LAMP/93/3

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

LAMP SERIES REPORT

(Laser, Atomic and Molecular Physics)

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THREE-PHOTON MICROMASERS

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ABSTRACT

A non-degenerate 3-photon micromaser is analyzed. A 4-level atom is taken and 3 modes of the field are considered. The model is solved for the case of resonance and the master equation for the density matrix is obtained. Semi-analytical solutions are obtained under specified approximations. The three modes can exist depending on the time of interaction.

MIRAMARE – TRIESTE
August 1993

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Preface

The ICTP-LAMP reports consist of manuscripts relevant to seminars and discussions held at ICTP in the field of Laser, Atomic and Molecular Physics (LAMP).

These reports aim at informing LAMP researchers on the activity carried out at ICTP in their field of interest, with the specific purpose of stimulating scientific contacts and collaboration of physicists from Third World Countries.

If you are interested in receiving additional information on the Laser and Optical Fibre activities at ICTP, kindly contact Professor Gallieno Denardo, ICTP.

1 Introduction

New methods of atomic physics, in particular high resolution laser spectroscopy have opened the field of matter-radiation interaction for experimental investigations, when it became possible to prepare simple physical systems with extremely large polarizabilities. Those include highly-excited states, the so-called Rydberg atoms, which can be strongly coupled to the radiation field, and can usually be treated as one-electron system [1]. The operation of micromasers [2] requires not only resonance of an atomic transition, and a low order cavity resonance, but also negligible damping of the cavity field. The coupled physical system of atom and field evolves without perturbation during observation time, and the radiation energy can be many times exchanged with the atomic system. Micromasers are operated with Rydberg atoms and resonators reaching very high-Q values. This situation realizes one of the fundamental theoretical models of radiation-matter interaction, namely the Jaynes-Cummings Model (JCM) [3]. In this model the cavity field retains only one degree of freedom, and the atom is a two-level system. In the rotating wave approximation (RWA), it allows exact solutions. The fluctuations of the micromaser field at low intensities are dominated by its discrete nature. Predicted properties of this model such as collapses and revivals [4] have been observed [5]. Also vacuum Rabi splitting for a single atom in an optical cavity [6] has been observed [7]. Furthermore, excited atoms injected into the micromaser not only pump the cavity field but also serve as probes to measure field quantities [8].

Micromasers have been operated for single and degenerate two-photon transitions [2]. The case of non-degenerate two-photon transition has been discussed recently [9]. In this article we analyze the case of a three-photon micromaser. As usual, it is assumed that atomic decay is neglected while traversing the cavity, i.e. the atomic life-times are much longer than the time taken by the atom to traverse the cavity.

The plan of the article as follows in Section 2, we introduce the model. The equation of motion for the density matrix is considered in Sec. 3. In Section 4, solutions to the mean photon numbers in the different modes are discussed under specified approximations. Some conclusions are drawn in Section 5.

2 The Model

We suppose an atom of 4 energy levels with upper-state $|e\rangle$, ground state $|g\rangle$, and two intermediate states ($|1\rangle$ and $|2\rangle$) with energies $\omega_e > \omega_1 > \omega_2 > \omega_g$ respectively (see Fig. 1). The transitions $|e\rangle \leftrightarrow |1\rangle$, $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |g\rangle$ are affected by single photons from different modes of the radiation field their energies are Ω_1, Ω_2 and Ω_3 respectively. This system can be described in the rotating wave approximation (RWA) by the following Hamiltonian in

$$H = \omega_e|e\rangle\langle e| + \omega_1|1\rangle\langle 1| + \omega_2|2\rangle\langle 2| + \omega_g|g\rangle\langle g| + \sum_{j=1}^{3}\Omega_j a_j^+ a_j$$

$$+ \{\lambda_1 a_1|e\rangle\langle 1| + \lambda_2 a_2|1\rangle\langle 2| + \lambda_3 a_3|2\rangle\langle g| + h.c.\} \quad (1)$$

where $a_i(a_i^+)$ is the radiaiton field annihilation (creation) operator of the ith mode, and λ_i are the coupling constants between the atom and the field. These coupling constants are proportional to the expectation values for the dipole moment operator between the states concerned. In what follows we assume exact resonance between the atom and the field, i.e.

