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LAMP SERIES REPORT

(Laser, Atomic and Molecular Physics)

INTERFERENCE IN A THICK PLATE AT LARGE ANGLE OF INCIDENCE

M.T. Tavassoli

and

F. Shah Shehany

MIRAMARE-TRIESTE

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International Atomic Energy Agency
and
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SERIES REPORT
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INTERFERENCE IN A THICK PLATE AT LARGE ANGLE OF INCIDENCE

M.T. Tavassoli *
International Centre for Theoretical Physics, Trieste, Italy

and

F. Shah Shehany
Department of Physics, University of Alzahra, Tehran, Iran.

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* Permanent address: Department of Physics, University of Tehran, Tehran, Iran.

ABSTRACT

A new approach to the interference in a plane parallel plate is introduced which is valid for any angle of incidence and any thickness. It is shown that the interference in a plate can be interpreted as the interference in a double-slit and the corresponding parameters are derived. It is also shown that for a particular angle of incidence, which depends only on the refractive index, the interfringes are minimum. It is proved theoretically and verified experimentally that the interference around this particular angle of incidence has several exploitable features which include: a) In thick plates large numbers of equidistant fringes are formed which are very adequate for producing interference gratings. b) It provides, in comparison to the conventional interferometric methods, an easier and more accurate means for direct measurement of wave-length. c) Multiple-beam interference at this particular angle improve the accuracy of the measurement of the fine structures of the atomic spectra, compared to other interferometric methods.

1. Introduction

In text-books of optics the study of interference in a plane parallel plate is usually limited to the near-normal incidence of light. It is, perhaps, generally accepted that the interference pattern is a set of concentric circular fringes whose interfringes decrease by the increase of incident angle. In some text-books (F.G.Smith and J.H.Thomson 1975 F.L.Pedrotti and L.S.Pedrotti 1987) it is mentioned that the interference in a plane parallel plate is similar to that of a double-slit, but no quantitative account is given. In this paper a new approach to the fringe formation in a plate is introduced which is not only valid for any angle of incidence but also establishes a quantitative relation between the corresponding parameters of a plate and a double-slit. Besides, this approach reveals that the interference in a plate around a special angle of incidence has some noteworthy features which can be of considerable values in interferometry, spectrometry, and the production of interference gratings.

2. Theory

If an expanded and ideally parallel beam of coherent light is incident on a transparent plate with precisely parallel plane surfaces, the reflected beams from both surfaces of the plate interfere in the overlapping region, but no fringes are observed. Fringes are formed only if, at least, one of the parameters in the optical path difference relation:

$$2\ ne \cos r + \lambda/2 = m\lambda \qquad (1)$$

is changed. In (1) n, e, r, λ and m are, refractive-index, plate thickness, angle of refraction, wave-length and a real number, respectively. But, if the plate is illuminated by a rather divergent beam of light, r varies in the interfering region and the fringes are formed. To find the fringe spacing, we differentiate (1) with respect to r :

$$-2\ n\ e \sin r\ \delta r = \lambda\,\delta m \qquad (2)$$

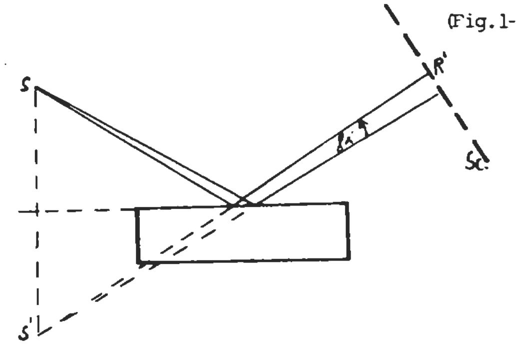
Then, we make use of Snells' law, $\sin i = n \sin r$, to express δr in terms of δi, the increment of incident angle,

$$\delta r = \cos i\ \delta i/n \cos r \qquad (3)$$

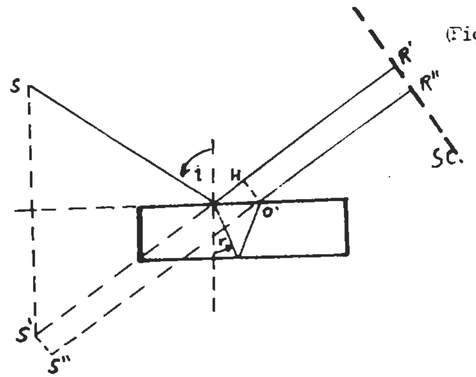
substituting (2) Into (3) and ignoring the minus sign, the following is obtained

$$\delta i = \lambda\,\delta m/2\ e \ \mathrm{tg}\ r \cos i \qquad (4)$$

If we put $\delta m=1$ in (4), δi implies the angular width of one fringe. Now if δi is multiplied by D, the distance between the observing screen and the intersection of the two rays reflected from the upper surface of the plate which enclose a fringe, R'S' in Fig 1-a, the inter-fringe I is obtained:



