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LAMP
SERIES REPORT
(Laser, Atomic and Molecular Physics)

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DYNAMICS OF TWO THREE-LEVEL ATOMS INTERACTING WITH TWO MODES OF RADIATION

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ABSTRACT

The system of two 3-level atoms in interaction with 2-modes of the radiation field is invesitgated. The (Ξ cascade) configuration for the two identical atoms is taken and exact resonance is considered. The wave function is calculated and the phenomena of collapses and revivals, sub-poissonian statistics, and two-mode anticorrelation are discussed. Subrevivals are noted for these phenomena.

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Preface

The ICTP-LAMP reports consist of manuscripts relevant to seminars and discussions held at ICTP in the field of Laser, Atomic and Molecular Physics (LAMP).

These reports aim at informing LAMP researchers on the activity carried out at ICTP in their field of interest, with the specific purpose of stimulating scientific contacts and collaboration of physicists from Third World Countries.

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1. Introduction:

One of the exactly solvable models in the field of quantum optics is the so called Jaynes - Cummings model (JCM) which describes quantum mechanically the interaction of a single mode of the radiation field with a two-level atom [1]. In this model damping is precluded and the rotating wave approximation is taken into consideration. The model lends itself to investigate methods of preparing non-classical states of radiation [2]. Although this model is simple, it is very rich with quantum phenomena such as collapses and revivals of the cavity Rabi oscillations, antibunching and squeezing [3]. In the last decade a lot of effort has been made to extend the basic (J.C) model and led to fruitful results.

The dynamics of a three-level atom interacting with two modes of the quantized cavity field has been studied [3-6]. Quantum collapses and revivals of the mean photon number as well as the atomic occupation probabilities have been predicted. Squeezing and higher-order squeezing in the three-level, two-mode system have been investigated [7]. Further generalizations included studies of dynamics of an M-level atom interacting with cavity fields [8]. Also the N-level and (N-1) modes with single photon transition and multiphoton transitions and intensity dependent coupling parameters have been discussed [9]. An interesting generalization of the JCM is the model with a limited number of two-level atoms in interaction with the quantized field of radiation. Analysis of the dynamics of two-level atoms interacting with a single-mode quantized electromagnetic field at resonance have been made for a single transition [10] and for the two-photon transition [11].In a number of studies exact solutions have been obtained for two two-level atom models with one-photon [12-17] and multi-photon transition [18,19]. The spontaneous emission spectrum of two two-level atoms with one-photon transitions in a nonideal cavity has been calculated [20] The effect of atomic cooperation on the emission spectrum of two atoms has been considered and the differences with that of a single atom has been indicated [21]. The collapses and revivals phenomenon in the atomic inversion for this system in the case of a single photon interaction and two-photon interaction and for the photon distribution for both transitions have been discussed [10,19]. The phenomena of normal squeezing and amplitude squared squeezing have been discussed for this system in both interactions for coherent inputs [19].

In this article we investigate the dynamics of a system of two three-level atoms and two different modes of the quantized electromagnetic field. Interaction with the field is considered to effect the transition between the atomic levels. The atomic levels configuration is considered to be the cascade (Ξ) configuration. The Hamiltonian is written for the system in section 2. Then a solution is given for the wave function. Once the wave function of the system is calculated, different phenomena can be discussed.

We discuss the effect of the cooperative interaction between the two identical atoms on the collapses and revivals phenomenon, and the time evolution of correlation functions which give rise to non-classical effects Furthermore phenomena of different squeezing are mentioned in the following sections.

2- Formulation of the problem:

The considered system consists of two three-levels atoms in interaction with two modes of the quantized field. The two atoms are close to each other so that we shall ignore any spatial dependence. Furthermore, we ignore the interaction between the two atoms.