$$\Omega_1 = \omega_e - \omega_1,\ \Omega_2 = \omega_1 - \omega_2 \text{ and } \quad \Omega_3 = \omega_2 - \omega_g\ . \tag{2}$$

Due to the structure of the Hamiltonian, and its constants of motion, it is observed that the Hilbert space of the system splits into subspaces spanned by the states

$$|e; N_1, N_2, N_3\rangle, |1; N_1+1, N_2, N_3\rangle, |2; N_1+1, N_2+1, N_3\rangle$$

and

$$|g; N_1+1, N_2+1, N_3+1\rangle \tag{3}$$

where $|N_i\rangle$ is the Fock state having N_i photons of the ith mode. In this subspace, we can expand the wavefunction of the system in the interaction picture, at any time $t > 0$ in the form

$$\begin{aligned} |\psi(t)\rangle \quad &= c_e(N_1, N_2, N_3, t)|e; N_1, N_2, n_3\rangle \\ &+ c_1(N_1+1, N_2, N_3, t)|1; N_1+1, N_2, N_3\rangle \\ &+ c_2(N_1+1, N_2+1, N_3, t)|2; N_1+1, N_2+1, N_3\rangle \\ &+ c_g(N_1+1, N_2+1, N_3+1, t\rangle|g; N_1+1, N_2+1, N_3+1\rangle \end{aligned} \tag{4}$$

where the c's stand for the time-dependent amplitudes. Their time evolution can be obtained from solving the Schrödinger in the interaction picture. The following set of simultaneous differential equations is obtained

$$i\frac{d}{dt}\mathbf{C} = \mathbf{MC} \tag{5}$$

where $\mathbf{C}$ is the column matrix

$$\begin{bmatrix} c_e \\ c_1 \\ c_2 \\ c_g \end{bmatrix} \tag{5a}$$

with the arguments subpressed for brevity and $\mathbf{M}$ is the square matrix

$$\mathbf{M} = \begin{bmatrix} 0 & \lambda_1\sqrt{N_1+1} & 0 & 0 \\ \lambda_1\sqrt{N_1+1} & 0 & \lambda_2\sqrt{N_2+1} & 0 \\ 0 & \lambda_2\sqrt{N_2+1} & 0 & \lambda_3\sqrt{N_3+1} \\ 0 & 0 & \lambda_3\sqrt{N_3+1} & 0 \end{bmatrix} \tag{5b}$$

The eigenvalues of $\mathbf{M}$ in this case are

$$\omega_\pm^2 = \frac{1}{2}\Omega_R^2 \pm \frac{1}{2}\sqrt{\Omega_R^2 - 4\lambda_1^2\lambda_3^2(N_1+1)(N_3+1)} \tag{5c}$$

with

$$\Omega_R^2 = \lambda_1^2(N_1+1) + \lambda_2^2(N_2+1) + \lambda_3^2(N_3+1)\ .$$

Thus Ω_R is the stimulated Rabi frequency for this model.

When the atomic system starts from its excited state $|e\rangle$ and the field modes in vacuo i.e.

$$c_e(N_1,N_2,N_3,0)=\delta_{N,0}\delta_{N_20}\delta_{N_30},\ c_1=c_2=c_g=0 \tag{6}$$

the set of equations (5) admits the solution

$$c_e(N_1,N_2,N_3,t)=A_+\cos\omega_+t+A_-\cos\omega_-t$$

$$c_1(N_1+1,N_2,N_3;t)=\frac{-i}{\lambda_1\sqrt{N_1+1}}(A_+\omega_+\sin\omega_+t-A_-\omega_-\sin\omega_-t)$$