(Fig.1-a) The interfringe at any angle of incidence is given by the product of the distance between the observing screen and the image of the source in the upper plane of the plate, S'R', in the angular width of a fringe, δi.



(Fig.1-b) The two sources of the equivalent double-slit are: S', the image of source S in the upper plane of the plate and S", the foot of the normal from S' to the direction of the corresponding reflected beam on the lower plane of the plate, i.e. S"R".

$$I = D\lambda / 2\ e\ tg\ r \cdot \cos i \qquad (5)$$

Now, comparing (5) with the Young formula

$$I = D\lambda / d \qquad (6)$$

We see that $2\ e\ tg\ r \cos i$ corresponds to d, the distance between the two slits. Thus,

$$d = 2\ e\ tg\ r \cos i \qquad (7)$$

or, expressing tag r in terms of sin r, one gets

$$d = \frac{e \sin 2i}{(n^2 - \sin^2 i)^{1/2}} \qquad (7')$$

To find out what is implied by (7) we consider Fig.1-b. Here, S' is the mirror image of S, the light source, in the upper plane of the plate. If we regard S as an illuminated slit, S' will be one of the required slits of the double-slit. The position of the other slit depends on the angle of incidence, and for each angle lies on the corresponding beam reflected on the lower plane of the plate, S" in Fig. 1-b. Because, simple Geometrical consideration in Fig.1-b shows that the separation S'S" = O'H of the two corresponding reflected beams OR' and O'R" is the same as given by (7).

From (7) it is obvious that d varies with i and it is zero for normal and grazing incidence. Thus, it must become maximum somewhere in between. To find the angle of incidence for which d is maximum, we derive $\delta d/\delta i$ from (7) and then put it equal to zero:

$$\delta d/\delta i = 2e \left| (1+tg^2 r) \cos i\ \delta r/\delta i - tg\ r \sin i \right| .$$

Making use of $\delta r/\delta i = \cos i / n \cos r$ and doing some simplification, we get

$$\delta d/\delta i = 2\ e\ (\sin^4 i - 2\ n^2 \sin^2 i + n^2)/n^3 \cos^3 r, \qquad (8)$$

and $\delta d/\delta i = 0$ leads to the equation

$$\sin^4 i - 2\ n^2 \sin^2 i + n^2 = 0 \qquad (9)$$

solution of this equation, for $n > 1$, gives

$$\sin i = \left| n^2 - n(n^2-1)^{1/2} \right|^{1/2} \qquad (10)$$

Thus, for the incidence angle satisfying (10) the separation of the slits is maximum and therefore the interfringes are minimum. In what follows we denote this angle and the corresponding slits' separation by i_{max} and d_{max} respectively. The curve in Fig.2 is the graph of $d/e = 2\ tg\ r \cos i$ versus the incidence angle i for $n = 1.52$. The deltas in Fig.2 belong to the experimental values of d calculated from the measurement of the fringe spacings and using Young's formula.

The maximum of the curve in Fig. 2 is rather flat. Therefore, one can assume that d remains constant in a rather broad angular interval around i_{max}. To estimate it's accuracy we use Taylor expansion of d at i_{max}. For this, we derive $\delta^2 d/\delta i^2$ from (8) and evaluate it's value at i_{max}; after some simplifying operation, this leads to

$$\delta^2 d/\delta i^2 = -4\ d_{max} \qquad (11)$$

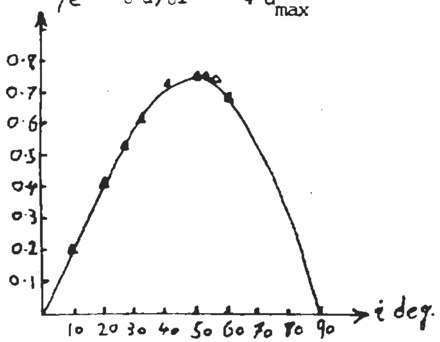


Fig. 2. The graph of $d/e = 2$ tg r cos i versus the angle of incidence for a plane parallel plate of thickness $e = 7.89$ mm and refractive index $n = 1.52$, the deltas are experimental values of d/e which are calculated from the measurement of the fringe spacings. Here d is the distance between the slits of the equivalent double-slit, r and i are the reflection and incidence angles.

