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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

# LAMP SERIES REPORT

(Laser, Atomic and Molecular Physics)

# THE JCM WITH INITIAL NUMBER-PHASE MINIMUM UNCERTAINTY STATE FIELD

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## THE JCM WITH INITIAL NUMBER-PHASE MINIMUM UNCERTAINTY STATE FIELD

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ABSTRACT

The Jaynes-Cummings interaction of a two-level atom with a quantized radiation field is studied when the field is initially in a Number-Phase Minimum Uncertainty State (NUS). The dynamic behaviour of atomic inversion and field statistics is demonstrated in detail by numerical calculations.

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## Preface

The ICTP-LAMP internal reports consist of manuscripts relevant to seminars and discussions held at ICTP in the field of Laser, Atomic and Molecular Physics (LAMP).

These reports aim at informing LAMP researchers on the activity carried out at ICTP in their field of interest, with the specific purpose of stimulating scientific contacts and collaboration of physicists from Third World Countries.

## 1. Introduction

The Jaynes-Cummings Model[1] (JCM) is one of the most important models in quantum optics. The model describes the interaction between a two-level atom and a single-model quantized radiation field. With the help of the JCM, many interesting nonclassical effects have been explored, such as the periodic collapse and revival of the Rabi nutation[2],[3] the producing of sub-Poissonian statistics field or antibunching[3] and squeezed state fields[4]. Recent experimental work with Rydberg atoms in a high-Q microwave resonantor[5] has made laboratory realization of this system possible. A growing interest in the detailed investigation of the JCM has been stimulated by the practical demonstration of some of these nonclassical effects.

Up to now the JCM has been studied thoroughly with different initial field states. These initial feld states are coherent, chaotic and superposition of the above two [6], binomial field state[7], logarithmic state [8] and squeezed state[9] (ordinary squeezed state that features different uncertainities in two quadrature amplitudes of field).

On the other hand, another kind of squeezed state-the amplitude-squeezed state or Number-Phase Minimum Uncertainty State (NUS) is currently drawing much attention. It is quite different from ordinary squeezed state and has been experimently observed [10].

In this paper the dynamic behavior of atomic inversion and the field statistics is studied for the JCM when an atom interacts initially with a NUS field.

## 2. The dynamic behavior of atomic inversion and field statistics

### A. The Number-Phase Minimum Uncertainty State[10] (NUS)

The NUS features reduced photon number uncertainty $\langle\Delta\hat{n}\rangle < \langle\hat{n}\rangle$ and an enhanced phase uncertainty $\langle\Delta\hat{\phi}^2\rangle > 1/(4\langle\hat{n}\rangle)$, while the product of these two uncertainties satisfies the minimum uncertainty relation $\langle\Delta\hat{n}\rangle\cdot\langle\Delta\hat{\phi}^2\rangle \sim 1/4$ (for the situation that the average photon number is much larger than unity).



The NUS can be generated by using a nonlinear Mach-Zehender interferometer. The interferometer includes an optical Kerr medium in one arm. The photon number distribution for NUS field is expressed by diagonal matrix elements of density operator of field on a number state basis :

$$P(n) = \rho_{nn} = e^{-(|\alpha|^2+|\xi|^2)} \frac{1}{n!}\left|\sum_{K=0}^{n}\sum_{m=0}^{\infty} \frac{n!\,\xi^K(-\xi^*)^m \alpha^{n-K+m}}{K!\,m!\,(n-K)!} e^{\frac{1}{2}\gamma(n-K+m)(n-K+m-1)}\right|^2 \qquad (1)$$

where $(|\alpha|^2+|\xi|^2)$ is the mean photon number, $\gamma$ is the nonlinear interaction parameter in Kerr medium.

### B. General results of the JCM

The Hamiltonian for the JCM in the RWA can be expressed in terms of the inversion, raising and lowering operators of the atom and the annihilation and creation operators of the field:

$$\hat{H} = \hbar\Omega\hat{\sigma}_z + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar g(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger) \qquad (2)$$

When the atom is initially in the excited state, the reduced density operator of the field, the time-dependent photon number distribution and the atomic inversion are respectively given by

$$\hat{\rho}_f(t) = \mathrm{Tr.}_{atom}\left\{\hat{U}_I(t)\begin{pmatrix}\hat{\rho}_f(0) & 0\\ 0 & 0\end{pmatrix}\hat{U}_I^\dagger(t)\right\} \qquad (3)$$

$$P(n,t) = P(n)\cos^2(gt\sqrt{n+1}) + P(n-1)\sin^2(gt\sqrt{n}) \qquad (4)$$

$$W(t) = \frac{1}{2}\sum_{n=0}^{\infty} P(n)\cos(2gt\sqrt{n+1}) \qquad (5)$$

When the atom is initially in the ground state, these quantities are respectively given by