$$c_2(N_1+1,N_2+1;N_3,t\rangle=\frac{1}{\lambda_1\lambda_2\sqrt{(N_1+1)(N_2+1)}}\{A_+(\omega_+^2-\lambda_1^2(N_1+1))\cos\omega_+t$$
$$+A_-(\omega_-^2-\lambda_1^2(N_1+1))\cos\omega_-t\}$$

and

$$C_g(N_1+1,N_2+1,N_3+1,t)=\frac{-i}{\lambda_1\lambda_2\lambda_3\sqrt{(N_1+1)(N_2+1)(N_3+1)}}\cdot$$
$$\cdot\begin{bmatrix} A_+\omega_+(\omega_+^2-\lambda_1^2(N_1+1)-\lambda_2^2(N_2+1))\sin\omega_+t \\ +A_-\omega_-(\omega_-^2-\lambda_1^2(N_1+1)-\lambda_2^2(N_2+1))\sin\omega_-t \end{bmatrix} \tag{7}$$

where

$$A_\pm=\pm\frac{1}{\omega_+^2-\omega^2}[\omega_\pm^2-\lambda_2^2(N_2+1)-\lambda_3^2(N_3+1)] \tag{7a}$$

When we let $\lambda_3\to 0$, we find that $\omega_-\to 0$ and $\omega_+\to\Omega_R$ of the 3-level atom and 2 modes and the formulae (7) coincide with those of Refs.[9].

3 The Density Matrix

With these amplitudes calculated, we can write down the equation of motion for the density matrix operator. When the time of flight (t_{int}) for the atom across the cavity is much shorter than the cavity damping time $\tau_i=\frac{1}{\gamma_i}$ for the mode i, the density matrix equation of motion takes the form [9], [10].

$$\frac{d\rho}{dt}=\left.\frac{d\rho}{dt}\right|_{gain}+L\rho \tag{8}$$

where the dissipation in the cavity with n_t thermal photons present is represented by

$$L\rho \quad =(n_t+1)\sum_{i=1}^{3}\frac{\gamma_i}{2}\{2a_i\rho a_i^+-a_i^+a_i\rho-\rho a_i^+a_i\}$$
$$+n_t\sum_{i=1}^{3}\frac{\gamma_i}{2}\{2a_i^+\rho a_i-a_ia_i^+\rho-\rho a_ia_i^+\}\ . \tag{8a}$$

When we take the pumping rate for the atoms to be r, and the distribution for the atoms traversing the cavity at time t to be $g(t)$, then the equation of motion for the probability distribution function $P(N_1, N_2, N_3, t)$ which is the diagonal term in the density matrix in the Fock state representation, is given by

$$\begin{aligned}
\frac{d}{dt}P(N_1, N_2, N_3, t). \quad &= r[\int g(t)dt\{(-1 + |c_e(N_1, N_2, N_3, t)|^2)P(N_1, N_2, N_3) \\
&+|c_1(N_1, N_2, N_3, t)|^2 P(N_1 - 1, N_2, N_3) \\
&+|c_2(N_1, N_2, N_3, t)|^2 P(N_1 - 1, N_2 - 1, N_3) \\
&+|c_g(N_1, N_2, N_3, t)|^2 P(N_1 - 1, N_2 - 1, N_3 - 1)\}] \\
&+(n_t + 1)[\gamma_1\{(N_1 + 1)P(N_1 + 1, N_2, N_3) - N_1 P(N_1, N_2, N_3)\} \\
&+\gamma_2\{(N_2 + 1)P(N_1, N_2 + 1, N_3) - N_2 P(N_1, N_2, N_3)\} \\
&+\gamma_3\{(N_3 + 1)P(N_1, N_2, N_3 + 1) - N_3 P(N_1, N_2, N_3)\}] \\
&+n_t[\gamma_1\{N_1 P(N_1 - 1, N_2, N_3) - (N_1 + 1)P(N_1, N_2, N_3)\} \\
&+\gamma_2\{N_2 P(N_1, N_2 - 1, N_3) - (N_2 + 1)P(N_1, N_2, N_3)\} \\
&+\gamma_3\{N_3 P(N_1, N_2, N_3 - 1) - (N_3 + 1)P(N_1, N_2, N_3)\}] \qquad (9)
\end{aligned}$$

This equation gives the time evolution of the probability distribution function. The structure is rather complicated, we shall apply some approximations in order to handle it.