The Hamiltonian for the system in the rotating wave approximation is written in the following form.

$$H = \sum_{j=1}^{2} \Omega_j a_j^+ a_j + \sum_{j=1}^{2} \{\omega_e^{(j)} (|e\rangle_{j\,j}\langle e|) + \omega_i^{(j)} (|i\rangle_{j\,j}\langle i|) + \omega_g^{(j)} (|g\rangle_{j\,j}\langle g|)\} + \sum_j \{k_1^{(j)} a_1 |e\rangle_{j\,j}\langle i| + k_2^{(j)} a_2 |i\rangle_{j\,j}\langle g| + h.c\} \qquad (2.1)$$

where $|e\rangle_j$, $|i\rangle_j$ and $|g\rangle_j$ are the excited, intermediate and the ground states of the j th atom with energies $\omega_e^{(j)}$, $\omega_i^{(j)}$, $\omega_g^{(j)}$ respectively. $|N_1\rangle, |N_2\rangle$ are the photon number states of the first and second modes respectively, $a_j (a_j^+)$ is the photon annihilation (creation)operator of the j th mode of energy Ω_j; $k_1^{(j)}$, $k_2^{(j)}$ are the atom-field coupling constants between the j th atom and the two modes respectively. The 3-level atom is taken in the cascade (Ξ) configuration. The mode 1 effects the transition between the intermediate state $|i\rangle$ and the upper-state $|e\rangle$, while mode 2 effects the transition between the intermediate and the ground state in both atoms. We shall consider the case of exact resonance, i.e. we have

$$\Omega_1 = \omega_e^{(j)} - \omega_i^{(j)}, \ \Omega_2 = \omega_i^{(j)} - \omega_g^{(j)} \tag{2.2}$$

The Hamiltonian (2.1) has the following constants of motion.

$$\hat{N}_1 + \sum_{j=1}^{2} (|e\rangle_j \langle_j e|), \quad \hat{N}_2 - \sum_{j=1}^{2} (|g\rangle_j \langle_j g|) \tag{2.3}$$

where $\hat{N}_i = a_i^+ a_i$ is the photon number operator for the mode i.

Consequently in the reduced subspace we may write the wave function of the system at any time t > 0, in the interaction picture in the following form

$$\begin{aligned} |\psi(t)\rangle = \ & X_1(t)|e_1,e_2;N_1,N_2\rangle + X_2(t)|e_1,i_2;N_1+1,N_2\rangle + \\ & + X_3(t)|e_1,g_2;N_1+1,N_2+1\rangle + X_4(t)|i_1,e_2;N_1+1,N_2\rangle + \\ & + X_5(t)|i_1,i_2;N_1+2,N_2\rangle + X_6(t)|i_1,g_2;N_1+2,N_2+1\rangle + \\ & + X_7(t)|g_1,e_2;N_1+1,N_2+1\rangle + X_8(t)|g_1,i_2;N_1+2,N_2+1\rangle + \\ & + X_9(t)|g_1,g_2;N_1+2,N_2+2\rangle \end{aligned} \tag{2.4}$$

Where the $X_i(t)$ are the time - dependent probability amplitudes for the corresponding eigenstates. Their dependence on the photon numbers N_1 and N_2 is suppressed. The time evolution of the wave function $|\psi(t)\rangle$ is given by the Schrödinger equation:

$$\frac{\partial|\psi(t)\rangle}{dt} = -i\, H_{in}|\psi(t)\rangle \tag{2.5a}$$

we can write the equation as

$$\frac{d\mathbf{X}(t)}{dt} = -\,i\,\mathbf{MX}(t) \tag{2.5b}$$

Where $\mathbf{X}$ is the column matrix with $X_i(t)$ (i = 1-9) as its components, and

$$M = \begin{bmatrix} 0 & k_1^{(2)}\gamma & 0 & k_1^{(1)}\gamma & 0 & 0 & 0 & 0 & 0 \\ k_1^{(2)}\gamma & 0 & k_2^{(2)}\gamma' & 0 & k_1^{(1)}\eta & 0 & 0 & 0 & 0 \\ 0 & k_2^{(2)}\gamma' & 0 & 0 & 0 & k_1^{(1)}\eta & 0 & 0 & 0 \\ k_1^{(1)}\gamma & 0 & 0 & 0 & k_1^{(2)}\eta & 0 & k_2^{(1)}\gamma' & 0 & 0 \\ 0 & k_1^{(1)}\eta & 0 & k_1^{(2)}\eta & 0 & k_2^{(2)}\gamma' & 0 & k_2^{(1)}\gamma' & 0 \\ 0 & 0 & k_1^{(1)}\eta & 0 & k_2^{(2)}\gamma' & 0 & 0 & 0 & k_2^{(1)}\eta' \\ 0 & 0 & 0 & k_2^{(1)}\gamma' & 0 & 0 & 0 & k_1^{(2)}\eta & 0 \\ 0 & 0 & 0 & 0 & k_2^{(1)}\gamma' & 0 & k_1^{(2)}\eta & 0 & k_2^{(2)}\eta' \\ 0 & 0 & 0 & 0 & 0 & k_2^{(1)}\eta' & 0 & k_2^{(2)}\eta' & 0 \end{bmatrix} \tag{2.6a}$$

