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SERIES REPORT

(Laser, Atomic and Molecular Physics)

THE EFFECT OF POLARIZATION OVER 2ND HARMONIC IN LASER LIGHT SCATTERING BY FREE ELECTRONS

V.P. Verma

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THE EFFECT OF POLARIZATION OVER 2ND HARMONIC IN LASER LIGHT SCATTERING BY FREE ELECTRONS

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ABSTRACT

In this work, the influence of linear as well as circular polarization on radiation at second harmonic frequency produced due to nonlinear scattering of intense laser light by free electrons is studied in detail. It is shown that if incident beam is linearly polarized, the scattered radiation has acquired the nearly same polarization but when the incident radiation field's polarization is turned over to circular one, the observed light is in general elliptically polarized. For scattering angle, θ is equal to $\pi/2$, the radiation becomes linearly polarized and when θ is small, there is no change in final polarization.

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INTRODUCTION

The laser sources of highly intense coherent light, characterized by high degree of monochromaticy, directionality, high intensity or brightness has revolutionized optics and opened up a new interesting and exciting field referred as a Nonlinear Optics[1,2]. The field is blooming since the first experimental demonstration of the production of optical harmonic waves in the nonlinear interaction of Ruby laser output of wavelength 6943 A with quartz crystal by Franken et al.[3] and also the advent of Opto-electronic techniques for detecting radiation of very low intensity in optical nonlinear effects[4,5] observed in the interaction of intense laser beams with matter.

On the other hand, the presence of harmonics in the scattering of intense monochromatic radiation by free[6-9] as well as bound[10,11] electrons were studied by several workers. It is predicted that such harmonic are generated due to nonlinearities available in the equation of motion, in the expression for scattered radiation field, and in the retardation effects[6-12]. The scattering cross-section has also been evaluated for various harmonics. Similar problems have been studied by many authors using semiclassical, and also quantum mechanical techniques[13→18]. It is reported that the results are reducible to the classical theories as predicted earlier by Vachaspati[6].

The fields of the generation of radiation at double frequency in the scattering of monochromatic laser light by free electrons

~~becomes important because~~ of recent experiment by Englert and Rinehart[17] who have the production of radiation at 2ω in nonlinear interaction of laser beam of wavelength, 1060 nm with free electrons and studied the variations of intensity. Good agreements are shown with the cross-section calculated by classical, semiclassical and quantum mechanical techniqes. It is argued that such type of process is purely classical phenomenon because the Planck's constant is not coming into the picture in Q E D -Theory.

Verma[19-22] studied the polarization characteristics of double frequency radiation generated in the nonlinear scattering of intense laser beams by electrons free and bound under different initial conditions. The findings of the paper should be verified experimentally by using a polarizer in front of the first stage detector in Rinehart and Englert experiment. As this should be easier to than to detect variations of intensity of second harmonic laser light.

In the present communication, it is intended to show the influence of polarization linear as well as circular of incident beam on the 2nd harmonic radiation emitted due to nonlinear interaction of driving laser field with free electrons.

THEORY

In order to study the effects of various polarizations on the harmonic radition emitted in the nonlinear interaction of intense laser light by free electrons, the equations of motion in the fields of linearly and circularly polarized beams of frequency ω are solved . The corresponding scattered electric field vectors oscillation will decide the nature of polarization of the observed radiations.

ELECTRON'S EQUATION OF MOTION :

Now considering a polarized e.m. wave of frequency ω and wave vector $\vec{k}$ travelling in the $\vec{z}$- direction with unit propagation vector $\vec{n}_o$ and impinging on the electrons having charge , e and mass, m respectively. Thus equation of motion of electron is as

$$m\, d^2\vec{z} / dt = e\, [\, \vec{E}(\vec{z},t) + \dot{\vec{z}} \times \vec{H}(\vec{z},t)\,] \qquad (1)$$

where symbols have their usual meaning and velocity of light is taken unity .

