R-mode Runaway in Rapidly Rotating Neutron Stars

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Abstract

In this article I discuss issues regarding the gravitational-wave driven instability of the r-modes of a rotating neutron star. After a brief overview of several of the difficult questions that face the theorist that wants to model/understand various r-mode scenarios, I discuss how we can confront our models with current and future observations. I show how the simplest r-mode scenarios agree quite well with data inferred from observations of both young neutron stars in supernova remnants and older stars that have been (or are still being) recycled in X-ray binaries. To conclude, I discuss the detectability of the gravitational waves emitted during the phase when the r-mode instability governs the spin-evolution of the neutron star.
1 The r-mode instability

In the last two years the r-modes in rotating neutron stars have attracted a lot of attention. The main reason for this is that they are unstable due to the emission of gravitational waves via a mechanism that was discovered by Chandrasekhar, Friedman and Schutz more than 20 years ago. Until recently the r-modes — which are essentially horizontal currents associated with very small density variations — had not been considered in this context. Hence the discovery that they are unstable at all rates of rotation in a perfect fluid star came as a slight surprise [1, 2]. Even more of a surprise was the subsequent realization that the unstable r-modes (which radiate mainly through the current multipoles) provide a much more severe constraint on the rotation rate of viscous stars (viscosity tends to counteract mode-growth due to gravitational radiation) than the f-modes (which are dominated by the mass multipoles). A direct comparison shows that the f-mode becomes unstable when the star is spun up to roughly 95% of the mass-shedding limit\(^1\), while the dominant r-mode becomes unstable already at 5% of the maximum spin rate (at some temperature) [3, 4]. The r-mode instability has emerged as a potential agent for spinning nascent neutron stars down to rotation rates similar to the initial period inferred for the Crab pulsar \((P \approx 19 \, \text{ms})\), in the process radiating an amount of gravitational waves that should be detectable with LIGO II for sources in the Virgo cluster [5]. It has also been suggested that the instability may operate in older, colder neutron stars and perhaps explain the clustering of spin periods in the accreting neutron stars in Low-Mass X-ray Binaries (LMXB) indicated by the observed kHz QPOs [6, 7].

The initial results for the unstable r-modes have created an atmosphere of great excitement in this research area, and a large number of interesting contributions have been made by numerous authors. In the limited amount of space available here I cannot possibly discuss this rapidly growing body of work in detail. For a more complete survey of the available literature the reader is referred to an exhaustive review on the subject [8] or Lindblom’s contribution to this volume.

2 Probing the extremes of physics

The potential relevance of the r-modes in neutron stars stems from the fact that the gravitational radiation that they emit (mainly through the current multipoles) drives the modes unstable, with a characteristic growth time (for the dominant \(l = m = 2\) r-mode)

\[
t_{gw} \approx 22 \left( \frac{1.4 M_*}{M} \right) \left( \frac{10 \, \text{km}}{R} \right)^4 \left( \frac{P}{1 \, \text{ms}} \right)^6 \, \text{s}.
\]  

This expression was obtained for uniform density stars, but it has been shown to be close (within a factor of two) to the results for \(n = 1\) polytropes [8]. \(M, R,\) and \(P\) are...

\(^1\)For a typical neutron star equation of state, the so-called Kepler limit corresponds to a rotation period in the range 0.8-2 ms.
$P$ represent the mass, radius and spin period of the star, respectively.

This result prompts us to ask a series of difficult questions:

*Given that viscosity tends to counteract mode-growth due to gravitational radiation, what are the main sources of viscosity and to what extent can they suppress the r-mode instability?*

In order to estimate the relevant time-scale for viscous dissipation (and, indeed, to determine (1) as well) we first assume that the true mode-solution is well represented by the solution to the non-dissipative perturbation equations. Then we use this solution to evaluate the effect of the various dissipation mechanisms and add their respective contributions to the rate of change of the mode energy $dE/dt$. This approach is reliable as long as the estimated growth/damping times are much longer than the oscillation period of the mode. This way it has been demonstrated that the two most commonly considered forms of viscosity, bulk and shear viscosity, do not completely suppress the instability [3, 4]. Since they are strongly temperature dependent — the shear viscosity dominates at low temperature, while the bulk viscosity is dominant at high temperature — they provide a “window of opportunity” for the r-mode instability to operate in the temperature range $10^8 - 10^{10}$ K. Given these results, one can then estimate that the r-modes will be unstable (at some temperature in this range) for spins above roughly 5% of the rate of rotation at which mass shedding sets in, see Figure 1.

