Statistics of Compact Objects
and Coalescence Rates

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Abstract

We review evolutionary scenarios, population statistics and merging rates of binary systems containing white dwarfs, neutron stars and black holes in the context of their impending detection as sources of gravitational waves. We begin in §1 with a census of the various systems currently known. In §2 we review the evolutionary scenarios which are thought to be the main routes for forming these systems. In §3 we look at the various selection effects at play in the observed samples, as well as some of the techniques used to correct for these biases in §4. Population syntheses are outlined in §5. In §6, we review recent merger rate determinations for various types of binary systems. We conclude the review with an optimistic look to the future in §7.
1 The Compact-Object Binary Census

This review is concerned with the statistics of compact objects in binary systems. Observationally, our knowledge of white dwarfs (WDs), neutron stars (NSs) and black holes (BHs) is improving all the time as state-of-the-art instruments (optical telescopes, revamped radio telescopes and new high-energy observatories) continue to discover new and exciting systems which can be studied in ever-increasing detail. Since many of these systems are expected to merge due to the emission of gravitational radiation, and emit gravitational waves upon coalescence, they are of great interest to the gravitational wave community given the imminent arrival of new detectors such as LIGO. We review the statistics of these sources and summarize the most recent estimates of the merging rates of the various binary systems.

Table 1 summarizes the sample sizes of the various types of binary systems including compact objects, which have been observed so far. For binary systems containing two compact objects, the notation used throughout this review is to list the objects in chronological order of their formation. The only case, so far, where this is in fact required is when we need to distinguish between a WD–NS and a NS–WD binary system (see §2).

<table>
<thead>
<tr>
<th>Type of System</th>
<th>We Observe</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>WD–WD</td>
<td>usually the brighter of the two white dwarfs</td>
<td>13</td>
</tr>
<tr>
<td>WD–NS</td>
<td>radio pulsar and sometimes the white dwarf</td>
<td>2</td>
</tr>
<tr>
<td>NS–WD</td>
<td>radio pulsar and sometimes the white dwarf</td>
<td>40</td>
</tr>
<tr>
<td>NS–NS</td>
<td>(so far) one neutron star as a radio pulsar</td>
<td>5</td>
</tr>
<tr>
<td>NS–Main Seq.</td>
<td>radio pulsar and the main-sequence star</td>
<td>2</td>
</tr>
<tr>
<td>NS–Giant</td>
<td>low-mass X-ray bin.; some QPOs and one XMSP</td>
<td>120</td>
</tr>
<tr>
<td>NS–Super Giant</td>
<td>high-mass X-ray bin.; sometimes X-ray pulses</td>
<td>70</td>
</tr>
<tr>
<td>BH–Giant</td>
<td>low-mass X-ray bin.</td>
<td>3</td>
</tr>
<tr>
<td>BH–Super Giant</td>
<td>high-mass X-ray bin.</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 1: Compact-object binaries known from radio, optical and X-ray observations. QPO stands for quasi-periodic oscillation seen in some low-mass X-ray binaries and XMSP is shorthand for the X-ray millisecond pulsar SAX J1808.4–3658 discussed in §2.

The systems in this table are a mixture of old friends (e.g. the low- and high-mass X-ray binaries), and more recent discoveries. The known population of WD–WD binaries is a good example of a growing sample which is benefiting from improved optical observations [34]. Radio pulsar observations provide a significant input to Table 1. The known pulsar population has doubled in the last few years thanks to two major surveys of the Galactic plane carried out using the multibeam system [46] on the Parkes radio telescope [15, 20]. Such a large haul of pulsars inevitably results in a number of interesting individual neutron stars, many of which are members of binary systems. Recent discoveries include a likely NS–NS binary [29], a likely WD–NS system in a 5-hr relativistic orbit [23] as well as two relativistic NS–WD binaries at intermediate Galactic latitudes [15].
Not all of the systems in Table 1 will coalesce due to the emission of gravitational radiation within a Hubble time, \( \tau_H \) (we assume here \( \tau_H = 10 \text{ Gyr} \) [19]). Fig. 1 shows this delineation for NS and WD binaries in the orbital eccentricity versus orbital period plane. The lines in Fig. 1 show gravitational-wave coalescence times \( T_{GW} = 10^8, 10^{10} \text{ and } 10^{12} \text{ yr} \) as a function of orbital period \( P \) and eccentricity \( e \) for two \( 1.4 \text{ M}_\odot \) neutron stars (top panel) and for two \( 1 \text{ M}_\odot \) compact objects in a circular orbit (lower panel). The nine binary systems to the left of the \( 10^{10} \text{ yr} \) line will merge due to gravitational radiation emission within \( \tau_H \).

