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The Interplay of Spin and Charge Channels in Zero Dimensional Systems

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LMU

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CENTER FOR THEORETICAL PHYSICS

SFB 631

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What is to be discussed:

Zero-dimensional system

Spin channel → **charge channel,**
conductance,
susceptibility

MK, Yuval Gefen, PRL 2006 (in press), cond-mat/0504751

Outline

- Universal Hamiltonian
- Zero-Bias Anomaly and Coulomb Blockade
- Mesoscopic Stoner Instability
- Gauge fluctuations and transport properties
- TDoS, dynamic transverse susceptibility
- Key results, conclusions and perspectives

Addressed Questions

- Influence of zero-mode interaction in spin channel on Tunneling DoS and dynamic transverse & longitudinal susceptibilities in the Coulomb valley regime
- Role of transverse gauge fluctuations in the vicinity of the Stoner Instability

Metallic quantum dots: many-electron system

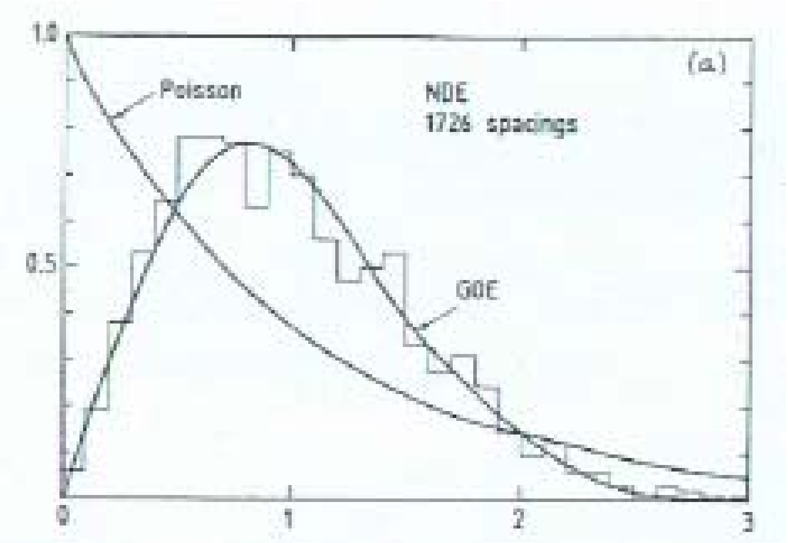
Random Matrix Theory

Quantum Dot

= artificial atom

GOE GUE

Wigner-Dyson statistics



$$P\left(\left\{E_\mu\right\}\right) \propto \exp\left(\frac{\beta}{2}\sum_{\nu\neq\mu}\ln\left[\frac{|E_\mu - E_\nu|}{\delta}\right]\right)$$

$\beta = 1$ Orthogonal (GOE)

$\beta = 2$ Unitary (GUE)

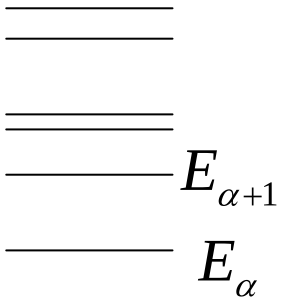
$\beta = 4$ Simplectic (GSE)

Metallic Quantum Dot: Universal Hamiltonian

Metallic grain or small island of electron gas

Quantum Dot

Electron-electron interactions in isolated metallic grains



Mean-level spacing $\Delta = \langle E_{\alpha+1} - E_{\alpha} \rangle$ (kinetic energy)

Thouless energy $E_T \sim D \cdot L^{-2}$ diffusive regime

$E_T \sim v_F L^{-1}$ ballistic regime

$g = E_T / \Delta \gg 1$ **metallic grain**

GUE

$$H_0 = \sum_{\alpha} E_{\alpha} n_{\alpha}$$

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2 - \cancel{\lambda_{BCS}\hat{T}^{+}\hat{T}}$$

$$E_c = \frac{e^2}{2C}$$

$$\hat{n} = \sum_{\alpha,\sigma} d^{+}_{\alpha\sigma} d_{\alpha\sigma}$$

charge

$$S^{\gamma} = \frac{1}{2} \sum_{\alpha,\sigma,\sigma'} d^{+}_{\alpha\sigma} \sigma^{\gamma}_{\sigma\sigma'} d_{\alpha\sigma'}$$

spin

$$T = \sum_{\alpha} d_{\alpha\uparrow} d_{\alpha\downarrow}$$

superconducting

Coulomb blockade

Kurland, Aleiner, Altshuler (2000)
Aleiner, Brouwer, Glazman (2002)