$$\hat{\rho}_f(t) = \mathrm{Tr.}_{atom}\left\{\hat{U}_I(t)\begin{pmatrix}0 & 0\\ 0 & \hat{\rho}_f(0)\end{pmatrix}\hat{U}_I^\dagger(t)\right\} \qquad (6)$$

$$P(n,t) = P(n)\cos^2(gt\sqrt{n}) + P(n+1)\sin^2(gt\sqrt{n+1}) \qquad (7)$$

$$W(t) = -\frac{1}{2}\sum_{n=0}^{\infty} P(n)\cos(2gt\sqrt{n}) \qquad (8)$$

where $\hat{U}_I(t) = \exp\{-\frac{i}{\hbar}\hat{H}_I t\}$ is the time evolution operator and $\hat{H}_I = \hbar g(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$ for the exact resonance ($\Omega = \omega$).

### C. The atomic inversion

For the JCM with initial NUS field, the atomic inversion can be easily obtained by substituting (1) to (5) and (8) Numerically calculating with computer, the graphs of the atomic inversion for the atom initially in its (a) lower state and (b) upper state are plotted in Fig. 1. There is no substantial difference between the two cases. But detailed features are obviously not the same.

The collapse and revival phenomena are demostrated clearly. There is close relationship between these phenomena and the parameter $\gamma$ (Fig.2). It is shown that the revival periods are the same for different $\gamma$ when $\langle \hat{n} \rangle$ is kept constant. Read out from the figure, the period is about 25; this corresponds well with Milburn's result[9], as $(gt)_R = 2\pi/\sqrt{\langle \hat{n} \rangle}$ for $\langle \hat{n} \rangle \gg 1$.

It is important to point out that the collapse envelop reconstructs well when Fano factor $F = \langle \Delta \hat{n}^2 \rangle / \langle \hat{n} \rangle$ is small for initial photon number distribution. When $\gamma = 0.10$, $\langle n \rangle = 16$ and $F = 0.8665$, the revival disappears after only two period. While for $\gamma = 0.15$, $\langle n \rangle = 16$ and $F = 0.0830$, the revival keeps well for sufficiently long time. This phenomenon can be well explained if we notice that $W(t)$ in (5) and (8) is just Fourier cosine series. It is not difficult to understant that when the photon number distribution $P(n)$ is norrow ( represented by small factor F), the revival in $W(t)$ lasts longer.

## D. The field statistics

The behavior of the field statistics is represented by the time-dependent photon number distribution $P(n,t)$ in (4) and (7). Some typical results are shown in Fig. 3.

First we look at the general aspects of evolution of the photon number distribution. We find that the detailed features of evolution are obviously not same for both the cases when the two-level system is initially in the excited and ground state respectively. But there are common aspects. The peak shifts in location and splits into two or more peaks corresponding to collapse of the atomic inversion. Then, at a certian time, the shape of the photon number distribution will be basically restored to the initial shape corresponding to revival of the atomic inversion. So $P(n,t)$ has the aspect of periodical restoration. In Fig.4 we can see it clearly.

Second, we find again that the initial NUS field with smaller factor F has the capability to reconstruct itself better. In other words, it is able to "remember" its initial shape for a longer time. Comparing Fig.5 with Fig.4, the fact is quite obvious. The correspondence between field statistics and atomic inversion reflects the coupled interaction between field and atom.

Lastly, we notice thatthe field statistics of the JCM with initial NUS can be kept sub-Poissionain. A better appreciation of the field statistics is possible by examining.

$$g^{(2)}(t) = \frac{\langle \hat{n}(t)^2 \rangle - \langle \hat{n}(t) \rangle}{\langle \hat{n}(t) \rangle^2} \tag{9}$$

or Mandel's parameter

$$Q(t) = \frac{\langle \Delta \hat{n}(t)^2 \rangle}{\langle n(t) \rangle} - 1 \tag{10}$$

Namely, $g^{(2)} < 1$ or $Q < 0$ corresponds to the sub-Poissonian distribution, $g^{(2)} > 1$ or $Q > 0$ to the super-Poissonian, and $g^{(2)} = 1$ or $Q = 0$ to the Poissonian.

In fig.6, we give some typical curves of $g^{(2)}(t)$ and $Q(t)$. It is shown that the field statistics always is sub-Poissonian. In addition, for the case $\langle n \rangle = 16$, $\gamma = 0.15$, the field is closer to the number state (for pure number state, $g^{(2)} = 1 - 1/\langle n \rangle$, $Q = -1$).