where $\gamma = \sqrt{N_1 + 1}$, $\eta = \sqrt{N_1 + 2}$

and $\gamma' = \sqrt{N_2 + 1}$, $\eta' = \sqrt{N_2 + 2}$ (2.6b)

The solution of equation (2.6) has the form

$$X_i(t) = \sum_{j=1}^{9} Q_{ij}(t)\, X_j(0) \tag{2.7}$$

The elements $X_j(0)$ specify the initial condition of the system.

In order to get some analytical solutions, we take into account two identical atoms interacting with the same coupling constants i.e we have $k_i^{(j)} = k_i$, and $\omega^{(j)} = \omega$, independent of the atomic site. Thus we consider the cooperative behaviour of the two atoms. Further, we take the ratio R as the ratio between the two coupling constants i.e $R = \frac{k_2}{k_1}$, and scale the time to the constant k_1, i.e the scaled time $\tau = k_1 t$ is used. With these assumptions into consideration, we give some of the time-dependent elements $Q_{ij}(N_1, N_2, t)$ in the following.

$$Q_{11} = \left(\frac{\lambda_1^2\cos\sqrt{\lambda_1}\;\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\lambda_2^2\cos\sqrt{\lambda_2}\;\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\lambda_3^2\cos\sqrt{\lambda_3}\;\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right)$$

$$- 2(2C^2R^2+2e^2+F^2R^2)\left(\frac{\lambda_1\cos\sqrt{\lambda_1}\;\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\lambda_2\cos\sqrt{\lambda_2}\;\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\lambda_3\cos\sqrt{\lambda_3}\;\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right)$$

$$+ (5C^4R^4+2C^2e^2R^2+4C^2F^2R^4+5e^4+6e^2F^2R^2)\left(\frac{\cos\sqrt{\lambda_1}\;\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\cos\sqrt{\lambda_2}\;\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}\right.$$

$$\left. + \frac{\cos\sqrt{\lambda_3}\;\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right) + (2C^6R^6+2C^4R^6F^2-2C^4R^4e^2+6C^2e^2F^2R^4-2C^2e^4R^2+2e^6$$

$$+ 4e^4F^2R^2)\left(\frac{1}{\lambda_1\lambda_2\lambda_3} - \frac{\cos\sqrt{\lambda_1}\;\tau}{\lambda_1(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} - \frac{\cos\sqrt{\lambda_2}\;\tau}{\lambda_2(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} -\right.$$

$$\left. - \frac{\cos\sqrt{\lambda_3}\;\tau}{\lambda_3(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right)$$

$$Q_{12} = Q_{14} = i\gamma\left[\frac{\lambda_1\sqrt{\lambda_1}\;\sin\sqrt{\lambda_1}\;\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\lambda_2\sqrt{\lambda_2}\;\sin\sqrt{\lambda_2}\;\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\lambda_3\sqrt{\lambda_3}\;\sin\sqrt{\lambda_3}\;\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\right.$$

$$- (3\gamma\backslash^2R^2+2\eta^2+2\eta\backslash^2R^2)\left(\frac{\sqrt{\lambda_1}\;\sin\sqrt{\lambda_1}\;\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\sqrt{\lambda_2}\;\sin\sqrt{\lambda_2}\;\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}\right.$$

$$\left. + \frac{\sqrt{\lambda_3}\;\sin\sqrt{\lambda_3}\;\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right) + (2\gamma\backslash^4R^4+3\gamma\backslash^2R^2\eta^2+2\gamma\backslash^2\eta\backslash^2R^4+\eta^4+2\eta\backslash^2\eta^2R^2)$$