Of course, this is just the first step towards a realistic model. Many additional mechanisms may be relevant in a real neutron star. As was first pointed out by Bildsten and Ushominsky [9], a viscous Ekman layer at the crust-core interface in a relatively cold neutron star will lead to a very strong dissipation mechanism. Given that the crust is likely to form already at a temperature of the order of $10^{10}$ K this mechanism is relevant for all but very young neutron stars. The dissipation time-scale associated with the Ekman layer can be estimated as (see [8] for a discussion)

$$t_{\text{Ek}} \approx 830 \left( \frac{T}{10^9 \text{K}} \right) \left( \frac{P}{1 \text{ms}} \right)^{1/2} \text{s},$$

for a canonical neutron star with mass $1.4M_\odot$ and radius 10 km. The r-mode instability window obtained from this estimate is compared to the normal fluid result in Figure 1. A solid crust has an influence that is far greater than any other dissipation mechanism considered so far. Our estimate suggests that all neutron stars with a crust are stable at rotation periods longer than roughly 5 ms [8, 10].

However, there are many complicating factors here. First of all, the neutron star crust is not solid as was assumed in the derivation of (2). In fact, it is rather like a jelly. This means that the crust nuclei will to a certain extent take part in the r-mode motion [11]. Furthermore, since it has a nonzero shear modulus the crust can support more or less distinct oscillation modes of its own. The interplay between the r-modes in the fluid core and toroidal modes in the solid crust is particularly interesting. Detailed calculations have shown that the unstable r-modes
Figure 1: The r-mode instability window for a canonical fluid neutron star (lower solid curve) is compared to that for a star with an Ekman layer operating at the base of the solid crust (upper solid curve). The $l = m = 2$ r-mode is unstable in the region above the relevant curve. The data were obtained for an $n = 1$ polytropic equation of state.

undergo a series of so-called avoided crossings with the crustal modes as the spin of the star increases [11, 12]. This is conceptually interesting and since it may have repercussions for many of our estimates regarding the r-mode instability it is an issue that should be investigated further. In addition, the fact that the r-modes extend into the crust mean that (2) is likely to overestimate the strength of the dissipation in the Ekman layer. Levin and Ushomirsky [11] have argued that the true dissipation may well be a factor of 100 weaker than (2).

Still, crucial pieces of physics may not yet have been included here. For example, it is quite possible that the heat released in the Ekman layer will be able to melt the crust. In a study of this issue, Lindblom, Owen and Ushomirsky [13] estimate that, for a star spinning at the Kepler limit, an r-mode with an amplitude exceeding $\alpha \approx 5 \times 10^{-3}$ (see [5] for the definition of the mode-amplitude) will be able to melt the crust. This means that if the r-mode instability is active in a young neutron star, the formation of the crust may be significantly delayed. But it is not yet clear what will happen if the r-modes do indeed manage to melt the crust. As soon as the crust melts the Ekman layer (that led to the excessive heating) disappears and the material will rapidly cool down to a level where the crust would begin to form again. A possible outcome is a solid-fluid mixed state [13] which is difficult to model in detail.

What is the mechanism that saturates the r-mode and prevents it from growing beyond some maximum amplitude?

Because the r-modes (and the notion of their instability) follow from a linear perturbation calculation we do not yet have a detailed picture of what happens once a mode has grown to a level where nonlinear effects come into play. A newly born
neutron star cools to the temperature at which the dominant r-mode goes unstable (a few times $10^{10}$ K) in a few seconds. Provided that the star spins fast enough the r-mode will then grow on a timescale given by (1) until it enters the nonlinear regime and... then what? In the first studies of the problem it was assumed that nonlinear effects (e.g. coupling to other modes) would lead to the mode saturating at some large amplitude [3, 4, 5]. The mode would continue to radiate angular momentum and the star would spin down from the mass-shedding limit to a period of 15-20 ms in a year or so. The amplitude of saturation is one of the crucial parameters of these models. In order for the instability to have a dramatic effect on the spin-evolution of a young neutron star, the r-mode must be allowed to grow to a reasonably large amplitude. One might expect non-linear effects to become relevant at much smaller mode-amplitudes than those considered in the early work. However, at present the indications are that the mode will be able to grow to a surprisingly large amplitude. This is demonstrated by very recent 3D time-evolutions (using a fully nonlinear relativistic hydrodynamics code with the spacetime “frozen”) of Stergioulas and Font [14]. The first results of investigations into the nonlinear coupling between r-modes and other modes seem to point in the same direction [15]. In these various studies, there are no signs of mode-saturation until at very large amplitudes. It should, of course, be noted that much work remains to be done on this problem before we can draw any firm conclusions.