![Figure 1: Top panel: orbital period versus eccentricity for eccentric binaries. Lower panel: orbital period distributions for circular-orbit systems. Filled circles denote NS–NS binaries, unfilled circles denote WD–NS binaries, filled stars denote NS–WD binaries and unfilled stars denote WD–WD binaries.](image)

To calculate \( T_{GW} \) in Fig. 1, we use the following simple formula which requires only the current mass, period and eccentricity of the binary system:

\[
T_{GW} \approx 10^7 \text{ yr} \left( \frac{P}{\text{hr}} \right)^{8/3} \left( \frac{m_1 + m_2}{\text{M}_\odot} \right)^{1/3} \left( \frac{\mu}{\text{M}_\odot} \right)^{-1} (1 - e^2)^{7/2}.
\]

Here \( m_{1,2} \) are the masses of the two stars and \( \mu = m_1 m_2 / (m_1 + m_2) \). This formula is a good analytic approximation (within a few percent) to the numerical solution of the exact equations for \( T_{GW} \) in the original papers by Peters & Mathews [39, 40].

2 Evolutionary Scenarios

Fig. 2 summarizes the main evolutionary scenarios that are thought to occur during the formation of most of the various compact-object binary systems we observe.
Starting with a binary star system, a compact object is formed at the end of the giant phase of the initially more massive star (the primary) which has an inherently shorter main sequence lifetime than the less massive star (the secondary). For stars with masses in the range 1–4 $M_\odot$ this is a relatively quiet collapse to form a WD and accompanying planetary nebula. For more massive stars this is a violent supernova which leaves behind a NS (for masses of 4–8 $M_\odot$) or a BH (for masses > 8 $M_\odot$). For most binary stars where this happens, from the virial theorem, it follows that the binary system gets disrupted if more than half the total presupernova mass is ejected from the system during the explosion [17]. In addition the fraction of surviving systems is affected by the magnitude and direction of the impulsive kick velocity the compact object receives at birth [5]. This high disruption probability explains why only a few percent of all canonical radio pulsars are observed with orbiting companions. Binary systems which disrupt produce a high-velocity compact object and an OB runaway star [9].

We are concerned in Fig. 2 with those binary systems that remain bound upon formation of the first compact object, and where the companion star is sufficiently massive to evolve into a giant star. For the case of the WD primary, a similar evolutionary path for the secondary would result in a WD–WD binary. Depending on the orbital parameters, however, the primary might overflow its Roche lobe onto the secondary before the primary becomes a WD. The secondary may now be massive enough to form a NS. Likely examples of such WD–NS systems are the binary pulsars B2303+46 and J1141−6545 [24, 49, 52].

In cases where the primary forms a NS or BH, Roche-lobe overflow onto the collapsed primary is expected to heat the accreted material up to such a degree that X-rays are liberated and the system is visible as an X-ray binary source. The accretion process transfers matter and angular momentum onto the primary. For NS primaries, this causes a spin up to shorter periods [8] forming a rapidly spinning "recycled" radio pulsar. X-ray binary systems can be categorized into two main groups depending on the mass of the secondary star. In low-mass X-ray binaries the secondary is not sufficiently massive to form a NS or BH and will evolve slowly, spinning the NS star primary up to periods as short as a few ms [1]. This model has recently gained strong support with the detection of Doppler-shifted 2.49-ms X-ray pulsations from the transient X-ray burster SAX J1808.4−3658 [11, 55]. At the end of the spin-up phase, the secondary sheds its outer layers to become a white dwarf in orbit around a rapidly spinning millisecond pulsar (NS–WD). A good example of this evolutionary path is PSR J1012+5307, a 5.35-ms pulsar in a 14-hr orbit around a bright white dwarf [38, 53]. Similar evolutionary paths are likely for binary systems with low-mass BH primaries. Currently there are seven low-mass X-ray binary systems where the primary masses inferred from optical and other observations clearly rule out NSs in favor of BHs [54].