4 The Mean Photon Number; and Steady State Solutions

Once the probability distribution function is evaluated, it is easy to calculate the equation of motion for any function of the field photon numbers. In effect we find that the mean photon number in the different modes of the field obey the following equations of motion:

$$\begin{aligned}
\langle \dot{N}_1 \rangle &= \gamma_1(n_t - \langle N_1 \rangle) + r\int g(t)dt\langle 1 - |c_e(N_1, N_2, N_3; t)|^2 \rangle \\
\langle \dot{N}_2 \rangle &= \gamma_2(n_t - \langle N_2 \rangle) + r\int g(t)dt\langle |c_2(N_1 + 1, N_2 + 1, N_3; t)|^2 \rangle \\
&\quad + |c_g(N_1 + 1, N + 1, N_3 + 1; t)|^2 \rangle \\
\langle \dot{N}_3 \rangle &= \gamma_3(n_t - \langle N_3 \rangle) + r\int g(t)dt\langle |c_g(N_1 + 1, N_2 + 1, N_3 + 1; t)|^2 \rangle \qquad (10)
\end{aligned}$$

where $\langle (\ldots) \rangle = \sum_{N_1, N_2, N_3}(\ldots)P(N_1, N_2, N_3)$ is the average over the photon numbers. The integral in these formulae represents the rate of gain of photons in the field mode due to the emission from the pumped atoms in the cavity. The first term represents the rate of loss of the photons from the cavity due to coupling with the thermal bath. This process depends on the excess of the number of the photons in each mode over the thermal photons in the cavity.

In order to get some semi-analytical solutions for the non-linear coupled set of equations (10), we shall resort to some approximations. When the semi-classical approximation $\langle N_i + 1 \rangle \simeq \langle N_i \rangle$ and the decoupling approximation [9,10]

$$\langle N_1^{S_i} N_2^{S_2} N_3^{S_3} \rangle = \langle N_1 \rangle^{S_1} \langle N_2 \rangle^{S_2} \langle N_3 \rangle^{S_3} \qquad (11a)$$

are used, the latter equations become tractable. When we further neglect the variation in the time of flight of the atoms across the cavity, i.e.

$$g(t) = \delta(t - t_{int}) \tag{11b}$$

we arrive at the following set of equation in the steady state case (i.e. $\langle \dot{N}_i \rangle = 0$)

$$\begin{aligned} \bar{n}_3 &= \frac{\gamma_3}{r} n_t + |c_g(\bar{n}_1, \bar{n}_2, \bar{n}_3, T)|^2 \\ \bar{n}_2 &= \frac{1}{r}(\gamma_2 - \gamma_3) n_t + \bar{n}_3 + |c_2(\bar{n}_1, \ barn_2, \bar{n}_3, T)|^2 \\ \bar{n}_1 &= \frac{1}{r}(\gamma_1 - \gamma_2) n_t + \bar{n}_2 + |c_1(\bar{n}_1, \bar{n}_2, \bar{n}_3, T|^2 \end{aligned} \tag{12}$$

where $\bar{n}_i = \frac{\gamma_i}{r}\langle N_i \rangle_s$ is the scaled photon number in the steady state, and the scaled time $T = \lambda\sqrt{\frac{r}{\gamma}} t_{int}$ is used (it is usually termed the tipping angle Θ for the atom in the cavity [9]), with λ and γ as idealized coupling and damping factors respectively. The ratio $(\frac{r}{\gamma})$ (usually termed N_{ex} [9]) represents the average number of atoms pumped during the cavity damping time. The amplitudes appearing in (12) are now given by