In the present case $|\vec{E}(\vec{z},t)| = |\vec{H}(\vec{z},t)|$

A coordinate system is needed in which the electron has no translatory motion and origin is taken at the mean position of the electron.

Let us take the following two cases :

(A) FOR LINEARLY POLARIZED INCIDENT WAVE :

The electric field vector in the direction of unit polarization vector $\vec{e}_1$(perpendicular to unit propagation vector $\vec{n}_o$) is expressed as

$$\vec{E}(\vec{z},t) = E_0\, \vec{e}_1 \cos(\omega\, t - \vec{k}\,.\,\vec{z}\,) \qquad (2a)$$

and

$$\vec{H}(\vec{z},t) = \vec{n}_o \times \vec{E}(\vec{z},t) \qquad (2b)$$

where propagation vector, $\vec{k}$ is related with $\vec{n}_o$ i.e.,

$$\vec{k} = \omega\, \vec{n}_o\, . \qquad (2c)$$

On using equations(2a,b,c) in expression (1) , we get the following solutions upto third order

$$\omega\, \vec{z}^{(1)} = -\, q\, \vec{e}_1 \cos \omega\, t \qquad (3a)$$

$$\omega\, \vec{z}^{(2)} = (1/8)\, q^2\, \vec{n}_o\, \sin 2\, \omega\, t \qquad (3b)$$

and

$$\omega\, \vec{z}^{(3)} = -(1/16)\, q^3\, \vec{e}_1\, [\, \cos\, \omega\, t + 1/3 \cos 3\, \omega\, t\,] \qquad (3c)$$

with $q = e\, E_0 / m\, \omega$

It is seen from these solutions that the electron is oscillating from its mean position with frequencies ω 2ω, and 3ω etc. This corresponds the process of harmonic genaration and it is due to nonlinearities available in the equations of motion because of strong intense laser field's interaction with free electron.

SCATTERED RADIATION FIELD

In the case of linearly polarized driving field signal, the expression for retarded electric field for scattering harmonics of an oscillating electron radiating in the direction of unit propagation vector ,$\vec{n}$ from its mean position at a long distance r at observation point $(x_0, \vec{x})$ turns out to be

$$\vec{E}_{scat} = \vec{H}_{scat} \times \vec{n} = e/R \left[\vec{n} \times \vec{M}\right] \times \vec{n} \quad \text{--- (4a)}$$

with $\vec{n} = \vec{x} \,/\, |\vec{x}|$ and

$$r = |\vec{x} - \vec{z}| \simeq |\vec{x}| \left(1 - \vec{x} \cdot \vec{z} / |\vec{x}|^2 + 1/2 \; z^2 / |\vec{x}|^2 \right)$$

$$\simeq |\vec{x}| - \vec{n} \cdot \vec{z} \quad \text{for large distance } |\vec{x}| \,,$$

where $\vec{M}$ (angular distribution of the radiation) can be defined as

$$(1 - u^2)^{-1} (1 - \vec{n} \cdot \vec{u})^{-2} [\vec{u} (1 - \vec{n} \cdot \vec{u})^{-1} ((\vec{u} \cdot \dot{\vec{u}}) (1 - \vec{n} \cdot \vec{u})$$

$$- (1 - u^2)(\dot{\vec{u}} \cdot \vec{n})) - (\dot{\vec{u}} - u^2 \dot{\vec{u}} + (\vec{u} \cdot \dot{\vec{u}}) \vec{u})] \quad \text{--- (4b)}$$

$\vec{u} = d\vec{z} / dt$ and $\dot{\vec{u}} = d^2 \vec{z} / d t^2$, are the initial velocity and acceleration for oscillating electron.