In high-mass X-ray binaries the secondaries are massive enough to explode themselves as a supernova, producing a second NS or BH. One example of such a binary system lucky enough to survive this second explosion is PSR B1913+16, a NS–NS binary where we observe a 59-ms radio pulsar with a spin-down age of $\sim 10^8$ yr.
which orbits its companion every 7.75 hr [18]. In this formation scenario, PSR B1913+16 is an example of the older, first-born, NS that has subsequently accreted matter from its companion. Due to the shorter lifetime of the more massive secondary, the NS spin period is longer than in the low-mass X-ray binary systems. Presently, there are no clear observable examples of the second-born NS in these systems. This is probably reasonable when one realizes that the observable lifetimes of recycled pulsars are 10–100 times larger than normal pulsars.

As we shall see, NS-NS binary systems are very rare in the Galaxy. This is another indication that the majority of binary systems get disrupted when one of the components explodes as a supernova and also implies that BH–NS systems will be very rare also. In addition, acceleration smearing will select against the detection of radio pulsars in tight BH–NS binaries (see §3.4). Likely precursors to these and BH–BH systems are the three currently known high-mass X-ray binary systems
which have a BH component [54]. The classic example is Cygnus X–1 where the implied BH mass is $16 \pm 5 \, M_\odot$ [16].

3 Observational Selection Effects

Having gotten a flavor for the likely evolutionary scenarios, we now turn our attention to the question of the completeness of the samples of compact-object binaries in the Galaxy. Apart from the X-ray binary systems, which are thought to be bright enough to be seen throughout the Galaxy [26], most samples suffer a number of selection effects. As a result, the true number of objects is likely to be substantially larger than the observed number. For samples of NSs studied as radio pulsars and WDs studied optically, the inverse square law discussed next is the main selection effect. For radio pulsars, there are a number of other more subtle effects which we discuss later in this section.

3.1 The Inverse Square Law

The most prominent selection effect at play in samples of astronomical objects is the inverse square law, i.e. for a given luminosity the observed flux density falls off as the inverse square of the distance. This results in the observed sample being dominated by nearby and/or bright objects. This effect is well demonstrated in the radio pulsar population by the clustering of sources around our location when projected onto the Galactic plane shown in Fig. 3. Although this would be consistent with Ptolemy’s geocentric picture of the heavens, it is clearly at variance with what we now know about the Galaxy, where the massive stars show a radial distribution about the Galactic center.

![Graphs](image)

Figure 3: Left: A sample of 706 radio pulsars projected onto the Galactic plane. The Galactic center is at $(0,0)$ and the Sun is at $(-8.5,0)$. Right: Cumulative number of observed pulsars [solid line] as a function of projected distance, $d$. The dashed line shows the expected distribution for a uniform-disk population [see text].
The extent to which the pulsar sample is incomplete is shown in the right panel of Fig. 3 where the cumulative number of pulsars is plotted as a function of the projected distance from the Sun. The observed distribution (solid line) is compared to the expected distribution (dashed line) for a uniform disk population in which there are errors in the distance scale, but no such selection effects. We see that the observed sample becomes strongly deficient in terms of the number of sources for distances beyond a few kpc.

3.2 Pulse Broadening Effects

Beyond distances of a few kpc from the Sun, the apparent flux density falls below the flux thresholds \( S_{\text{min}} \) of most surveys which, following Dewey et al. [14], we quantify as:

\[
S_{\text{min}} = \sigma_{\text{min}} \left( \frac{T_{\text{rec}} + T_{\text{sky}}}{K} \right) \left( \frac{G}{KJy^{-1}} \right)^{-1} \left( \frac{\Delta \nu}{\text{MHz}} \right)^{-\frac{1}{2}} \left( \frac{t_{\text{int}}}{\text{s}} \right)^{-\frac{1}{2}} \left( \frac{W}{P-W} \right)^{-\frac{1}{2}} \text{ mJy.}
\]

In this expression \( \sigma_{\text{min}} \) is the threshold signal-to-noise ratio, \( T_{\text{rec}} \) and \( T_{\text{sky}} \) are the receiver and sky noise temperatures, \( G \) is the gain of the antenna, \( \Delta \nu \) is the observing bandwidth, \( t_{\text{int}} \) is the integration time, \( W \) is the detected pulse width and \( P \) is the pulse period.