$$\left. \left(\frac{\sin\sqrt{\lambda_1}\;\tau}{\sqrt{\lambda_1}(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\sin\sqrt{\lambda_2}\;\tau}{\sqrt{\lambda_2}(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\sin\sqrt{\lambda_3}\;\tau}{\sqrt{\lambda_3}(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right)\right]$$

$$Q_{51} = -2\gamma\eta\left[\frac{-\lambda_1\cos\sqrt{\lambda_1}\;\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} - \frac{\lambda_2\cos\sqrt{\lambda_2}\;\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} - \frac{\lambda_3\cos\sqrt{\lambda_3}\;\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\right.$$

$$+ 2(\eta^2+\eta\backslash^2R^2)\left(\frac{\cos\sqrt{\lambda_1}\;\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\cos\sqrt{\lambda_2}\;\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\cos\sqrt{\lambda_3}\;\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right)$$

$$+ (\eta^4+2\gamma\backslash^2\eta\backslash^2R^4+2\eta^2\eta\backslash^2R^2-\gamma\backslash^4R^4)\left(\frac{1}{\lambda_1\lambda_2\lambda_3} - \frac{\cos\sqrt{\lambda_1}\;\tau}{\lambda_1(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}\right.$$

$$\left.\left. - \frac{\cos\sqrt{\lambda_2}\;\tau}{\lambda_2(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} - \frac{\cos\sqrt{\lambda_3}\;\tau}{\lambda_3(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} \right)\right]$$

$$Q_{13} = Q_{17} = R\gamma\gamma'\Big[\frac{\lambda_1\cos\sqrt{\lambda_1}\,\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\lambda_2\cos\sqrt{\lambda_2}\,\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\lambda_3\cos\sqrt{\lambda_3}\,\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}$$

$$- (3\gamma'^2R^2-\eta^2+2\eta'^2R^2)\Big(\frac{\cos\sqrt{\lambda_1}\,\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\cos\sqrt{\lambda_2}\,\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}$$

$$+ \frac{\cos\sqrt{\lambda_3}\,\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\Big) - (2\gamma'^4R^4 + 2\gamma'^2\eta'^2R^4 + 2\eta^2\eta'^2R^2 - 2\eta^4)$$

$$\Big(\frac{-\cos\sqrt{\lambda_1}\,\tau}{\lambda_1(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} - \frac{\cos\sqrt{\lambda_2}\,\tau}{\lambda_2(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} - \frac{\cos\sqrt{\lambda_3}\,\tau}{\lambda_3(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} + \frac{1}{\lambda_1\lambda_2\lambda_3}\Big]$$

$$Q_{16} = Q_{18} = i3\gamma\gamma'\eta R\Big[\frac{\sqrt{\lambda_1}\,\sin\sqrt{\lambda_1}\,\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\sqrt{\lambda_2}\,\sin\sqrt{\lambda_2}\,\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\sqrt{\lambda_3}\,\sin\sqrt{\lambda_3}\,\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}$$

$$-(\gamma'^2R^2+\eta^2)\Big\{\frac{\sin\sqrt{\lambda_1}\,\tau}{\sqrt{\lambda_1}(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\sin\sqrt{\lambda_2}\,\tau}{\sqrt{\lambda_2}(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\sin\sqrt{\lambda_3}\,\tau}{\sqrt{\lambda_3}(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\Big\}$$

$$Q_{19} = 6\gamma\gamma'\eta\eta'R^2\Big[\frac{\cos\sqrt{\lambda_1}\,\tau}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} + \frac{\cos\sqrt{\lambda_2}\,\tau}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} + \frac{\cos\sqrt{\lambda_3}\,\tau}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}$$

$$+ (\gamma'^2R^2+\eta^2)\Big(\frac{1}{\lambda_1\lambda_2\lambda_3} - \frac{\cos\sqrt{\lambda_1}\,\tau}{\lambda_1(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)} - \frac{\cos\sqrt{\lambda_2}\,\tau}{\lambda_2(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)} -$$

$$- \frac{\cos\sqrt{\lambda_3}\,\tau}{\lambda_3(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\Big)\Big] \qquad (2.8a)$$