## 3. Conclusion

We have discussed the dynamic behavior of atomic inversion and field statistics in the JCM with an initial NUS field. It is shown that the collapse and revival phenomena, the evolution of photon number distribution and statistical properties of the field have no essential distinction between cases when atom is initially in the excited or ground state. But detailed features are obviously not the same. We have also indicated that corresponding to collapse of the atomic inversion, the time-dependent photon number distribution $P(n,t)$ can be basically restored to its initial shape. Furthermore, the periodic collapse and restoration can last longer for smaller F factor. It is also shown that the field statistics of the NCM initial NUS field is always sub-Poissonian.


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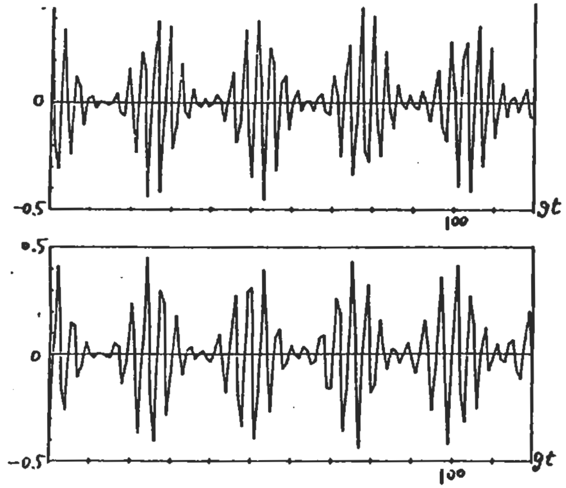



(a)

(b)

Fig.1 The atomic inversion W(t). The atom is initially in (a) upper level, (b) lower level. ($\langle n \rangle$=16, $\gamma$=0.15 )

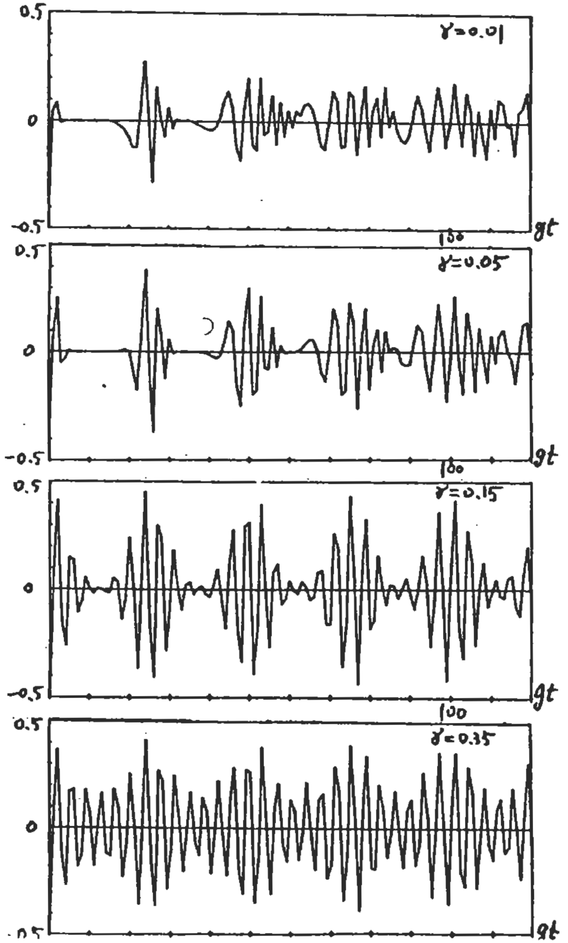
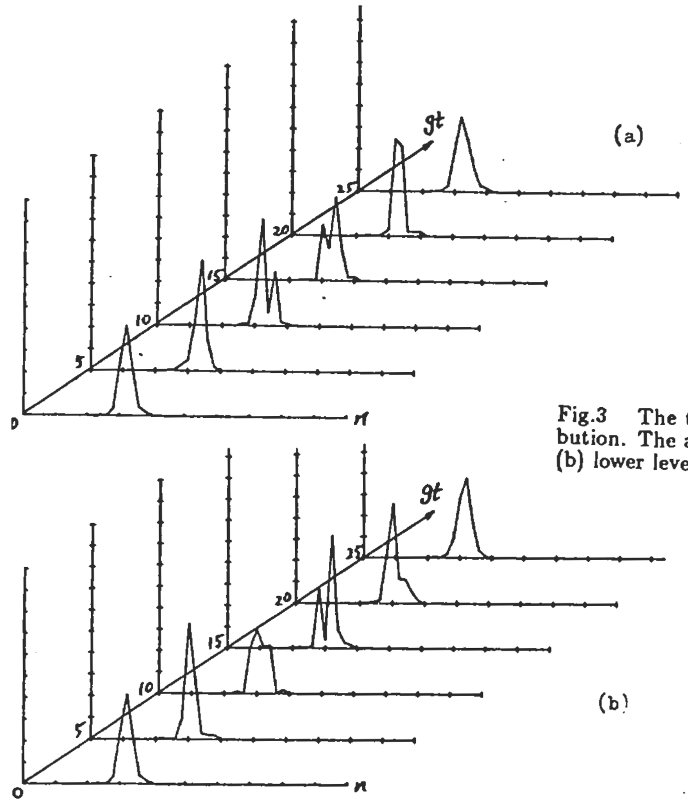