Scaling:

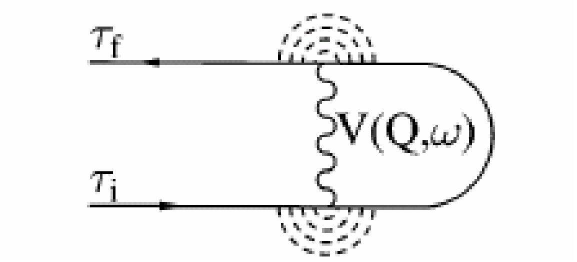
Short-range interaction $E_c = 4|J| \sim \Delta$

Coulomb interaction $E_c = r_s (k_F L)^{d-1} \Delta \gg |J|$

What is a zero-mode interaction?

Electron-electron interaction

$$H_{\text{int}} = \frac{1}{2}\sum_{Q} V(Q)\rho(Q)\rho(-Q) \qquad \vec{Q} = \frac{2\pi}{L}\vec{n}$$



$$V(\mathbf{Q},\omega_n) = \frac{V_0(\mathbf{Q})}{1+V_0(\mathbf{Q})\Pi(\mathbf{Q},\omega_n)}$$

$$\Pi(\mathbf{Q},\omega_n) = \nu_0 \frac{D\mathbf{Q}^2}{D\mathbf{Q}^2+|\omega_n|}$$

Q=0 contribution = zero mode. No screening!!!

$$H_{\text{int}} = \frac{1}{2}V(0)\left[\hat{n}-N\right]^2 \qquad V(0) = \frac{e^2}{C}$$

Nazarov (1989)
Levitov, Shytov (1996)
Kamenev, Gefen (1996)

Zero-mode interaction requires
a non-perturbative treatment at low temperatures!

Zero-bias anomaly in zero-dimensional systems

Quantum Dot

$$H_{\text{int}} = E_c(\hat{n} - N)^2$$

"Orthodox" theory of the Coulomb Blockade

R.I.Shekhter (1975)
Ben-Jacob, Gefen (1985)
Mullen, Gefen, Ben-Jacob (1988)
Averin, Likharev (1991)

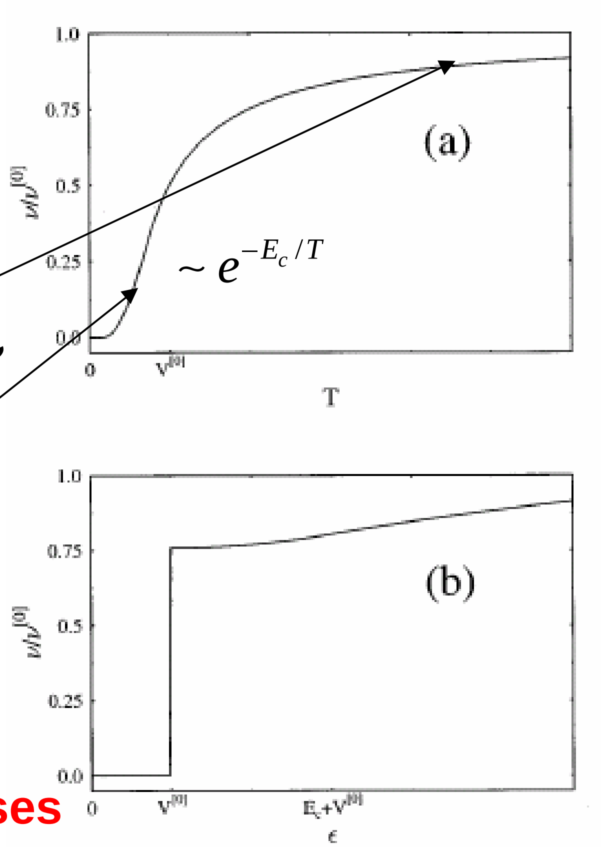
ZBA

perturbative

non-perturbative

$$\nu(\epsilon)/\nu^{[0]}(\epsilon) = 1 - \frac{V}{4T}\text{sech}^2\left(\frac{\epsilon}{2T}\right)$$

$$\nu(\epsilon)/\nu^{[0]}(0) = \cosh\left(\frac{\epsilon}{T}\right)\exp\left(-\frac{E_c}{T}\right)$$