with

$$\lambda_1 = [\gamma'^2R^2 + \eta^2]$$

$$\lambda_2 = \frac{\delta+2\gamma^2+3\gamma'^2R^2+3\eta^2+2\eta'^2R^2}{2}$$

$$\lambda_3 = \frac{-\delta+2\gamma^2+3\gamma'^2R^2+3\eta^2+2\eta'^2R^2}{2} \qquad (2.8b)$$

where

$$\delta^2 = [4\gamma^4 - 4\gamma^2(\gamma'^2R^2 - \eta^2 + 2\eta'^2R^2) + \gamma'^4R^4$$
$$+ 2\gamma'^2R^2(17\eta^2 + 2\eta'^2R) + \eta^4 - 4\eta^2\eta'^2R^2 + 4\eta'^4R^4]$$

It is apparent that when we take R = 0 in this case, we get the case of the cooperative two two-level atoms in interaction with a single mode [10,11,19]. This because λ_1 and $\lambda_3 \longrightarrow \eta^2$ and all the oscillatory coefficients related to these vanish identically in the last expressions, while $\lambda_2 \longrightarrow \lambda_+$ [see 19]. Once the wave function (2.4) is calculated, all expectation values for the field or the atomic operators can be computed. In the following discussion we use the calculated wave function to discuss the dynamics of the system as well as some phenomena related to the state of the field. Because of the complex structure of the amplitude coefficients Q_{ij}, we recourse to numerical investigations in what follows

3- Discussion:

Now let us specify the initial condition for the system. We suppose both atoms to be initially in the excited state of energy ω_o and the field modes in coherent states i.e

$$X_1(0) = \sum_{N_1, N_2} q_1(N_1)\, q_2(N_2) \quad , \; X_J(0) = 0 \text{ for } J \neq 1 \tag{2.9}$$

Where $q_i(N_i) = \exp(-|\alpha_i|^2/2)\alpha_i^{N_i} / \sqrt{N_i!}$ $\qquad (i = 1,2)$

specifies the coherent state distribution for the mode . In the following discussion we take $\bar{N}_i = |\alpha_i|^2 = 10$ for $i = 1,2$

We shall discuss the following phenomena:

collapses and revivals, sub-Poissonian and super-Poissonian distribution, two-mode correlative and mention different types of squeezing.

3.1 Collapses and Revivals:

We look for this phenomenon in the pho t on number operator, we plot the time evolution $\langle\hat{N}_1(t)\rangle - \bar{N}_1$ and $\langle\hat{N}_2(t)\rangle - \bar{N}_2$ against $\tau = k_1 t$.

These are given by

$$\langle\hat{N}_1(t)\rangle - \bar{N}_1 = 1 + \sum_{N_1, N_2} |q_1(N_1)|^2 |q_2(N_2)|^2 \{|Q_{51}|^2 + 2|Q_{16}|^2 + |Q_{91}|^2 - |Q_{11}|^2\} \tag{3.1}$$

$$\langle\hat{N}_2(t)\rangle - \bar{N}_2 = 2 \sum_{N_1, N_2} |q_1(N_1)|^2 |q_2(N_2)|^2 \{|Q_{31}|^2 + |Q_{16}|^2 + |Q_{91}|^2\} \tag{3.2}$$

In Figs (1a-c) we plot the quantity $\langle\hat{N}_1(t)\rangle - \bar{N}_1$ for different values of the ratio R of the coupling constants k_1 and k_2($R = \frac{k_2}{k_1}$) namely for R = 0.5, 0.8 and 1.0. In fig (1a) - the plot for R = 0.5 - we note that after the collapse of the damped oscillatory behaviour there is a revival but with very small amplitude. The amplitude of this revival in particular increases and splits into two revivals as we increase R to 0.8 (fig 1b), the splitting becomes apparent for R = 1 (fig 1c). Following this a close revival (fig 1a) with bigger amplitude. However this revival is quite complex. This is because of the nature of the expression (3.1). As we increase R this diminishes and for R= 1 we note that this revival breaks up into subrevivals. As time develops we note that revivals start to interfere and no clean collapses are shown. It is observed that as R increases the collapse period decreases and also the revival times. The oscillation amplitudes decrease as the time passes. The appearance of subrevivals is noted in this case for R = 1.