$\vec{M}$ is evaluated at retarded time

$$t = x_0 - r = x_0 - |\vec{x}| + \vec{n} \cdot \vec{z} \quad \text{--- (4c)}$$

On using solutions(3a,b,c) , radiation angular distribution at observation point $(x_0, \vec{x})$ turns out to be

$$\vec{M} = \vec{M}^{(1)} \cos \psi + \vec{M}^{(2)} \sin 2\psi + \vec{M}^{(3)} \cos 3\psi \quad \text{--- (5a)}$$

with wave function $\psi = \omega \, (x_0 - x)$

$$\vec{M}^{(a)} = \vec{e}_1 \, P^{(a)} + \vec{n}_0 \, Q^{(a)} \,, \quad \text{index } a = 1,2,3 \text{ etc.}$$

$$P^{(1)} = - \omega q [1 - (q/4)^2 (5 - \cos\theta + 2 \cos^2\alpha)]$$

$$P^{(2)} = - 2 \omega q^2 \cos\alpha$$

$$P^{(3)} = (9/16) \, \omega q^3 (1 - \cos\theta - 6 \cos^2\alpha) \quad \text{--- (5b)}$$

$$Q^{(1)} = (1/8) \, \omega q^3 \cos\alpha$$

$$Q^{(2)} = - (1/2) \, \omega q^2$$

$$Q^{(3)} = (9/8) \, \omega q^3 \cos\alpha$$

$\cos\alpha = (\vec{n} \cdot \vec{e}_1) = - \sin\theta \sin\phi_0$, θ = scattering angle, and ϕ_0 is the minimum angle that electric vector of incident light makes with the plane of scattering.

Thus we see that the vibrating electrons emit radiations of frequency ,ω of applied field, 2nd harmonic frequency , 2ω and 3rd harmonic,3ω etc. We take into account only double frequency radiation field because its intensity is detectable experimentally[17] and higher order radiation field's strength is very small in the optical region . Therefore , that can be neglected here in our studies.

Hence $\vec{M}_{2\omega}$ can be calculated with help of $P^{(2)}$ and $Q^{(2)}$ i.e.,

$$\vec{M}_{2\omega} = - \omega q^2 (2 \cos\alpha \, \vec{e}_1 + 1/2 \, \vec{n}_0) \sin 2\psi \quad \text{---- (5c)}$$

and corresponding electric field vector now becomes

$$\vec{E}_{2\omega}^{\,scat} = -\omega\, q^2\, (e/r)[\, 2 \cos \alpha\, \vec{e}_1 + 1/2\, \vec{n}_o - (\, 2 \cos^2 \alpha\, \vec{e}_1$$

$$+\, 1/2 \cos \theta\, \vec{n}_o)\, \vec{n}\,] \sin 2\psi \qquad \text{---- (5d)}$$

The light scattering cross-section for 2nd harmonic $d\sigma$, for scattering of linearly polarized light into a solid angle $d\Omega$, can be calculated with help of the time average of the square of the magnetic field,

$$\vec{H}_{2\omega}^{\,scat} = e^2/\, x^2\, [\, \vec{n} \times \vec{M}_{2\omega}\,]^2_{av.} \qquad \text{---- (6)}$$

where $\vec{M}_{2\omega}$ is given in equation (5c) .

Now

$$[\, \vec{n} \times \vec{M}_{2\omega}\,]^2_{\text{time av.}} = (1/8)(e^2/\, m\,\omega^2\,)\, [\, 4 \sin^2 2\alpha\, E_o^4$$

$$+ \sin^2\theta\, E_o^4 - 8\, E_o^4 \cos^2\alpha \cos\theta\ ]$$

and

$$[\, d\sigma^{(2\omega)}/\, d\Omega\,]_{\text{pol.light}} = (1/4)\, (e^2/m)^2 (\, eE_{o}/m\omega\,)^2 \sin^2\theta\, [\, 1+$$

$$16 \cos^2 \phi_o\, (\, 1 - \sin^2\theta \cos^2\phi_o) - 8 \cos^2\phi_o \cos\theta\ ] \qquad \text{---- (7)}$$

The angular variation of cross-section is depicted in figure--1 .It is seen that the cross-section is largest near $\phi_o \simeq \pi/4$ and $\theta = \pi/2$. For these values of ϕ_o and θ, scattered light at 2ω frequency has the polarization angle ,$\phi \simeq 116.5^\circ$.

