBs follow the star formation rate. Schmidt’s most recent [8] estimate for the current local rate $R_{\text{iso}} = 2 \times 10^{-6}\text{Mpc}^{-3}\text{yr}^{-1}$, where anticipating the discussion of beaming the subscript $\text{iso}$ indicates that this quantity is calculated assuming isotropic emission. The current local rate, which is of interest to us here is lower than the rate in the past which was higher when the star formation rate was higher. This rate corresponds to one burst per $5 \times 10^6\text{yr}$ per galaxy for a galaxy density of $0.01\text{Mpc}^{-3}$. Note that this rate is larger by a factor of 10 than Schmidt’s previous estimate reported a year ago [7]. This difference could serve as an indication to the possible error in these numbers. The luminosity of GRBs is highly variable spanning two orders of magnitude. The average energy released in a burst (assuming isotropic emission), $\dot{E}_{\text{iso}}$ is $10^{52}\text{ergs}$. The bursts with measured red shifts range from $z = 0.433$, for GRB990712 to $z = 4.2$ (excluding $z = 0.05$ of SN1998bw which may or may not be associated with GRB980425), the most distant one observed so far is at $z = 4.5$. Schmidt’s analysis suggests that if the bursts are isotropic then once per year we see a burst as near as 1Gpc ($z = 0.22h_{60}$).

2.2 Short and Long Bursts

The duration distribution of the bursts divides GRBs into two populations: long and short bursts according to $T > 2\text{sec}$ or $T < 2\text{sec}$. About one quarter of the observed bursts are short. The $\log N/\log S$ distribution clearly shows that the observed short bursts are not detected from as far as the long ones. This means that on average short bursts are weaker and hence the observed short bursts are nearer to us than the long ones [9, 10]. The best estimate so far suggests that all short bursts are at $z < 0.5$. So far afterglow was not detected from any short burst and we don’t have a redshift measurement to any short burst hence there is no independent confirmation of this expectation. Given about 250 short bursts per year and assuming that they all come from within $z < 0.5$ we find that the current rate of observed short bursts is $R_{\text{iso-short}} = 2 \times 10^{-8}\text{Mpc}^{-3}\text{yr}^{-1}$ about ten times larger than the rate of long GRBs.

2.3 Jets and Beaming

There are various indications arising from GRB afterglows that GRBs are beamed with beaming angles ranging from $2^\circ$ to $20^\circ$ [11, 12, 13, 14]. Beaming would have immediate implications on both the rate and the energy of these events. With beaming angles of $\theta = 0.1\theta_{0.1}$ the overall current rate of GRBs is $R_\theta = 8 \times 10^{-7}\theta_{0.1}^{-2}\text{Mpc}^{-3}\text{yr}^{-1}$ or a burst per $10^4\theta_{0.1}^{-2}\text{yr}$ per galaxy, with one burst per year at a distance of $135\theta_{0.1}^2\text{Mpc}$ and a redshift of 0.03. Note, however, that this “nearby” burst will, most likely, not be beamed in our direction and hence its
gamma-ray emission won’t be observed. It could possibly be detected later as an orphan afterglow in the optical or the radio bands if its position would be known with a good enough accuracy. The overall energy $E_{\gamma}$ is reduced of course by exactly the same factor $\tilde{E}_0 (\theta^2/4) \tilde{E}_{\text{iso}} = 2.5 \times 10^{49} \theta_{0.1}^2$ ergs.

It is not known if short bursts are beamed. Assuming that they are beamed with beaming angles similar to those seen in long bursts we find that the rate of these bursts is one per $10^7 \theta_{0.1}^{-2}$ yr per galaxy with one burst per year at a distance of $800 \theta_{0.1}^2$ Mpc.

3 Mergers

I consider here both binary neutron star mergers and black hole-neutron star mergers under the single category of mergers. These sources are the “canonical” sources of gravitational radiation emission. Both LIGO and VIRGO aim in detecting these sources. Specifically the goal of these detectors is to detect the characteristic “chirping” signals arising from the inspiraling phase of these events. The possibility of detection of such signals has been extensively discussed (see e.g. [3]) and we won’t repeat this here. Such events could be detected up to a distance of $\sim 20$ Mpc with LIGO I and up to $\sim 300 - 600$ Mpc with LIGO II.