The same situation for $\langle \hat{N}_2(t)\rangle - \bar{N}_2$ of equation (3.2) is illustrated in figs (2a-c). Fig (2a) for R = 0.5 shows a different behaviour from the plot for $\langle \hat{N}_1(t)\rangle$ in fig (1a). But as we increase R (fig 2b and 2c) they become similar especially when R = 1 (fig 2c) which is expected, since in this case the coupling constants are equal. In this later case we note again the subrevivals. The general trend for the collapses and revivals periods is the same for the 1st mode. Further, we note that the oscillations for $\langle \hat{N}_1(t)\rangle - \bar{N}_1$ are around 1.2 and for $\langle \hat{N}_2(t)\rangle - \bar{N}_2$ around 0.5. This means that the atomic system losses its energy to the field modes, and most of the time more energy is stored in the modes of the field than its initial energy.

3-2 Subpoissonian and Superpoissonian Distributions:

The distribution of the photons in the field modes can be considered through the second order correlation function which is defined through [22]

$$g_i^2(t) = \frac{\langle \hat{N}_i^2(t)\rangle - \langle N_i(t)\rangle}{(\langle N_i(t)\rangle)^2} \qquad i = 1,2 \tag{3.3}$$

where $\langle \hat{N}_i^2(t)\rangle$ can be found by choosing s_1 and s_2 from the following general expression

$$\langle N_1^{s_1} N_2^{s_2}\rangle = \sum_{N_1=0}\sum_{N_2=0} |q(N_1)|^2|q(N_2)|^2\{ N_1^{s_1} N_2^{s_2} |Q_{11}|^2$$

$$+ (N_1+1)^{s_1}(N_2)^{s_2}(|Q_{21}|^2 + |Q_{41}|^2) + (N_1+1)^{s_1}(N_2+1)^{s_2}(|Q_{31}|^2$$

$$+ |Q_{71}|^2) + (N_1+2)^{s_1} N_2^{s_2}(|Q_{51}|^2) + (N_1+2)^{s_1}(N_2+1)^{s_2}(|Q_{16}|^2$$

$$+ |Q_{81}|^2) + (N_1+2)^{s_1}(N_2+2)^{s_2}(|Q_{91}|^2)\} \tag{3.4}$$

The distribution is said to be sub-poissonian if $g_i^{(2)} < 1$ and super-poissonian for $g_i^{(2)} > 1$. The temporal development of $g_1^{(2)}(t)$ and $g_2^{(2)}(t)$ for various values of R and the initial conditions specified earlier is shown in figs (3 and 4). The shapes of the curves are seen to be similar to that of the photon number but with its first revival split into two as reminiscent from JCM [23] and its generalization [24]. The correlation function $g_1(t)$ oscillates but the distribution tends to be sub-poissonion for R < 1 but as R = 1 we note that the field oscillates from super-poissonion to sub-poissonion distribution (fig 3.c). The correlation function $g_2^{(2)}(t)$ for the second mode on the other hand shows super-poissonian behaviour as shown in figs (4a,b) for R = 0.5 and 0.8 respectively. However for R = 1 we find that the field oscillates between super- and sub-poissonion behaviour in a similar behaviour to the 1 st mode in this case.

3.c Anticorrelation :

In fig 5 we have plotted the cross correlation function

$$\Delta_{cross}(t) = \langle N_1(t)N_2(t)\rangle - \bar{N}_1\bar{N}_2 \tag{3.5}$$

The two modes are said to be anticorrelated when $\Delta_{cross}(t) < 0$ and correlated for > 0. Collapses and revivals are shown in the figures for $\Delta_{cross}(t)$ (fig 5a-c). The system shows anticorrelation for R = 0.5 and 0.8 (see fig 5a and b). However for R = 1 we find the system exhibiting correlation rather than anticorrelation for any time larger than the collapse time.

3.d Squeezing :

Fluctuations in the two quadratures [25]and [26]

$$Z = \frac{a_1 + a_2 + a_1^+ + a_2^+}{2\sqrt{2}} \quad \text{and } Z = \frac{a_1 + a_2 - a_1^+ - a_2^+}{2\sqrt{2}}$$

determine the state of the normal two mode squeezing of two modes of the field.