POLARIZATION OF SCATTERED 2ND HARMONIC RADIATION :

In order to analyse the influence of polarization on the scattered radiation field, let us take the direction of electric fields of incident intense and scattered signals as the direction of polarization and also a plane (the plane of the scattering)containing the incident and emitted radiations in the direction of unit polarization vectors $\vec{n}_o$ and $\vec{n}$ respectively . Incident intense signal can be expressed in terms of unit vectors $\vec{\beta}_1$ and $\vec{\beta}_2$ perpendicular to each other and also to $\vec{n}_o$ in such a way that these unit vectors form a right handed triad system . Explicitly,it is seen that $(\, \vec{n} \times \vec{n}_o)$ is directly proportional to $\vec{\beta}_2$ and $\vec{\beta}_1$ is perpendicular to it . Therefore , $(\vec{n} \times \vec{n}_o) \times \vec{n}_o$ is directed along $\vec{\beta}_1$,i.e.,

$$(\, \vec{n} \times \vec{n}_o) \times \vec{n}_o \propto \vec{\beta}_1$$

or

$$\vec{\beta}_1 = -\, c\, (\, \vec{n} - \cos\theta\, \vec{n}_o) \quad \text{and}$$

$$\vec{\beta}_2 = c\, (\, \vec{n} \times \vec{n}_o) \quad \text{with} \quad c = [\, 1 - (\vec{n}\cdot\vec{n}_o)^2\,]^{1/2} \qquad \text{---- (8a)}$$

We define another set of unit vectors $\vec{\alpha}_1$ and $\vec{\alpha}_2$ (which are perpendicular to each other) and also $\vec{n}$,which form a right handed coordinate system in order to define the scattered radiation i.e.,

$$\vec{\alpha}_1 = c\,(\vec{n}_o - \cos\theta\,\vec{n}) \quad \text{and}$$

$$\vec{\alpha}_2 = c\,(\vec{n} \times \vec{n}_o) \qquad \text{—— (8b)}$$

Unit vectors $\vec{\beta}_1$, $\vec{\beta}_2$, $\vec{\alpha}_1$ and $\vec{\alpha}_2$ are defined in detail in References(18, 19) [See Figure - 2]

The study of polarization effects in the light scattered by free electrons does, however, give interesting information on the nature of the electrons and of the light itself. In order to investigate the polarization characteristics of the second harmonic scattered light, we resolve $\vec{e}_1$ and $\vec{n}_o$ along three axes, i.e., $\vec{\alpha}_1$, $\vec{\alpha}_2$ and $\vec{n}$. Initial polarization vector $\vec{e}_1$ can be written in terms of unit vectors $\vec{\beta}_1$ and $\vec{\beta}_2$, i.e.,

$$\vec{e}_1 = \vec{\beta}_1 \cos\phi_o + \vec{\beta}_2 \sin\phi_o$$

and also in terms of $\vec{\alpha}_1$, $\vec{\alpha}_2$ and $\vec{n}$

$$\vec{e}_1 = (\vec{e}_1 \cdot \vec{\alpha}_1)\,\vec{\alpha}_1 + (\vec{e}_1 \cdot \vec{\alpha}_2)\,\vec{\alpha}_2 + (\vec{e}_1 \cdot \vec{n})\,\vec{n} \qquad \text{—— (9a)}$$

where

$$\vec{e}_1 \cdot \vec{\alpha}_1 = \vec{\beta}_1 \cdot \vec{\alpha}_1 \cos\phi_o = \cos\theta \cos\phi_o \quad \text{with } \vec{\beta}_2 \cdot \vec{\alpha}_1 = 0$$

$$\vec{e}_1 \cdot \vec{\alpha}_2 = \sin\phi_o \quad \text{with } \vec{\beta}_1 \cdot \vec{\alpha}_2 = 0 \text{ and } \vec{\beta}_2 \cdot \vec{\alpha}_2 = 1$$