$$\begin{aligned} c_e &= A_+ \cos u_+ T + A_- \cos u_- T \\ c_1 &= \frac{-i}{\sqrt{a_1 \bar{n}_1}} [A_+ u_+ \sin u_+ T + A_- \sin u_- T] \\ c_2 &= \frac{1}{\sqrt{a_1 a_2 \bar{n}_1 \bar{n}_2}} [A_+ (u_+^2 - a_1 \bar{n}_1) \cos u_+ T + A_- (u_-^2 - a_1 \bar{n}_1) \cos u_- T] \end{aligned}$$

and

$$\begin{aligned} c_g = \frac{-i}{\sqrt{a_1 a_2 a_3 \bar{n}_1 \bar{n}_2 \bar{n}_3}} &[A_+ u_+ (u_+^2 - a_1 \bar{n}_1 - a_2 \bar{n}_2) \sin u_+ T \\ &+ A_- u_- (u_-^2 - a_1 \bar{n}_1 - a_2 \bar{n}_2) \sin u_- T] \end{aligned} \tag{13}$$

with $a_i = \frac{\lambda_i^2}{\lambda^2} \frac{\gamma}{\gamma_i}$ parametrizing the difference in coupling constants and damping factors for the different transitions and modes; while

$$2u_\pm^2 = a_1 \bar{n}_1 + a_2 \bar{n}_2 + a_3 \bar{n}_3 \pm \sqrt{(a_1 \bar{n}_1 + a_2 \bar{n}_2 + a_3 \bar{n}_3)^2 - 4 a_1 a_3 \bar{n}_1 \bar{n}_3}$$

and

$$A_\pm = \pm \frac{1}{u_+^2 - u_-^2} \{u_\pm^2 - (a_2 \bar{n}_2 - a_3 \bar{n}_3)\} \tag{14}$$

We look at solutions to the set (12) in what follows in the case of a very cold cavity (i.e. $n_t = 0$) and equal coupling constants ($\lambda_i = \lambda$ for $i = 1,2,3$) and equal damping factors for the different modes ($\gamma_i = \gamma$ for $i = 1,2,3$) (i.e. $a_i = 1$). It is easy to show that the set of equations (12) under the above mentioned conditions admits the following solutions:

(i) $\bar{n}_1 = \bar{n}_2 = \bar{n}_3 = 0 \quad \text{for} \quad 0 < T < T_0$

where T_0 is the threshold, in this regime the upper state $|e\rangle$ is populated while the rest are empty

(ii) $\bar{n}_1 = \sin^2 \sqrt{\bar{n}_1} T \quad \bar{n}_2 = 0 = \bar{n}_3 \quad T_0 \leq T < T_1 = \frac{\pi}{2}$.

Only the first mode is excited during this regime, and the level $|e\rangle$ starts to depopulate until it is empty at $T = T_1$ when the state $|1\rangle$ starts to be populated

(iii) $\bar{n}_1 = 1$, $\bar{n}_2 = -\cos\sqrt{1+\bar{n}_2}T$ and $\bar{n}_3 = 0$; $T_1 \leq T < T_2 = \frac{\pi}{\sqrt{2}}$.

Here we have photons in the two modes and the level $|e\rangle$ is depopulated along this period, the level $|1\rangle$ starts to depopulate until it is empty at T_2, thence level $|2\rangle$ starts to populate

(iv) $n_1 \neq 0$, $n_2 \neq 0$, $n_3 \neq 0$ for $T > T_2$.

Now the three modes are present in the cavity and other levels start to repopulate until

(v) At the special value $T_3 = \frac{2\pi}{\sqrt{5\bar{n}}}$, we have $\bar{n}_1 = \bar{n}_2 = \bar{n}_3 = \bar{n} \equiv 0.973$, a local minimum for $\bar{n}_1$ and $\bar{n}_2$ at which the two intermediate levels 1 and 2 are empty while most of the population rests in the ground level $|g\rangle$ with very small population in the uppermost excited level $|e\rangle$, see Fig.2.

This latter case has no counterpart in the two-photon micromaser [9]. After that the number of photons in the three modes fall down rather slowly as shown in figure 2.