The field is said to be in a squeezing state if

$$\Delta^2 Z \text{ or } \Delta^2 Z < \frac{1}{4}$$

Using the wave function (2.4) we have computed the normal two-mode squeezing parameter $\Delta^2 Z$ for different values of R and we found that a very small amount of squeezing occurs for a very short interval and then it is lost. Also consideration of different types of squeezing: Frequency sum or difference did not give encouraging results for the initial conditions specified in this paper.

4. Conclusions:

We have considered a system of two identical three-level atoms in interaction with two quantized modes of the radiation field. The atomic schematic representation is the (Ξ) cascade configuration, and the case of exact resonance is considered.The wave function of the system in the interaction picture is calculated. The case of cooperative interaction between the two atoms and the field is taken into consideration.

The atoms are supposed to be in their upper-most excited states and the field modes assume coherent distributions with equal mean photon number. The phenomenon of collapses and revivals in the photon numbers of the two modes are discussed. Under the specified initial conditions the atoms lose some of their energy to the field modes. The collapses and revivals are sensitive to the ratio R between the coupling constants of the two modes. Consequently, subrivivals appear when this ratio increases. The autocorrelation function $g^{(2)}$ and the cross correlation function show collapses and revivals and the photon distribution is sensitive to the ratio R. Sub-poissonian distribution occurs in the two modes as well as anticorrelation between the two modes, for some values of R.

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Captions to the figures

Fig 1: Time development for $\langle N_1(t)\rangle - \overline{N}_1$ of the first mode

a) for $R = 0.5$ b) values shifted by 2.3 for $R = 0.8$

c) values shifted by 4.5 for $R = 1.0$

Fig 2: Time evolution for $\langle N_2(t)\rangle - \overline{N}_2$ of the 2nd mode