Thus the first two non zero terms of Taylor expansion of d are:

$$d = d_{max} - 2\ d_{max} (\Delta i)^2 \qquad (12)$$

where Δi is the angular distance from i_{max}. If we assume that d is constant in an angular internal of $2\Delta i$, with i_{max} at its middle, the relative error is about:

$$\delta d/d \simeq 2(\Delta i)^2 \qquad (13)$$

For an angular interval of 4 arc degrees i.e. $\Delta i = 2^o$ this is about 0.2%.

It is interesting to see that the relations for the limiting cases: interference at normal and grazing incidence, which are developed in the text-books can be easily derived from (5). For para normal incidence we proceed as follows. By puting tg $r \simeq r = i/n$ and cos $i \simeq 1$ in (5), we get

$$I \simeq D\ n\lambda/2\ e\ i \qquad (14)$$

For incidence symmetrical to the normal of the plate, the fringes are circular. If we denote the radius of the kth fringe by ρ_k, then the interfringe I is $\delta\rho_k$ for $\delta k = 1$. Therefore, one can write:

$$\delta\rho_k/\delta k = D\,n\,\lambda/2\ e\ i \qquad (15)$$

On the other hand, for the small angle of incidence, we have:

$$i \simeq \rho_k/D \qquad (16)$$

Substituting (16) in (15) we get

$$2\rho_k \delta\rho_k \approx D^2 n\lambda\, \delta k$$

which by integration from $\rho = 0$ to $\rho = \rho_k$ it is obtained

$$\rho_k = D(kn\lambda/e)^{1/2} \qquad (17)$$

which is exactly the same as given in text - books.

For the grazing angles, $i = \pi/2$, one can write:

$\sin r \approx 1/n$ and $\cos i = \cos(\pi/2-g) \approx g$

where g is the angle between the reflected beam and the surface of the plate. Making use of these quantities in (5), it is obtained

$$I = D\lambda\sqrt{n^2-1}/2\; e\, g \qquad (18)$$

Again, replacing I by $\delta\rho_k/\delta k$ and g by ρ_k/D in (18) and following the same procedure as was followed for the para normal incidence we get

$$\rho_k = D\sqrt[4]{n^2-1}\sqrt{\lambda k/e} \qquad (19)$$

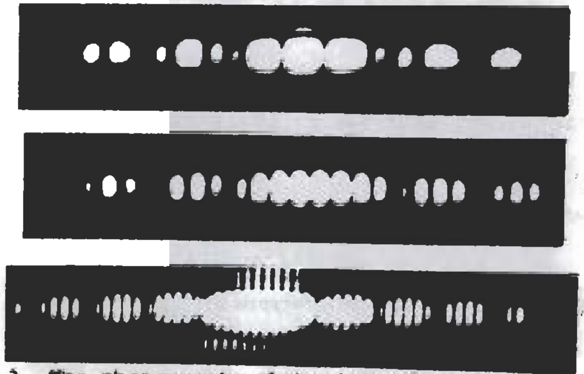
Here, ρ_k is the distance of the kth fringe from the surface of the plate. Equ. (19) is exactly the same as is given for Lummer - Gehrcke plate in text-books (M. Born and E. Wolf 1975).

3. Experimental results and conclusions.

In one series of experiments a plane parallel plate of thickness e = 7.89 mm and refractive index n = 1.52 was illuminated by a He-Ne laser beam through a narrow slit. The observing screen was held perpendicular to the direction of the reflected beam for each

angle of incidence. The angle of incidence varied from zero to 70 degrees. For each angle, the interfringe was measured and used for calculation of d, the slits' distance. The deltas in Fig.2 are experimental values of d/e against the angle of incidence, which are neatly fitted on the theoretical curve.

As the angle of incidence increases from zero, the number of fringes increases and the interfringe decreases rapidly. This is clear from the photographs of Fig.3-a, b, and c which are recorded for the incident angles, of $\approx 2.0^\circ$, $\approx 4.4^\circ$ and $\approx 10^\circ$ respectively.