and

$$\vec{e}_1 \cdot \vec{n} = -\sin\theta \cos\phi_o \quad \text{with } \vec{\beta}_1 \cdot \vec{n} = \cos(\pi/2 + \theta)$$

and

$$\vec{\beta}_2 \cdot \vec{n} = 0$$

Now equation (9a) can be replaced by

$$\vec{e}_1 = \cos\theta \cos\phi_o\,\vec{\alpha}_1 + \sin\phi_o\,\vec{\alpha}_2 - \sin\theta \cos\phi_o\,\vec{n} \qquad \text{—— (9b)}$$

Similarly

$$\vec{n}_o = (\vec{n}_o \cdot \vec{\alpha}_1)\,\vec{\alpha}_1 + (\vec{n}_o \cdot \vec{\alpha}_2)\,\vec{\alpha}_2 + (\vec{n}_o \cdot \vec{n})\,\vec{n} \qquad \text{—— (10a)}$$

where $(\vec{n}_o \cdot \vec{\alpha}_1) = \sin\theta$; $(\vec{n}_o \cdot \vec{\alpha}_2) = 0$ and $(\vec{n}_o \cdot \vec{n}) = \cos\theta$ Expression (10a) can be reduced as

$$\vec{n}_o = \sin\theta\,\vec{\alpha}_1 + \cos\theta\,\vec{n} \qquad \text{—— (10b)}$$

On substituting equations (10a, b) in (5d), we obtain

$$\vec{E}_{2\omega}^{\,scat} = A\,\vec{S}_{2\omega}^{\,scat} \qquad \text{—— (11a)}$$

with $\vec{S}_{2\omega}^{\,scat} = [\cos\phi\,\vec{\alpha}_1 + \sin\phi\,\vec{\alpha}_2]\sin 2\psi$ (11b) and

A (amplitude at 2ω) $= 2/r\,(e^3 E_0^2/\omega m^2)\sin\theta$ as this is varied linearly to the square of the amplitude of applied field, E_o. Equations (11a) and (11b) gives us

$$\tan \phi = \sin 2\phi_o / 2\cos\theta \cos^2\phi_o - 1/2 ; \quad 0 \leq \phi \leq \pi . \qquad (12)$$

where ϕ is the final polarization minimum angle that scattered electric vector is viewed by an observer facing it, makes with that plane of scattering. The variation of ϕ is shown in figure-3 for different values of the parameters, ϕ_o and θ.

Let us conclude that the following :

(i) When $\phi_o = 0$ then $\phi = 0$ for all θ means that when incoming signal is polarized in the plane of paper, scattered radiation obtained, is also polarized in the same plane of scattering irrespective of the values of scattering angle.

(ii) For $\phi_o = \pi/2$ then $\phi = \pi$ for all θ values, the scattered scattered light at 2ω frequency would be polarized in the plane of the paper for scattering angle provided the incident light is polarized perpendicular to the plane of the paper.

(iii) For particular values of ϕ_o, the observed beam would be perpendicularly polarized at the critical angle of scattering i.e., $\theta_c \approx 75.5^0$. But when $\theta >> \theta_c$, there is no perpendicular polarization. There is always perpendicular polarization for $\theta << \theta_c$. This jump in polarzation from zero to $\pi/2$ can not be measured because scattering cross-section is vanished, unfortunately i.e.,

$$[d\sigma^{(2\omega)}/d\Omega]_{1p} = 0 \quad \text{at } \theta = \theta_c \approx 75.5^0.$$

(iv) We see from expression(7), which is plotted in figure --1 that the peak value of cross-section is at $\phi_o \approx \pi/4$ for scattering angle, $\theta = \pi/2$. SO FAR AS EXPERIMENT IS CONCERNED, IT

IS SUGGESTED THAT THE SAME WOULD BE DONE AT $\phi_o \approx \pi/4$ and $\theta = \pi/2$ to get good response.