The detection of the chirping merger signal is based on fitting the gravitational radiation signal to pre-calculated templates. Kochaneck and Piran [15] suggested that the detection of a merger gravitational radiation signal would require a lower S/N ratio if this signal coincides with a GRB. This could increase somewhat the effective sensitivity of LIGO and VIRGO to such events.

It is expected that mergers (either binary neutron star or a black hole-neutron star mergers) produce the short GRBs (see [16]). Considering the isotropic rate of short GRBs estimated earlier we find that there should be one short burst per year within $\sim 450$ Mpc. This is just at the sensitivity level of LIGO II. As already mentioned it is not clear if short GRBs are beamed. If they are beamed, with the same beaming factor as long GRBs we should expect several hundred mergers events per a single observed burst. This would put one merger event per year at $\sim 800 \theta_{0.1}^2$ Mpc.

The corresponding distances to long GRBs are much longer. The nearest (long) GRB detected within a year would be a $1$ Gpc. This is far beyond the sensitivity of even LIGO II. If GRBs are beamed then the nearest (long) event would be much nearer, at $1350 \theta_{0.1}$ Mpc, well within the sensitivity of LIGO II. However, this burst would be directed away from us. Still a GRB that is beamed away from us is expected to produce an “orphan” afterglow and the gravitational radiation signal could trigger a search for this afterglow.
4 Collapsars

The Collapsar model [17, 18] is based on the collapse of the core of a massive star to a black hole surrounded by a thick massive accretion disk. The accretion of this disk onto the black hole, is accompanied by the acceleration of ultra relativistic jets along the rotation axis and powers the GRB. The jets first have to punch a hole in the stellar envelope. The GRB forms only after the jets have emerged from the envelope. Due to the relatively long time that it takes for the jets to punch a hole in the envelope it is expected that Collapsars can produce only the long bursts.

As far as gravitational radiation is concerned this system is very similar to a regular supernova. Rotating gravitational collapse has been analyzed by Stark and Piran [4]. They found that the gravitational radiation emission emitted in a rotating collapse to a black hole is dominated by the black hole’s lowest normal modes, with a typical frequency of $\sim 20c^3/GM$. The total energy emitted is:

$$\Delta E_{GW} = \epsilon Mc^2 = \min(1.4 \cdot 10^{-3}a^4, \epsilon_{max})Mc^2,$$

where $a$ is the dimensionless specific angular momentum and $\epsilon_{max}$ is a maximal efficiency which is of the order a few $\times 10^{-4}$. The expected amplitude of the gravitational radiation signal, $h$, would be of the order of $\sqrt{cGM/c^2d}$ where $d$ is the distance to the source. Even LIGO II won’t be sensitive enough to detect such a signal from a distance of 1Gpc or even from 100 Mpc. Furthermore, this signal would be rather similar to a supernova gravitational radiation signal. As regular supernovae are much more frequent it is likely that a supernova gravitational radiation signal would be discovered long before a Collapsar gravitational radiation signal.

5 Gravitational Radiation from the GRB

I turn now to examine the gravitational radiation that would arise from the GRB process itself. According to the fireball model the “inner engine” accelerates a mass $m = E/\Gamma c^2$ to a Lorentz factor $\Gamma$. Typical values for $\Gamma$ are 100 or more (see e.g. [1, 2]). Then a fraction of this energy is converted via internal shocks to the gamma-rays. The rest of the energy is dissipated later while it is slowing down by the surrounding matter, producing the afterglow.

The most efficient generation of gravitational radiation could take place here during the acceleration phase, in which the mass is accelerated to a Lorentz factor $\Gamma$. To estimate this emission we follow Weinberg’s [19] analysis of gravitational radiation emitted from a relativistic collision between two particles. This estimate is different from the “Christodoulou effect” of a permanent signature from a source moving at the speed of light [20]. We consider the following simple toy model. Consider two particles at rest with a mass $m_0$ that are accelerated instantly at $t = 0$ to a Lorentz factor $\Gamma$ and energy $E$. Conservation of energy requires that some (actually most) of the rest mass was converted to kinetic energy during the acceleration and the rest mass of the accelerated particle is $m = E/\Gamma = m_0/\Gamma$. Using
the formalism developed by Weinberg [19] to estimate the gravitational radiation generated in particle collisions, we calculate the gravitational radiation emitted by this system. Prior to the acceleration the two particles have momenta $m_0(1, 0, 0, 0).