It is also relevant to mention the work of Wu, Matzner and Arras [16] here. They argue that the crust-core boundary layer is likely to be turbulent and provides a mechanism for saturation. Under some circumstances this would lead to mode-saturation at rather small amplitudes, but one can infer that the resultant saturation amplitude is of order unity for rapidly rotating stars. This could well indicate that the modes saturate due to some alternative, as yet unspecified, mechanism in a newly born neutron star.

What happens when the r-mode saturates: Is the mode destroyed by the saturation mechanism or does it prevail and spin the star down?

The original spin-down scenarios were based on the assumption that, once the mode has saturated it survives virtually unchanged (at the saturation amplitude) and the excess angular momentum is radiated away as gravitational waves. The star then evolves along a sequence of uniformly rotating equilibrium models as it loses angular momentum [5]. Recent work indicates that the true behaviour is likely to be more complicated than this. First of all, one might expect that a large amplitude unstable mode will lead to differential rotation in the stellar fluid [14, 17]. It is well-known that this is the case for the bar-mode instability in the Maclaurin spheroids. Once spun up to the point where the bar-mode becomes unstable, the Maclaurin spheroids evolve through a sequence of differentially rotating ellipsoids. One might expect an analogous evolution for stars governed by the r-mode instability. Evidence in favour of this possibility has been presented by Rezzolla, Lamb and Shapiro [18]. They argue that the r-mode leads to a nonlinear differential drift of the various fluid elements. Their calculation is based on inferring higher order (in the mode-
amplitude) results from established linear results, and may not be quantitatively 
reliable, but it provides an indication that nonlinear effects will severely alter the 
fluid motion. This result is supported both by the time-evolutions of Stergioulas 
and Font [14] and a shell toy model studied by Levin and Ushomirsky [17]. In the 
latter case the nonlinear effects can be determined exactly, and they lead to the 
anticipated differential drift. Furthermore, the shell toy-model shows that, once 
radiation reaction is implemented, another source of differential rotation comes into 
play. Thus it would seem almost certain that differential rotation will play a key 
role in any realistic r-mode scenario. In addition it is not clear that the r-mode will 
survive once it reaches saturation. It could well be that its energy cascades into 
a set of other modes, or its character might change drastically through (say) the 
development of shocks.

What is the role of the magnetic field?

Differential rotation immediately brings magnetic field effects into focus. While 
effects due to electromagnetic waves generated by an oscillation mode are typically 
small, differential rotation may lead to a twisting of the field lines and a dramatic 
increase in the field strength. In the case of the r-modes the instability scenario 
may lead to the generation of a very strong toroidal magnetic field. Although there 
have been some initial studies of this problem [18, 19] (see Rezzolla’s contribution 
to this volume), we are likely far away from any quantitative answers at the present 
time. This is not too surprising since many aspects of neutron star magnetic fields 
remain to be understood.

To what extent are the details of the supranuclear equation of state relevant?

Despite intense efforts in recent years, the detailed supranuclear equation of state 
remains a mystery. At the present time, observations provide some constraints on 
the theoretical models, but few of the proposed equations of state can be ruled out. 
Given that there are numerous possibilities, and that the corresponding predictions 
for the nuclear reaction rates that dictate the various viscosity coefficients differ 
widely, this provides a serious challenge for the r-mode models. For example, the 
bulk viscosity result would be significantly different if the presence of hyperons in 
the neutron star core was accounted for [20]. Not only is the associated viscosity 
coefficient stronger than normally assumed, the temperature dependence is also 
different in the hyperon case (the coefficient scales as $T^{-2}$ rather than $T^{6}$). This 
makes the hyperon bulk viscosity relevant also at low temperatures, and it may 
well provide the most severe damping mechanism for the r-modes. In order to 
assess the extent to which hyperons affect the estimated instability time-scales, 
calculations that allow for the presence of exotic particles (as predicted by most 
modern equations of state) need to be performed. At the time of writing, no such 
results are available but it is clear that this is an issue that may turn out to be of 
great importance.