If the detected pulse width equals the pulse period, the pulsar is of course no longer detectable as a periodic radio source. The detected pulse width can be larger than the intrinsic width for a number of reasons: finite sampling effects, pulse dispersion and as scattering due to the presence of free electrons in the interstellar medium. Scattering is particularly important for pulsars close to the Galactic plane where the increased column density of free electrons can significantly broaden the pulse, reducing the effective signal-to-noise ratio and overall search sensitivity. Fortunately, the scale sizes involved mean that scattering decreases significantly at higher observing frequencies. This is one of the main reasons for the success of the Parkes multibeam surveys which are conducted at 1.4 GHz, where scattering is not nearly as severe as at the “classical” pulsar searches at 430 MHz.

3.3 Radio Pulsar Beaming

A selection effect that we simply have to live with is beaming: the fact that the emission beams of radio pulsars are narrow means that only a fraction of 4\( \pi \) steradians is swept out by the radio beam during one rotation. A first-order estimate of the so-called “beaming fraction” is 20%; this assumes a beam width of 10 degrees and a randomly distributed inclination angle between the spin and magnetic axes of the NS.

More detailed studies show that short-period pulsars have wider beams and therefore larger beaming fractions than their long-period counterparts [7, 30, 36, 48]. It must be said, however, that a consensus on the beaming fraction as a function of period has yet to be reached. This is shown in Fig. 4 where we compare the period dependence of the beaming fraction as given by a number of models. Adopting the
Figure 4: Four different pulsar beaming models: Lyne & Manchester [1988; LM88 [30]], Tauris & Manchester [1998; TM98 [48]], Biggs [1990; JDB90 [7]] and Narayan & Vivekanand [1983; NV83 [36]].

Lyne & Manchester model [30], pulsars with periods $\sim 0.1$ s beam to about 30% of the sky compared to the Narayan & Vivekanand model [36] in which pulsars with periods $\leq 0.1$ s beam to the entire sky.

3.4 Acceleration Smearing

Standard pulsar searches use Fourier techniques [32] to search for $a$-priori unknown periodic signals and usually assume that the apparent pulse period remains constant throughout the observation. For searches with long integration times (e.g. the inner-plane survey at Parkes [29] uses 35-min pointings), this assumption is only valid for solitary pulsars, or those in binary systems where the orbital periods are longer than about a day. For shorter-period binary systems, as noted by Johnston & Kulkarni [20], the Doppler-shifting of the period results in a spreading of the signal power over a number of frequency bins in the Fourier domain, leading to a reduction in signal-to-noise ratio. To quantify this effect, consider a pulsar with a fixed spin period $P$ in orbit with another star. An observer will see the period shift by an amount $aT/(Pc)$, where $a$ is the (assumed constant) line-of-sight acceleration $a$ during the observation of length $T$ and $c$ is the speed of light. Given that the width of a frequency bin is $1/T$, we see that the signal will drift into more than one spectral bin if $aT^2/(Pc) > 1$. This implies that the survey sensitivities to rapidly-spinning pulsars in tight orbits are significantly compromised when the integration times are large.

It is clearly desirable to employ a technique to recover the loss in sensitivity due to Doppler smearing. The Parkes surveys are using a technique to search for drifting signals in spectra from subsets of the total 35-min integration. For $N$ subsets, the width of the frequency bins is $N$ times larger than in the spectrum of the entire transform and the frequency drift is reduced by a factor of order $N$. Acceleration
smearing is therefore reduced by a factor $\sim N^2$ when the spectra are shifted and stacked (added) for a variety of trial shifts. This incoherent approach is well suited for larger projects such as the multibeam search.