(a)

(b)

(c)

Fig.3. The photographs of the interference pattern in a plane parallel plate of thickness e = 7.89 mm and refractive index n=1.52 when it was illuminated by He-Ne laser beam through a narrow slit at incidence angles a) $\approx 2.0^\circ$ b) $\approx 4.4^\circ$ and c) $\approx 10^\circ$

In another series of experiments, the laser beam was expanded into a divergent beam and directed onto the plate at incidence angle close to i_{max}. A large number of equidistant fringes were observed. Fig.4 shows the fringes contained in an angular interval of $\Delta i \simeq 0.9^{\circ}$, when a plate of thickness 7.89 mm and refractive index 1.52 was illuminated by an expanded He-Ne laser beam at incident angle $i = 48^{\circ}$.

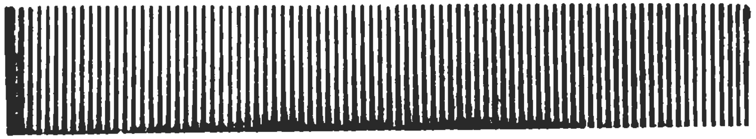


Fig. 4. The number of fringes contained in an angular interval of $\Delta i \simeq 0.9^{\circ}$ when a plate of thickness $e = 7.89$ mm and refractive index $n = 1.52$ was illuminated by an expanded He-Ne laser beam at incident angle $i = 48^{\circ}$.

There are three noteworthy points about these results.

a) The number of equidistant fringes can be made so large that one can produce interference gratings. To appreciate this, we try to estimate the number of the fringes. The angular width of a fringe at i_{max} is $\delta i = \lambda/d_{max}$. Then, the number of fringes m contained in an angular interval $2\Delta i$ is

$$m = 2\Delta i/\delta i = 2\, d_{max}\Delta i/\lambda \qquad (20)$$

According to (10) for a refractive index $n = 1.52$ we have $i_{max} = 49.05^{\circ}$. Substituting the corresponding numerical values for tg r and cos i in (7) we get

$$d_{max} \simeq 0.750\ e$$

putting it in (20) gives

$$m = 1.5\ (e/\lambda)\ \Delta i \qquad (21)$$

For $e = 50$ mm, $\lambda = 632.8$ nm and $\Delta i \simeq 4^{\circ}$, we have $m \simeq 8200$, which is a large number.

Using this technique the authors have produced interference gratings with widely different numbers of lines per unit length-from 5 to 300 lines per millimeter (M.T. Tavassoli and F.Shah-Shehany 1986).

It may be practical, by employing multiple-beam interference in a plate with the adequate reflective coating, to control the ratio of the areas of the transparent and the opaque strips. This is of considerable value in the production of interference gratings.

b) The interference at angles close to i_{max} provides an easy and accurate means for measuring wave-length. To show this, we rewrite (20) in the following form:

$$\lambda = 2\ d\ \Delta i/m \qquad (20')$$

the relative error of λ can be evaluated in the following manner:

$$\delta\lambda/\lambda = \delta d/d + \delta i/\Delta i - \delta m/m \tag{21}$$

where δi is the absolute error in measuring Δi. If we take $\delta m = 0$, no mistake in counting the fringes, and replace $\delta d/d$ from (13). we get:

$$\delta\lambda/\lambda = 2(\Delta i)^2 + \delta i/\Delta i \tag{22}$$

Now, to minimize $\delta\lambda/\lambda$ we assume δi, the absolute error in the angular interval measurement, is constant for each set up. Then, we put the derivative of (22) equal to zero. Thus,

$$4\Delta i - \delta i/(\Delta i)^2 = 0$$

which gives :

$$\Delta i = (\delta i/4)^{1/3} \tag{23}$$

substituting it in (22) leads to

$$\delta\lambda/\lambda = 6(\delta i/4)^{2/3} \tag{24}$$

For $\delta i = 1$ arc second we have $\delta\lambda/\lambda = 7 \times 10^{-4}$.

c) The last noteworthy point is that multiple-beam interference is extensively used in spectroscopy. For instance, Fabry-Perot and Lummer-Gehrcke interferometers are widely used to resolve fine structures of spectra. In these devices, light strikes the corresponding Plates normally and tangentially respectively. As was shown in section 2, (14) and (18), the interfringes in

these two cases are inversely proportional to the incident and grazing angles. Therefore the separations of the corresponding spectral lines' fringes depends on the angle of incidence and averaging over many fringe spacings is not straight-forward. But, if the interference at i_{max} is exploited the fringe spacing for each spectral line is constant over an angular interval of few degrees. This improves the accuracy of the measurement considerably.

References

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