A related issue regards the fact that the bulk of a neutron star is expected
to become superfluid once it cools below a few times $10^9$ K. At this point some rather exotic dissipation mechanisms come into play, and it turns out (somewhat paradoxically) that a superfluid star is more dissipative than a normal fluid one. The most important new mechanism is the so-called mutual friction which has been shown to completely suppress the instability associated with the r-modes [21]. The initial expectations were that mutual friction would also have a strong effect on the r-modes. Detailed calculations by Lindblom and Mendell have shown that this is not necessarily the case [22]. The outcome seems to depend rather sensitively on the detailed superfluid model (the parameters of the so-called entrainment effect), and only in a small set of the models considered by Lindblom and Mendell does mutual friction affect the r-modes in a significant way. It would thus seem as if the r-mode instability may prevail also in superfluid stars. Additionally, it is worth pointing out that the inner crust of a neutron star (extending between $0.6\rho_{\text{nuclear}}$ out to neutron drip) will likely be permeated by superfluid neutrons. It is not at all clear at the moment whether one should expect these neutrons to be strongly pinned to the crust nuclei or not. But if the superfluid is at all free to move relative to the crust it will likely lead to a significantly weaker Ekman layer dissipation on the r-modes.

At this point it should be quite clear that, in order to understand the astrophysical role of the r-mode instability, we need to probe the very extremes of physics. We need to understand the nonlinear fluid dynamics of the modes themselves, the backreaction on the fluid from (for example) gravitational radiation, the role of a dynamic magnetic field, and on top of all this resolve the many issues regarding the equation of state. So... given that this is a meeting on gravitational-wave sources... will we be able to do all this in a year or so and provide the analysis groups for LIGO, GEO600, TAMA and VIRGO with reliable templates\(^2\) that can be used to search their data streams for r-mode signals? Now, I don’t want to come across as too pessimistic, but phrases like “never in a million years” come to mind. However, it may be that the question I am asking is unreasonable. Maybe this is the point where we should realize that our power to provide theoretical predictions\(^3\) is limited and it is time to ask our observational colleagues for help. In other words, maybe the relevant question is:

*Can we use observations to test and/or constrain various theoretical r-mode scenarios?*

I think this is a crucial question, and as I will argue later, we already have interesting data in this respect. For our fellow “gravitational-wave astronomers” the challenge is to invent a pragmatic detection strategy based on general principles rather than detailed theoretical information. This is a difficult task, but it is of great importance since an actual detection may help improve our understanding of general relativity, supranuclear physics, magnetic fields, superfluidity et cetera. This information is certainly worth waiting for, even if it takes a few years and requires

\(^2\)Comparable to the accurate post-Newtonian results for inspiralling binaries, for example.

\(^3\)Unless we find a mechanism that completely kills the r-mode instability, that is.
the development of a suitably advanced generation of detectors.

3 Nascent neutron stars

A few years ago it was generally assumed that neutron stars were born spinning at (or at least near to) the mass shedding limit. This would seem to be a natural outcome as long as angular momentum is conserved as the stellar core undergoes gravitational collapse following a supernova explosion. Some support for this picture was provided by data for young pulsars associated with supernova remnants. In the ideal case, observations provide estimates of the present rotation period, the spindown rate and also the age of the pulsar. Given this data one can use the standard magnetic braking model and (by assuming that the neutron star has been spinning down at the current rate during its entire life) estimate the initial spin rate. The best studied young pulsar is the Crab (PSR0531+21) which is the remnant of the supernova that was observed by Chinese astronomers in 1054. From the data for the Crab one can estimate an initial spin period of about 19 ms. Unfortunately, we are uncommonly lucky in the case of the Crab. In the typical case it is very difficult to obtain an accurate estimate of the age of a supernova remnant so the estimated initial spin periods come with large errorbars.

There have recently been several exciting ideas and observations regarding the spin at which neutron stars are born. On the theoretical side, Spruit and Phinney [23] have argued that magnetic locking between the core and the envelope of the progenitor star may prevent the collapsing core from spinning rapidly. In fact, Spruit and Phinney propose that the neutron star spin is mainly due to the birth kicks that also (or alternatively) produce the large linear velocities observed for pulsars. In their model most pulsars are born spinning slowly, but there is still a fraction (perhaps 10%) that spin faster than a few ms. The serendipitous discovery of the 16 ms pulsar PRS J0537-6910 in N157B [24] provides clear observational support for the existence of such rapidly spinning young neutron stars. One can estimate that the neutron star in N157B had an initial period of a few ms at birth (assuming a braking index typical of young pulsars).