(B) FOR INCIDENT SIGNAL CIRCULARLY POLARIZED :

Considering that the intense cicularly polarized laser beam is incident on free electron in the direction of $\vec{z}$ with wave vector $\vec{k}$ whose electric field vector is represented as

$$\vec{E}^{inc} = E_o [\vec{e}_1 \cos(\omega t - \vec{k}.\vec{z}) + \vec{e}_2 \sin(\omega t - \vec{k}.\vec{z})] \quad — (13)$$

where $\vec{e}_1$ and $\vec{e}_2$ are two mutually perpendicular polarization unit vectors which can be resolved in terms of $\vec{\beta}_1$ and $\vec{\beta}_2$ such that $\vec{e}_1.\vec{\beta}_1 = \vec{e}_2.\vec{\beta}_2 = \cos\phi_o$, $\vec{e}_1.\vec{\beta}_2 = \vec{e}_2.\vec{\beta}_1 = \sin\phi_o$. E_o is the amplitude of incident signal. The sulution of the equation of motion of free electron in the field of expression (13) can be had in the increasing powers of amplitude of signal which are

$$\omega \vec{z}^{(1)} = -q[\vec{e}_1 \cos\omega t + \vec{e}_2 \sin\omega t] \quad — (14)$$

On using the solution in expression (4b), we get the following radiation angular distributions at retarded time

$$\vec{M}_\omega = \omega q (7/8\, q^2 - 1)(\vec{e}_1 \cos\psi + \vec{e}_2 \sin\psi)$$
$$-(1/8)\omega q^3 [(\cos^2\alpha - \cos^2\beta)(\vec{e}_1 \cos\psi - \vec{e}_2 \sin\psi) \quad — (15a)$$
$$+ 2\cos\alpha\cos\beta(\vec{e}_1 \sin\psi + \vec{e}_2 \cos\psi)]$$

and

$$\vec{M}_{2\omega} = 2\omega q^2 [(\cos\beta\, \vec{e}_1 + \cos\alpha\, \vec{e}_2) \cos 2\psi \qquad \text{——— (15b)}$$

$$- (\cos\alpha\, \vec{e}_1 - \cos\beta\, \vec{e}_2) \sin 2\psi \quad]$$

where

$\cos\alpha = -\sin\theta \cos\phi_o$ and $\cos\beta = \sin\theta \sin\phi_o$

Second harmonic scattered electric field vector can be evaluated by using above expression for $\vec{M}_{2\omega}$ as

$$\vec{E}_{2\omega}^{scat} = (2/r) e\, \omega\, q^2 [(\cos\beta\, \vec{e}_1 + \cos\alpha\, \vec{e}_2 - \cos\alpha \cos\beta\, \vec{n}) \cos 2\psi$$

$$-(\cos\alpha\, \vec{e}_1 - \cos\beta\, \vec{e}_2 + (\cos^2\beta - \cos^2\alpha)\, \vec{n}) \sin 2\psi \quad] \qquad \text{—— (16)}$$

This can be simplified as

$$\vec{E}_{2\omega}^{scat} = A' \, \vec{S}_{2\omega}^{scat} \qquad \text{——— (17)}$$

where second harmonic field amplitude is given as

$$A' = (2/r)\, e\, \omega\, q^2 \qquad \text{——— (17a)}$$

and

$$\vec{S}_{2\omega}^{scat} = \sin\theta [(\sin\phi_o \vec{e}_1 - \cos\phi_o \vec{e}_2 + \sin\theta \sin 2\phi_o\, \vec{n}) \cos 2\psi$$

$$+ (\cos\phi_o \vec{e}_1 + \sin\phi_o \vec{e}_2 + \sin\theta \cos 2\phi_o\, \vec{n}) \sin 2\psi \quad] \qquad \text{——— (17b)}$$