a) for $R = 0.5$ b) values shifted by 1.2 for $R = 0.8$

c) values shifted by 3.1 for $R = 1.0$

Fig 3: dependence of $g_1^{(2)}$ for the first mode

a) for $R = 0.5$ b) values shifted by 0.07 for $R = 0.8$

c) values shifted by 0.14 for $R = 1.0$

Fig 4: Time development for $g_2^{(2)}(t)$ for the second mode

a) for $R = 0.5$ b) values shifted by 0.03 for $R = 0.8$

c) values shifted by 0.07 for $R = 1.0$

Fig 5: Time dependence of $\Delta_{cross}(t)$

a) for $R = 0.5$ b) values shifted by 4.0 for $R = 0.8$

c) values shifted by 9.0 for $R = 1.0$

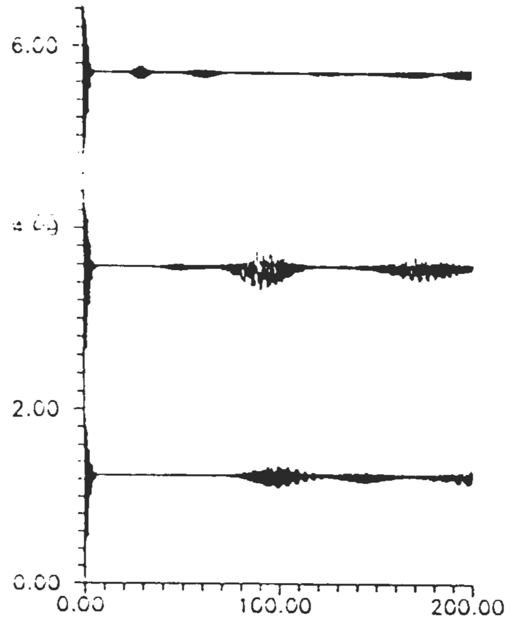

Fig.1

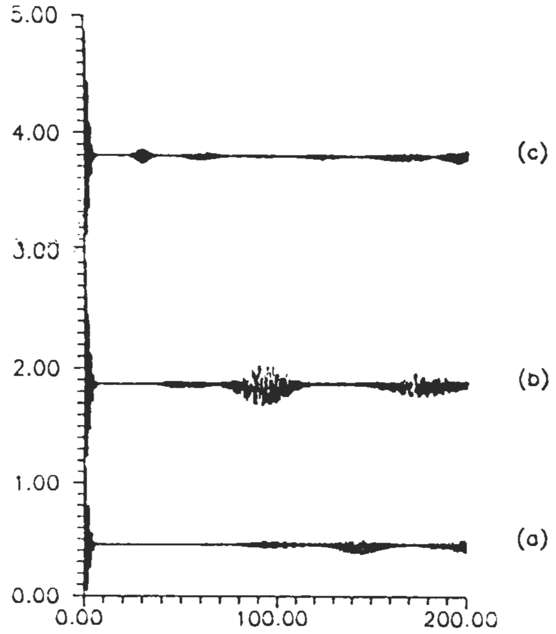

Fig.2

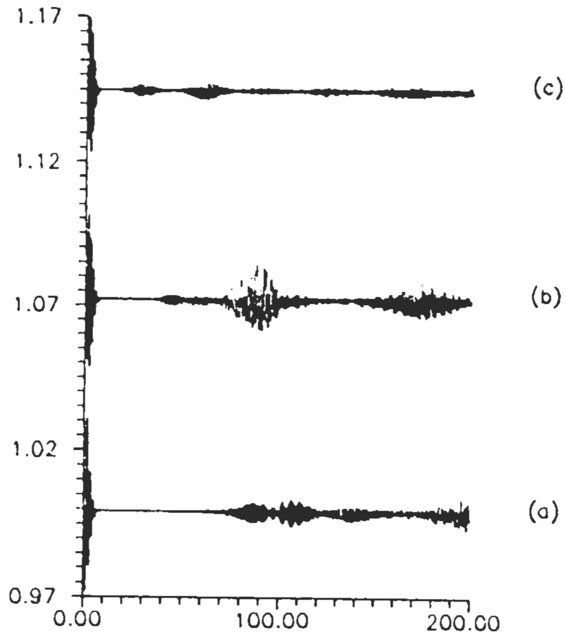

Fig.3

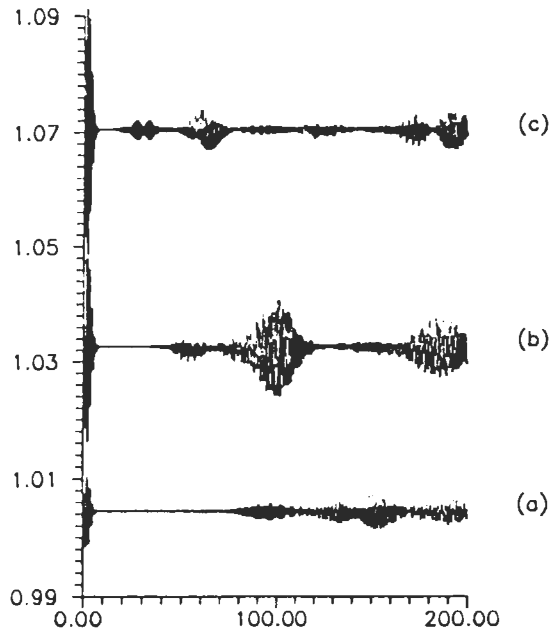

Fig.4

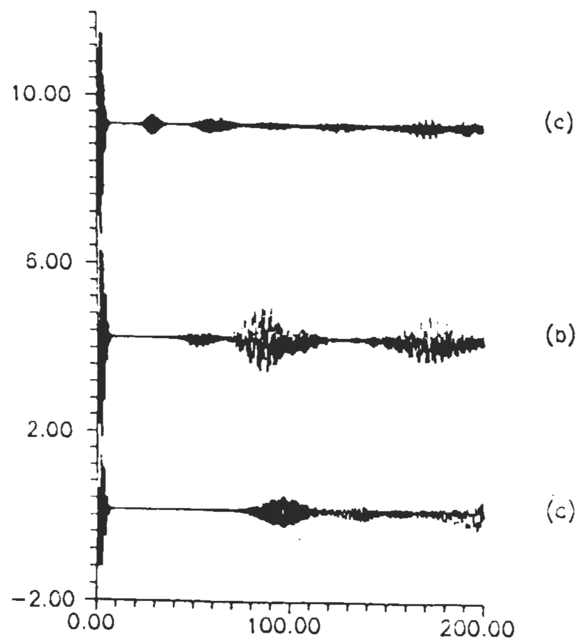

Fig.5