The current evidence suggests that, even though it is not clear that all neutron stars are born spinning fast, a subset certainly forms with periods shorter than (say) 10 ms. Given the recent estimates for the r-mode instability it would seem that this mechanism may operate in these stars, and could play a role in determining their rotation rate. If we bring the phenomenological spin-evolution model suggested by Owen et al [5] to bear on the problem we find that the key parameter is the saturation amplitude $\alpha_s$. Provided that $\alpha_s$ is sufficiently large, the r-modes will spin a young neutron star down appreciably. If we assume that the neutron star is born with core temperature well above $10^{10}$ K and that it initially spins at the Kepler limit, the r-mode instability comes into play within a few seconds as the star cools and enters the instability window. The mode then grows from some small initial amplitude to the saturation level in a few minutes. Once the mode has saturated, the star spins down. At some point the star has cooled (or spun
down) sufficiently that the r-mode is again stable. Then the mode amplitude decays and the star presumably enters a phase where magnetic braking takes over and dominates the spin-evolution. The “final” spin period depends on several factors. The most important of these are the saturation amplitude, the cooling rate and whether a crust forms during the evolution. If we consider a simple perfect fluid model the r-mode spin-down leads to $P \approx 12-22$ ms for $\alpha_s$ in the range $0.01-1$ (and canonical neutron star parameters), see Figure 2. This result is clearly consistent with observations for many young pulsars (in particular the Crab).

![Figure 2](image.png)

**Figure 2**: Left: Spin-down evolutions for a fluid neutron star. We assume that the star is isothermal and that it cools entirely due to the modified URCA process. Two different evolutions corresponding to an r-mode saturated at $\alpha_s = 0.01$ and 1, respectively, are shown as solid lines. The dashed curve indicates the evolution that would follow if the r-mode was initially excited to the saturation level rather than given a very small initial amplitude ($\alpha = 10^{-6}$) at the onset of the instability. It is notable that the initial amplitude has little effect on the period at the end of the spin-down phase. Right: Spin-down evolutions for a neutron star in which a solid crust forms before the r-mode has grown appreciably. The presence of an Ekman layer at the base of the crust leads to a strong dissipation of the mode energy and the final spin-period after the instability phase is much shorter than in the crustless case.

However, as discussed in the previous section, the r-mode instability may be strongly suppressed in neutron stars with a solid crust. Given that the melting temperature of the crust could be as high as $10^{10}$ K, the crust may form shortly after the neutron star is born and we need to discuss the effect that this may have on the r-mode scenario. The interplay between a growing mode and the formation of the crust leads to complicated questions that require much further study. For example, it is not at all clear to what extent the melting temperature of the crust is affected by large scale surface waves in the star. Brushing such complicating issues aside, I will assume that the r-mode is not able to prevent the crust from forming (and that the melting temperature is at the high end of the anticipated range). The main dissipation mechanism is then due to the presence of the Ekman layer (at least above the superfluid transition temperature). In this scenario, the period reached after the spin-down phase lies in the range $P \approx 2.5-4.5$ ms. Just as in the perfect fluid case, this is an interesting prediction since it agrees quite well with the data.
for the recently discovered 16 ms pulsar in N157B.

These simple examples show that the r-mode instability can lead to young neutron stars being spun down to rotation rates that agree with extrapolations from current observations. But are there really two distinct avenues of formation for nascent neutron stars? This is certainly not impossible, but it is likely to be a serious oversimplification. After all, we should not draw too many conclusions from the fact that the two r-mode scenarios (based on the very simplest models) lead to results that match the two available data points. On the other hand, the current evidence does not rule out the possibility that the r-mode instability plays a role in the evolution of a newly born neutron star. We simply need more detailed modelling and better observational data in order to be able to draw reliable conclusions.