After the acceleration the particles’ momenta are $m \Gamma (1, \pm \beta)$. The energy emitted per unit frequency per unit solid angle in the direction at an angle $\alpha$ relative to $\beta$ is:

$$\frac{dE}{d\Omega d\omega} = \frac{Gm^2 \beta^2}{c^2} \left[ \Gamma^2 \left( \frac{\beta^2 - \cos^2 \alpha}{1 - \beta^2 \cos^2 \alpha} \right) + \frac{\cos^2 \alpha}{\Gamma^2 (1 - \beta^2 \cos^2 \alpha)} \right].$$

(2)

The result is independent of the frequency, implying that the integral over all frequencies will diverge. This non-physical divergence arises from the non-physical assumption that the acceleration is instantaneous. In reality this acceleration takes place over a time $\delta t$, which is of order $0.01 \text{sec}$. This would produce a cutoff $\omega_{\text{max}} \sim 2\pi/\delta t$ above which Eq. 2 is not valid. The angular distribution found in Eq. 2 is disappointing. The EM emission from the ultra-relativistic source is beamed forwards into a small angle $1/\Gamma$, enhancing the emission in the forward direction by a large factor ($\Gamma^2$). Here we find that the gravitational radiation from this relativistic ejecta is spread rather uniformly in almost all $4\pi$ directions. Instead of beaming we have “anti-beaming” with no radiation at all emitted within the forward angle $\gamma^{-1}$ along the direction of the relativistic motion.

Integration of the energy flux over different directions yields:

$$\frac{dE}{d\omega} = \frac{Gm^2}{c^2} \left[ 2\Gamma^2 + 1 + \frac{1 - 4\Gamma^2}{\Gamma^2 \beta} \tan^{-1}(\beta) \right].$$

(3)

As expected the total energy emitted is proportional to $E^2 = m^2 \Gamma^2$. Further integration over frequencies up to the cutoff $2\pi/\delta t$ yields:

$$E \approx \frac{2Gm^2 \Gamma^2}{c^2 \delta t}.$$

(4)

In reality the situation is more complicated than the one presented here. First, the angular width of the emitted blobs is larger than $\gamma^{-1}$. The superposition of emission from different directions washes out the no emission effect in the forward direction. Additionally, according to the internal shocks model the acceleration of different blobs goes on independently. Emission from different blobs should be combined to get the actual emission. Both effects reduce the effective emission of gravitational radiation and makes the above estimate an upper limit to the emission that is actually emitted.

The gravitational signal is spread in all directions (apart from a narrow beam along the direction of the relativistic motion the GRB). It ranges in frequency from 0 to $f_{\text{max}} \approx 100 \text{Hz}$. The amplitude of the gravitational radiation signal at the maximal frequency, $f_{\text{max}} \approx 100 \text{Hz}$, would be: $h \approx (Gm \Gamma^2 / c^2 d)$. For typical values of $E = m \Gamma = 10^{51} \text{ergs}, \delta t = 0.01 \text{sec}$ and an optimistic distance of 100 Mpc, $h \approx 2.5 \cdot 10^{-25}$, far below the sensitivity of the planned gravitational radiation detectors. Even if we consider a burst which is ten times nearer this “direct” gravitational radiation signal would still be undetectable.
6 Conclusions

GRBs produce gravitational radiation in two phases. The first is during the formation of the compact object (most likely a black hole) believed to be associated with the “inner engine of the GRB. The second is from the acceleration phase of the ultra relativistic ejecta. This second emission is “directly” associated with the GRB phenomenon and would occur in any GRB model which is based on the rather well established fireball concept. Our conclusions are somewhat disappointing. In spite of the ultra relativistic nature of the fireballs arising in GRBs they are not potential sources of detectable gravitational radiation signals. Gravitational radiation may be, however, detected from the stellar processes associated with the energy generation for GRBs. Here the situation is also disappointing. If GRBs are associated with mergers we would expect that once a year there would be a sufficiently nearby merger that could be detected. These chances are larger if mergers are associated with short GRBs which are weaker and have larger event rate than long ones. However, because of beaming it is most likely that we won’t observe the associated GRB. Gravitational radiation produced within the Collapsar model are even more disappointing. This emission is very similar to the emission expected from regular supernovae. Typical distances to these GRBs are hundred Mpcs. The gravitational radiation from such sources is practically undetectable. With the much larger rate of regular supernovae it is by far more likely to detect a gravitational radiation signal from a regular supernova than from a Collapsar.

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References