Where $\vec{e}_1$ and $\vec{e}_2$ are given by

$$\vec{e}_1 = \cos\theta \cos\phi_o\, \vec{\alpha}_1 + \sin\phi_o\, \vec{\alpha}_2 - \sin\theta \cos\phi_o\, \vec{n}$$

and

$$\vec{e}_2 = -\cos\theta \sin\phi_o\, \vec{\alpha}_1 + \cos\phi_o\, \vec{\alpha}_2 + \sin\theta \sin\phi_o\, \vec{n}$$

Thus we see that this is not a simple expression .One has to analyse it to find out the exact nature of this received signal so far as this must be written in a suitable form to exhibit ellipticity by introducing a phase new ζ parameter :

$$\vec{S}_{2\omega}^{scat} = \vec{S}_c \cos(2\psi + \zeta) + \vec{S}_s \sin(2\psi + \zeta) \qquad \text{—— (18)}$$

On using $\vec{S}_c \cdot \vec{S}_s = 0$, the orthogonality condition , the parameter ζ can be determined. We find

$$\tan 2\zeta = \tan 4\phi_o \text{ ; i.e., } \zeta = 2\phi_o \text{ .} \qquad \text{—— (19)}$$

After simplification ,we write $\vec{S}_c$ and $\vec{S}_s$ in a very compact form

$$\vec{S}_c = -\sin\theta\, \vec{\alpha}_2 \quad \text{and}$$

$$\vec{S}_s = -\sin\theta \cos\theta\, \vec{\alpha}_1 \qquad \text{—— (20)}$$

Now general equation (18) for second harmonic field is reduced as

$$\vec{S}_{2\omega}^{scat} = \sin\theta \, [\cos\theta \, \vec{\alpha}_1 \sin 2(\psi + \phi_o) - \vec{\alpha}_2 \cos 2(\psi + \phi_o)] \qquad (21)$$

This is clear from the above expression that the scattered electric vector traces out an ellipse and consequently second harmonic light is elliptically polarized,in general .

DISCUSSION :

(i) **When scattering angle ,θ** is small i.e., $\cos\theta \rightarrow 1$, the expression (21) reduces to that for circularly polarized light and that can be written as

$$\vec{S}_{2\omega}^{scat} = \sin\theta \, [\sin 2(\psi + \phi_o) \, \vec{\alpha}_1 - \cos 2(\psi + \phi_o) \, \vec{\alpha}_2] \qquad (22)$$

(ii) When $\theta = \pi/2$, equation (21) turns out to be

$$\vec{S}_{2\omega}^{scat} = - \cos 2(\psi + \phi_o) \, \vec{\alpha}_2 \qquad (23)$$

This is the expression for straight line ,and radiation at 2ω becomes linearly polarized.

The change of polarization of light from circular to linear at second harmonic frequency should not be difficult to detect experimentally.

CONCLUSIONS

As it is shown that when incident laser beam is plane polarized, the scattered light at 2ω is analysed carefully and found nearly plane polarized. The final polarization angle, ϕ dependence on the initial polarization angle, ϕ_o and scattering angle , θ is given in equation(12) and variation is depicted in figure-3 . It is found that the polarized scattering cross-section is maximum at $\phi_o \simeq \pi/4$ for $\theta \simeq \pi/2$. So far as the experiment is conc it is done at the same initial polarization and scattering angles where scattering of laser output high and detection of polarization effects on the emitted radiations would be a little easier . One can turn over the state of polarization from linear to circular in order to see the influence . In this case of circular polarization ,the scattered electric field vector oscillates to trace out an ellipse and obtained light is,in general, elliptically polarized .Ellipticity vanishes for very small values of θ and then there is no change in the final polarization . But for $\theta = \pi/2$ the scattered light becomes linearly polarized .Peak values of polarized cross-section for scattering radiation in the case of circularly polarized incident laser light is observed at scattering angle,$\theta = \pi/2$.