4 The enigmatic Low-mass X-ray binaries

While the data for young neutron stars remains sketchy, a wealth of information has been gathered for accreting neutron stars in binary systems in the last five years. Detailed observations (mainly with the Rossi X-ray Timing Explorer (RXTE)) of quasiperiodic phenomena at kHz frequencies in more than a dozen Low-Mass X-ray Binaries (LMXB) strongly suggest that these systems contain rapidly spinning neutron stars [25]. This data provides strong support for the standard model for the formation of millisecond pulsars via spin-up due to accretion. Models aimed at explaining the recent RXTE observations suggest that the neutron stars in LMXB may have spin frequencies in the narrow range 260-590 Hz. Three different models have been proposed to explain this clustering. The first model (due to White and Zhang [26]) is based on the standard magnetosphere model for accretion induced spin-up, while the remaining two models are based on the idea that gravitational radiation balances the accretion torque. In the first such model for the LMXB, the gravitational waves are due to a quadrupole deformation induced in the deep neutron star crust because of accretion generated temperature gradients [7]. In the second gravitational-wave model the unstable r-modes dissipate excess angular momentum from the neutron star [6].

The r-mode model follows original suggestions by Papaloizou and Pringle [27] and Wagoner [28] (although they focussed on the instability in the f-modes). They suggested that an accreting star in which a mode-instability is active would reach a spin-equilibrium, with the emitted gravitational waves balancing the accreted angular momentum. Should this happen, the neutron stars in LMXB would be prime sources for detectable gravitational waves. However, as was independently pointed out by Levin [29] and Spruit [19], this idea may not be viable. In addition to generating gravitational waves that dissipate angular momentum from the system, the r-modes will heat the star up (via the shear viscosity). Since the shear viscosity gets weaker as the temperature increases, the mode-heating triggers a thermal runaway and in a few months the r-mode would spin an accreting neutron star down to a rather low rotation rate.

As can be seen in Figure 3, the estimated instability curve for r-modes damped
by dissipation in a viscous boundary layer agrees well with the fastest observed neutron star spin frequencies. This figure indicates that the r-mode instability may well play a role in determining the maximum spin-rate an accreting neutron star can achieve. To model the evolution of an accreting neutron star further we can use the spin-down model of Owen et al. [5]. The accreting neutron stars in LMXB are expected to have core temperatures of the order of $10^8$ K. At this temperature, the star reaches the period at which the r-mode instability sets in after accreting and spinning up for something like $10^5$ years. For the particular estimates used in this article this corresponds to a critical period of 1.5 ms. Once the r-mode becomes unstable (point A in Figure 3), viscous heating (mainly due to the energy released in the viscous boundary layer) rapidly heats the star up to a few times $10^8$ K (note that I am assuming that this will not lead to crust melting). The r-mode amplitude increases until it reaches the prescribed saturation amplitude $\alpha_s$ at which unspecified nonlinear effects halt further growth (point B in Figure 3), and the neutron star rapidly spins down as excess angular momentum is radiated as gravitational waves. When the star has spun down to the point where the mode again becomes stable (point C in Figure 3), the amplitude starts to decay and the
mode plays no further role in the spin evolution of the star (point D in Figure 3) unless the star is again spun up to the instability limit. Two examples of such r-mode cycles (corresponding to \( \alpha_s = 0.1 \) and 1, respectively) are shown in Figure 1.

In the proposed model, the accretion rate only affects the time it takes the star to complete one full cycle. As soon as the mode becomes unstable the spin-evolution is dominated by gravitational radiation and viscous heating. Once the star has gone through the brief phase when the r-mode is active it has spun down to a period in the range 2.8-4.8 ms (corresponding to \( 0.01 \leq \alpha_s \leq 1 \)). At this point the mode is again stable and continued accretion may resume to spin the star up. Since the star must accrete roughly \( 0.1 M_{\odot} \) to reach the instability point, and the LMXB companions have masses in the range \( 0.1 - 0.4 M_{\odot} \), it can pass through several “r-mode cycles” during its lifetime.

5 The formation of millisecond pulsars

The fastest known pulsar is PSR1937+21 which spins with a period of 1.56 ms. It is generally assumed that this pulsar, and the many other old pulsars that spin significantly faster than (say) 10 ms, have been spun up by accreting matter from a binary companion. This means that these stars may have evolved through one or several of the r-mode cycles discussed above. Of course, before we make this assumption we should query whether there are other possible ways that a millisecond pulsar can be formed. With the current estimates for the r-mode instability it seems likely that the answer is no. The main alternative model — that millisecond pulsars are formed by accretion-induced collapse of a white dwarf — is inconsistent with the r-mode scenario [4]. The collapse would form a star hot enough to spin down beyond a period of several milliseconds because of the instability, cf. Figure 2.