Fig.2 The atomic inversion W(t) for different parameter $\gamma$ . The atom is initially in lower level. ($\langle n \rangle$=16)






(a)

(b)

Fig.3 The time-dependent photon number distribution. The atom is initially in (a) upper level, (b) lower level. ($\langle n \rangle$=16, $\gamma$=0.15)

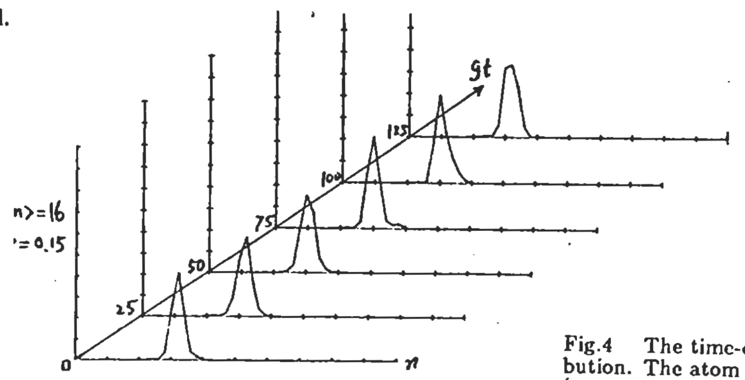
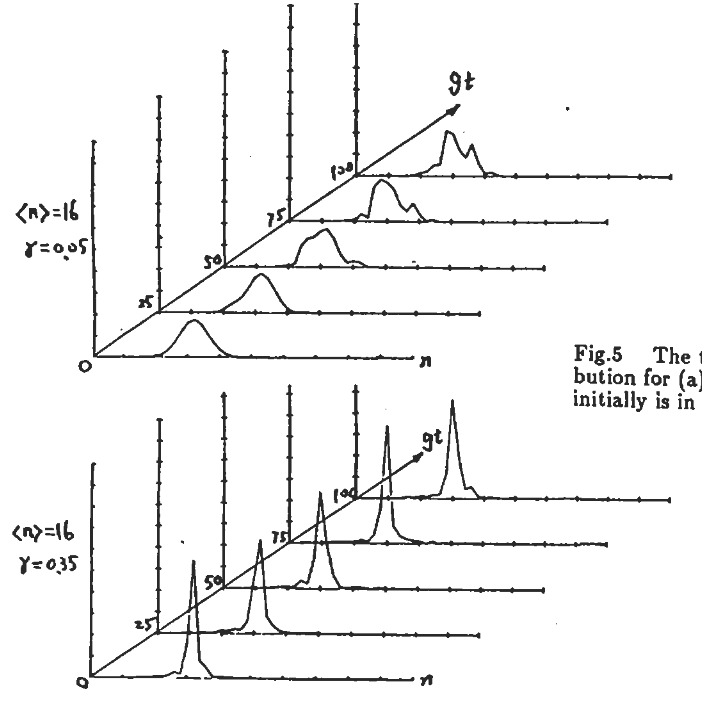



Fig.4 The time-dependent photon number distribution. The atom is initially in lower level, ($\langle n \rangle$=16, $\gamma$=0.15)




(a)

(b)

Fig.5 The time-dependent photon number distribution for (a) $\gamma=0.05$, (b) $\gamma=0.35$. The atom initially is in lower level. ($\langle n\rangle=16$)

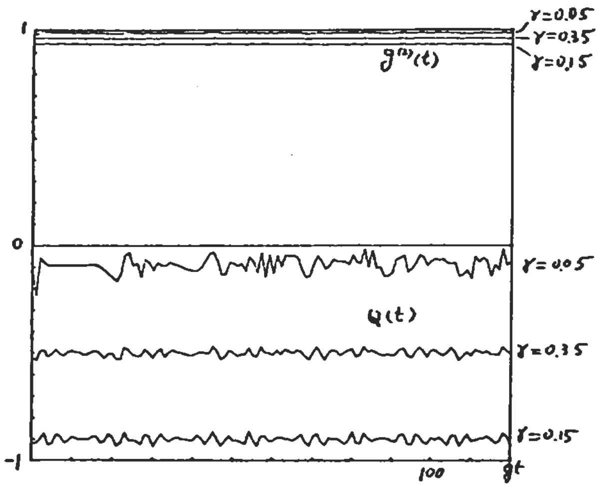



Fig.6 The $g^{(2)}(t)$ and $Q(t)$ of the field. ($\langle n\rangle=16$)