For smaller scale pulsar search projects, such as deep targeted searches of globular clusters, a coherent technique can be employed to improve the sensitivity yet further. In the so-called acceleration search [35], the pulsar is assumed to have a constant acceleration during the integration. Each time series can then be resampled to refer it to the frame of an inertial observer using the Doppler formula to relate a time interval in the pulsar frame, $\tau$, to that in the observed frame at time $t$, as $\tau(t) = \tau_0(1 + at/c)$, where $\tau_0$ is a normalizing constant. Searching over a range of accelerations is required to find the time series for which the trial acceleration most closely matches the true value. In the ideal case, a time series is produced with a signal of constant period for which full sensitivity is recovered. Anderson et al. [2] used this technique to find PSR B2127+11C, a double neutron star binary in M15 which has parameters similar to B1913+16. Camilo et al. [16] have recently applied the same technique to 47 Tucanae to discover 9 binary pulsars including one in a 96-min orbit around which is currently the shortest binary period for a radio pulsar.

For the shortest orbital periods, the assumption of a constant acceleration during the observation clearly breaks down. Ransom et al. [43] have developed a particularly efficient algorithm for finding binaries whose orbits are so short that many orbits can take place during an integration. This phase modulation technique exploits the fact that the pulses are modulated by the orbit to create a family of periodic sidebands around the nominal spin period of the pulsar. This technique has already been used to discover a 1.7-hr binary pulsar in NGC 6544 [43]. The existence of these short-period radio pulsar binaries, as well as the 11-min X-ray binary X1820–303 in NGC 6624 [47] implies that there must be many more short-period binaries containing radio or X-ray pulsars in globular clusters that are waiting to be discovered by more sensitive searches.

4 Correcting for Selection Effects

Given a well-defined sample of objects from a number of surveys, the most straightforward technique to correct for selection effects is to calculate a scale factor for each object in the sample. The scale factor is in essence an estimate of the number of similar objects in a region of space where selection effects are well understood. For each object, a Monte Carlo simulation is used to seed a given region of space with $N$ identical objects (i.e., identical luminosity and other relevant parameters which define the object’s detectability). Then, using a realistic model for the survey(s), the number of objects $n$ that are actually detectable is found. The scale factor is then simply the ratio $N/n$. Since the scale factor is principally a function of a source’s luminosity, more luminous sources are detectable over larger volumes of space. For these sources $n$ is larger and therefore the scale factor is smaller than for fainter sources which are harder to detect. This approach is similar to the classic $V/V_{\text{max}}$.
technique first used to correct observationally-biased samples of quasars [45].

Model Galactic Population

\[ N_G \]

Flux-limited sample

\[ N_{\text{obs}} \]

Model Detected Population

\[ <N_{\text{obs}}> \]

Figure 5: Small-number bias of the scale factor estimates derived from a synthetic population of sources where the true number of sources is known. Left: An edge-on view of a model Galactic source population. Right: The thick line shows the true number of objects in the model Galaxy, \( N_G \), plotted against the number detected by a flux-limited survey, \( N_{\text{obs}} \). The thin solid line shows the median sum of the scale factors, \( N_{\text{est}} \), as a function of \( N_{\text{obs}} \) from a large number of Monte-Carlo trials. Dashed lines show 25 and 75% percentiles of the \( N_{\text{est}} \) distribution. (Courtesy, V. Kalogera)

The power of this technique is that it requires a relatively small number of assumptions (essentially just the underlying distribution of objects) and has been shown to give self-consistent results when applied to large samples of objects [27]. For smaller samples, however, the detected sources are likely to be those with larger-than-average luminosities; the derived scale factors are therefore on the small side. This "small-number bias" was first pointed out by Kalogera [21] for the sample of NS–NS binaries where we know of only two sources which will merge within \( \tau_1 \) — PSRs B1534+12 [56] and B1913+16 [18]. Only when the number of sources in the sample gets past 10 or so does the sum of the scale factors become a good indicator of the true population size. In Fig. 5 we infer, for a sample of two objects, a bias of up to an order of magnitude due to this effect.

Armed with a robust estimate of the true number of sources, the merging rate can be calculated by dividing the scale factors by a reasonable estimate for the merging time. This is discussed in detail in §6.2 in context of the NS–NS population.