It is interesting to compare the data for the observed millisecond pulsars to the predictions of the r-mode runaway model for accreting neutron stars in LMXB. This model predicts that an accreting neutron star that has been spun up beyond 5 ms or so must remain in the rather narrow range of periods \( 1.5 - 5 \) ms until it has stopped accreting and magnetic braking slows it down. Since a given star can go through several r-mode cycles before accretion is halted one would expect observations to indicate a clustering of millisecond pulsars in this specific range of rotation rates. As is clear from Figure 4, this prediction agrees well with the millisecond pulsar data. Millisecond pulsars are mainly found in the range \( 1.56 - 6 \) ms, which is in good agreement with the proposed model.
6 Can we hope to detect the gravitational waves from r-modes?

At the present time it seems possible that the r-modes of a rapidly spinning neutron star, whether newly born or spun up to a short period by accretion, may govern the star’s spin-evolution. Given this possibility it is relevant to ask whether the gravitational waves that carry away much of the stars angular momentum are detectable. This is a particularly relevant question given the generation of large-scale interferometers (GEO600, LIGO, VIRGO, TAMA) that is about to come into operation. The detectability of the emerging gravitational waves has been assessed by Owen et al [5] for the case of young neutron stars (see also [31]). By assuming that a data analysis strategy can be tailored to the r-mode signal, and that this leads to results comparable to those of matched filtering, one can estimate that the r-modes provide a promising source for an advanced generation of detectors such as LIGO II, see Figure 5. One can estimate that the gravitational waves from a hot young neutron star will be detectable with a signal-to-noise ratio of order ten [5, 31]. It is unlikely that the r-modes will be observed by the first generation detectors, however. This will only happen if we are lucky enough to witness a unique event, e.g. a supernova in the galaxy or the local group. Such events are, however, expected to occur only every 30 years or so. In order to reach a reasonable event rate of several per year, a detector needs to be sensitive enough to observe gravitational waves from the Virgo cluster (at a distance of about 20 Mpc).
So what about the gravitational waves from the r-mode runaway in accreting neutron stars? Well, the fact that the r-mode is active only for a small fraction of the lifetime of the system (something like one month out of the $10^7$ years it takes to complete one full cycle) means that even though these sources would be supremely detectable from within our galaxy the event rate is far too low to make them relevant. However, the spin-evolution is similar to that of a hot young neutron star once the r-mode has reached the saturation amplitude. This means that the detectability of the emerging gravitational waves is comparable to that of waves from hot young neutron stars. Thus these events can potentially be observed in rather distant galaxies. For a source in the Virgo cluster these gravitational waves could be detected with a signal to noise ratio of a few using LIGO II, cf. Figure 5. However, even at the distance of the Virgo cluster these events would be quite rare. By combining a birth rate for LMXB of $7 \times 10^{-6}$ per year per galaxy with the fact that the the volume of space out to the Virgo cluster contains $\sim 10^3$ galaxies, and the possibility that each LMXB passes through (say) four r-mode cycles during its lifetime we deduce that one can only hope to see a few events per century in Virgo (an event rate comparable to the supernova rate in our galaxy). In order to see several events per year the detector must be sensitive enough to detect these gravitational waves from (say) 150 Mpc. This would require a more advanced detector configuration such as a narrowband LIGO II, cf. Figure 5. But still, this possibility should not be ruled out.
7 Questions questions questions...

As I discussed in the beginning of this article, our modelling of the r-mode instability has reached a level where we need to address a large number of truly difficult questions. In order to arrive at reliable answers we need to probe the very extremes of physics. Given the many uncertainties associated with, for example, the supranuclear equation of state it seems quite likely that we will not have the desired answers any time soon. This means that we may not be able to provide "accurate" theoretical templates that will be useful in searching the data streams once the new generation of gravitational-wave detectors come on-line. In fact, I think it is very likely that we need observations to guide us towards a better understanding of the various relevant pieces of physics. It is with this attitude that I have written this article. I wanted to sketch how we can use constraints from observations to guide us through the maze of difficult theoretical issues. Of course, the actual models I have used here are rather naive. Still it is satisfying to see how the various proposed scenarios agree well with much of the available observational data. And I think our understanding could be advanced significantly if we knew the answer to three questions:

*Are there different evolution routes for a newly born neutron star?*

*Do accreting neutron stars undergo r-mode runaway cycles?*

*And does this lead to an associated clustering of the millisecond pulsars in a narrow range of spin periods?*

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