5 Conclusions

A three non degenerate photon micromaser has been investigated. The equation of motion for the density matrix is obtained, and the probability distribution function is calculated. The mean-photon numbers in the different modes are obtained. The equations are coupled and highly non-linear. The steady state solutions are obtained. When semiclassical and decoupling approximations are considered, the set of equations become tractable and a set of semi-analytical solutions are discussed. Depending on the interaction time t_{int} (through the scaled time T) the three modes can be present at the same time. The mean photon number in the two modes 1 and 2, show local minimum at $T_3 = \frac{2\pi}{\sqrt{5\bar{n}}}$ with $\bar{n} = 0.973$ which is not present in the two-photon micromaser [9].

Acknowledgments

One of the authors (A.-S.F.O.) wishes to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He would also like to thank Professor G. Denardo for discussions.

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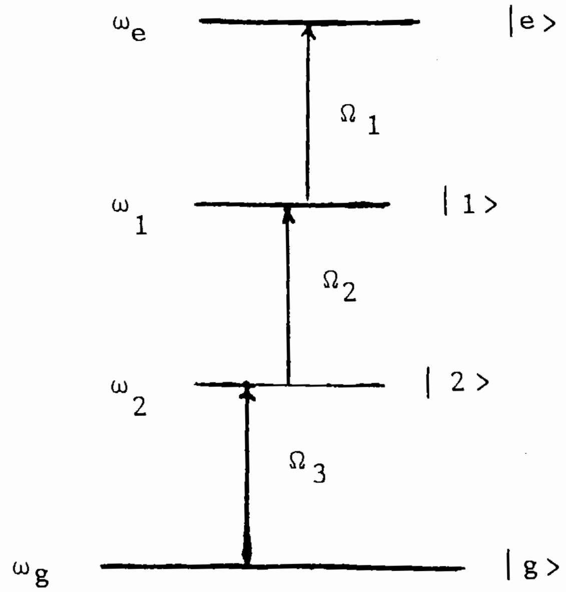

Fig. 1 - Schematic representation for the atomic system and the field modes

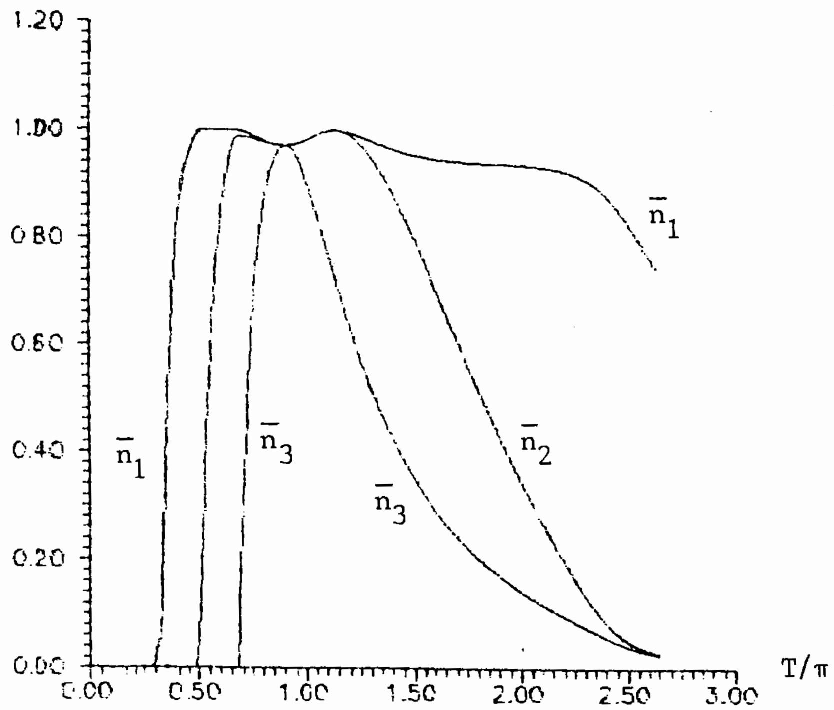

Fig. 2 - Time evolution for the mean photon numbers (Eqs.12) for $n_t = 0$ and $a_1 = a_2 = a_3 = 1$.