5 Population Syntheses

An alternative approach to the empirical methods described above is to undertake a full-blown Monte Carlo simulation of the most likely evolutionary scenarios described in §2. In this “scenario-machine” approach, a population of primordial binaries is synthesized with a number of underlying distribution functions: primary
mass, binary mass ratio, orbital period distribution etc. The evolution of both stars is then followed to give a predicted sample of binary systems of all the various types. Since the full range of binary parameters is known, the merger rates of each type of binary are then automatically predicted by this model without the need to debate what the likely coalescence times will be. Selection effects are not normally taken into account in this approach since the final census is usually normalized to the star formation rate, or only relative formation rates are required.

Numerous examples of this approach (most often to populations of binaries where one or both members are NSs) can be found in the literature [13, 44, 50]. Although this method is extremely instructive in its approach, the large numbers of input assumptions about initial conditions, the physics of mass transfer and the kicks applied to the compact object at birth result in a wide range of predicted event rates.

6 Recent Coalescence Rate Estimates

6.1 WD–WD binaries

Nelemans et al. [37] have applied a scenario-machine approach to estimate the WD–WD coalescence rate. Following reasonable modeling assumptions, they find that this is a surprisingly large contribution to the compact-object coalescence rate, with a likely value of $5 \times 10^{-2} \, \text{yr}^{-1}$. Using the predicted orbital distributions, they also calculate the likely spectrum of gravitational radiation emitted by this population. This turns out to be extremely interesting and potentially detectable in a $10^6$ s LISA observation. There is even a potential source confusion in the spectrum below 3 mHz. While the authors caution that there is some uncertainty in this approach due to uncertainties in input assumptions, further WD-WD population studies, particularly those which attempt to account for selection effects, seem worthwhile given the promising event rates calculated so far.

6.2 NS–NS binaries

Only two NS–NS binaries which will merge in a reasonable time-scale are currently known in the Galactic disk: PSRs B1534+12 and B1913+16. Ever since the discovery of the original binary pulsar B1913+16, numerous estimates of the Galactic population and merger rate of NS–NS binaries have been published. Since the pulsar surveys are fairly well understood, this population lends itself well to an empirical determination of the event rate where the scale factor of an object is divided by its most likely lifetime.

Some debate exists about what is the most reasonable estimate of the lifetime. Phinney [41] estimated individual merger rate contributions for NS–NS binaries by dividing their scale factor by the merger time $\tau_M$ which he defined as the sum of the pulsar's spin-down age plus $\tau_{GW}$ defined above. In another paper, van den Heuvel and myself [51] argued that a more likely estimate of the mean merging time can
be obtained by appealing to steady-state arguments where we expect sources to be created at the same rate at which they are merging. The mean lifetime was then found to be about three times its current spin-down age. This argument does, however, depend on the luminosity evolution of radio pulsars which is currently only poorly understood. Arzoumanian, Cordes & Wasserman ([3]; hereafter ACW) have recently published an extensive discussion on this topic and they use kinematic data to constrain the most likely ages of the pulsars and note that the remaining detectable lifetime should also take account of the reduced detectability at later epochs due to acceleration smearing as the NS–NS binary becomes more compact.

Taking all these factors into account, ACW revise earlier scale factors estimates [12] and detectable lifetimes of B1534+12 and B1913+16 to be 40/(710 Myr) and 20/(245 Myr) respectively leading to a combined merger rate of $1.3 \times 10^{-7}$ yr$^{-1}$. This should be viewed as a lower limit since it does not take into account pulsar beaming or the small-number bias discussed above. For a beaming fraction of 30% and a small-number correction of a factor of 10, we estimate the merger rate of NS–NS binaries to be $4 \times 10^{-6}$ yr$^{-1}$. Extrapolating this number out to include NS–NS binaries detectable by LIGO–II in galaxies within 200 Mpc à la Phinney [41] we find an expected event rate of 1 merger per year.

Although this event rate sounds fairly pessimistic, it is important to realize that considerable uncertainties exist. To appreciate this, consider an upper limit to the NS–NS merger rate presented by Bailes [6]; his argument goes as follows: since we expect one of the members of a NS–NS binary to be a normal (unrecycled) radio pulsar but so far we only observe the recycled member, the NS–NS birth-rate may not exceed $1/N$ times the birth-rate of normal pulsars where $N$ is the number of normal pulsars currently known. ACW revised Bailes’ estimate to take into account current numbers and acceleration smearing effects and find that the NS–NS merger rate must not exceed $10^{-4}$ yr$^{-1}$. The expected LIGO–II upper limit of NS–NS mergers is then 25 mergers per year. Even a more stringent upper limit presented by Kalogera and myself [22] which is based on the scale factors of radio pulsars that are likely to be the result of disrupted binaries after the second supernova explosion allows up to 10 mergers per year detectable by LIGO–II.