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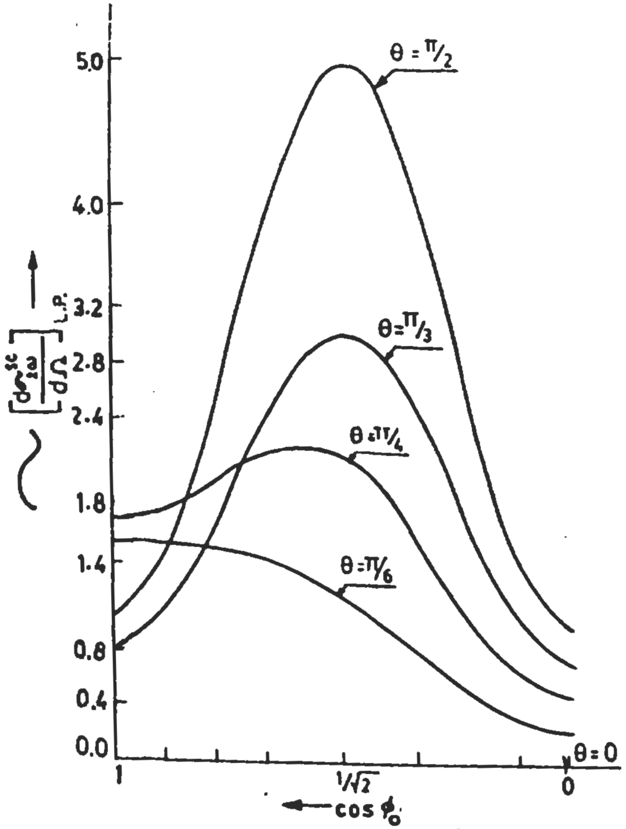
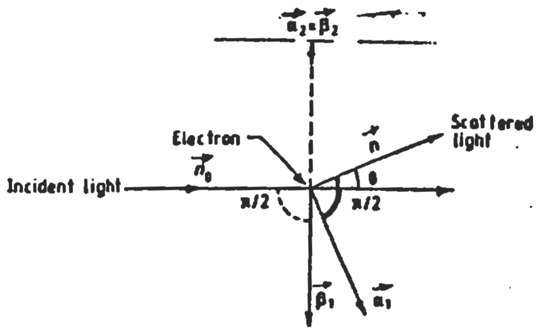
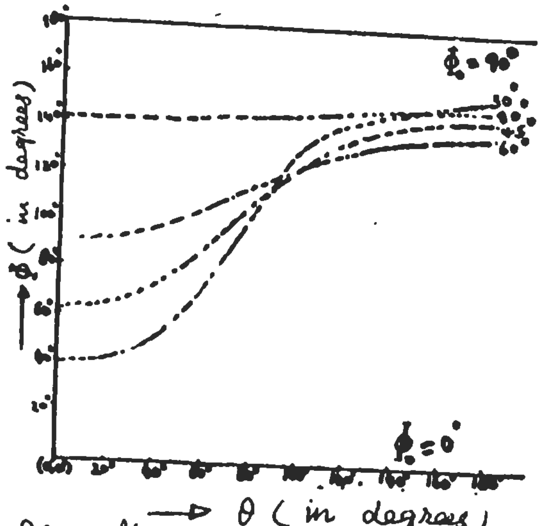


Fig. 1: Angular variation of differential scattering cross-section.



The electric vector of incident light is resolved along mutually perpendicular unit vectors ($\vec{\beta}_1$, $\vec{\beta}_2$ and $\vec{n}_0$) and that of the scattered 2ω light is resolved along the unit vectors ($\vec{\alpha}_1$, $\vec{\alpha}_2$ and $\vec{n}$). Unit vectors $\vec{\beta}_1$, $\vec{\alpha}_1$, $\vec{n}_0$ and $\vec{n}$ are in the plane of the scattering and $\vec{\beta}_2 = \vec{\alpha}_2$ is perpendicular to the plane of scattering pointing into the paper.

Fig. 2



Polarization of scattered light, Φ vs scattered angle for various incident Polarization, Φ_0.

Fig. 3.