### 6.3 WD–NS binaries

This population of sources has only recently been identified by observers following the identification of a WD companion to the binary pulsar B2303+46 by van Kerkwijk and Kulkarni [52]. Previously, this eccentric binary pulsar was thought to be an example of a NS–NS binary in which the visible pulsar is the second-born neutron star [28]. The optical identification rules this out and now strongly suggests a scenario in which the WD was formed first [49] as outlined in §2. Population syntheses by Tauris & Sennels [49] suggest that the formation rate of WD–NS binaries is between 10–20 times that of NS–NS binaries. Based on the merging rate estimates for NS–NS binaries discussed in the previous section, this translates to a steady-state merging rate of WD–NS binaries of $8 \times 10^{-5}$ yr$^{-1}$. 
Although PSR B2303+46 has a long orbital period and will not contribute significantly to the overall merger rate of NS–WD binaries, the new discovery of PSR J1141–6545 [24] which will merge in 1.3 Gyr is suggestive of a large population of similar binaries. This is particularly compelling when one considers that the radio lifetime of the visible pulsar is only a fraction of total lifetime of the binary before coalescence due to gravitational-wave emission. Edwards & Bailes [15] estimate there to be 850 WD–NS binaries within 3 kpc of the Sun which will merge within a Hubble time. Further optical work on the newly-discovered WD–NS binary systems is required to form a more complete picture. In addition, continued timing of the radio pulsars observed so far will undoubtedly lead to more stringent mass determinations and useful tests of strong-field gravity.

6.4 Some remarks on BH binaries

Although we expect them to exist, no BH–NS systems are currently known. Scenario machines predict birth-rates between $10^{-6}$ and $10^{-4} \text{yr}^{-1}$. Given that 1200 pulsars are currently known, with a mean birth-rate of 0.01 yr$^{-1}$ [31], we can apply Bailes’ upper limit argument discussed in §6.2 to limit the BH–NS birth-rate to $0.01/1200 \sim 8 \times 10^{-6} \text{yr}^{-1}$. The ongoing Parkes multibeam pulsar search of the Galactic plane presently has the best chances of finding such systems if they exist in the Galaxy. Currently, the most massive radio pulsar binary system found in the Parkes survey is PSR J1740–3052 where the orbiting companion is likely to be a K–supergiant star with a mass of at least 11 M$_\odot$ [33].

Bound BH–BH binaries in the Galactic disk are expected to be even rarer than BH–NS binaries. Portegies Zwart & McMillan [42] have recently suggested that globular clusters might be excellent breeding grounds for BH–BH binaries. The idea is that massive BHs are formed and then, due to dynamical relaxation, tend to sink to the cores of the clusters where they interact with other stars in the dense stellar core forming binaries with other BHs [25] which eventually get ejected during close encounters with single BHs. A conservative estimate of $10^{-4} N$ ejected BH binaries per N-star cluster leads to a large population of BH–BH binaries in the local Universe with a detection rate of about 1 event/day with LIGO–II!

7 Summary and Outlook

The new window on the Universe that will be opened by gravitational-wave astronomy will undoubtedly throw up a number of surprises in the near future. In addition, radio pulsar searches are providing more and more extreme examples of binary systems involving neutron stars and will undoubtedly reveal new treasures in the coming years... NS–BH binaries and dual-line NS–NS binaries are all exciting prospects that are being actively searched for by groups using large radio telescopes around the world. As we have seen, there are a number of factors (observational selection, small-number statistics, population modeling assumptions) which lead to considerable uncertainties in the estimates for compact-object coalescence rates.
As a result, observers should not be too discouraged by some of the numbers in this table. We close by making the remark that, given all the uncertainties in the current estimates, gravitational-wave detectors may ultimately provide far better constraints on the various merging rates discussed here.

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