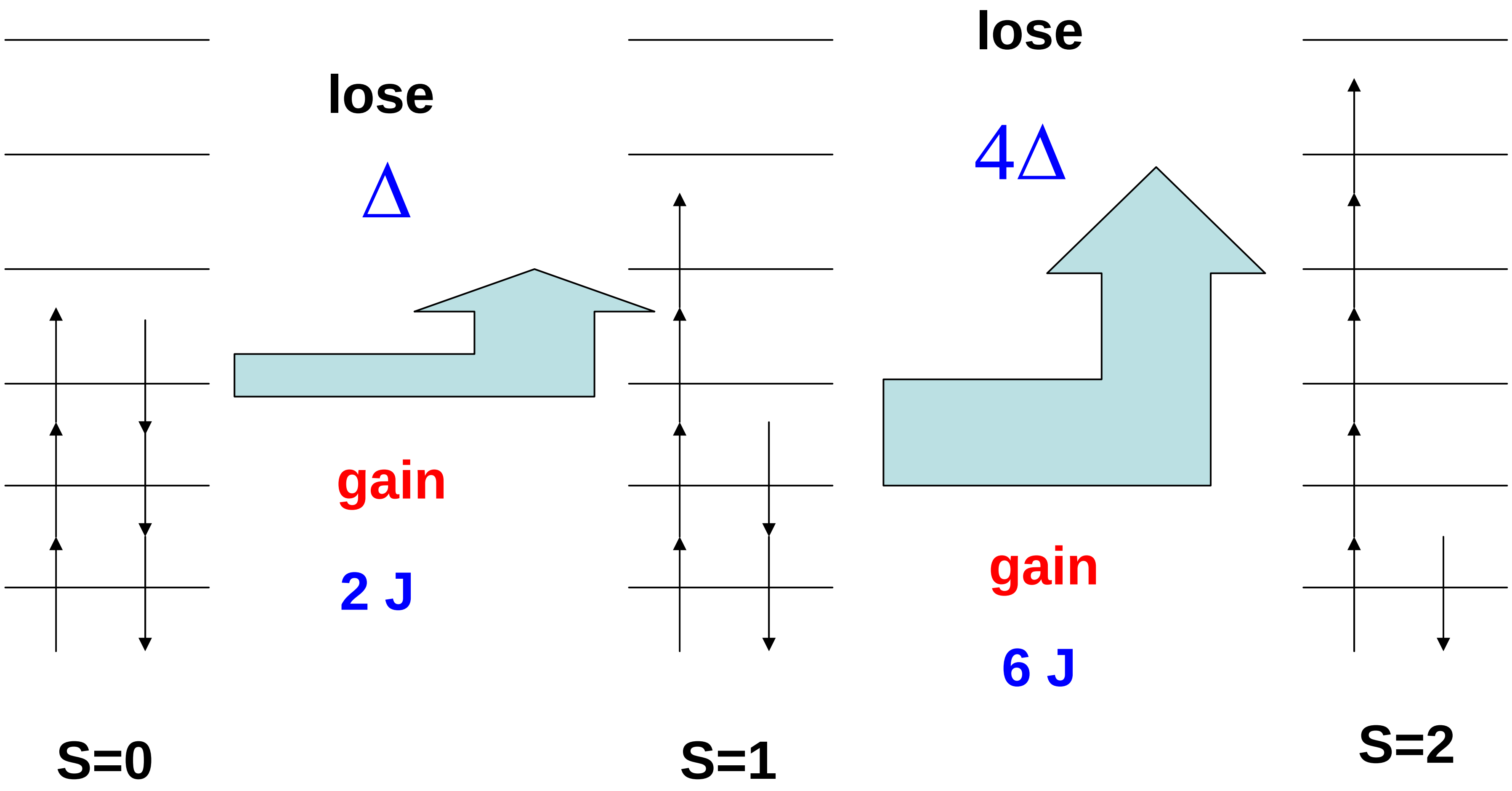


ZBA and Coulomb blockade are two limiting cases of the same theory

Mesoscopic Stoner Instability

$$H_{\text{int}} = E_c(\hat{n}-N)^2 - J(\vec{S})^2$$

integer spin $(\vec{S})^2 = S(S+1)$



lose

Δ

gain

2 J

lose

4Δ

gain

6 J

S=0

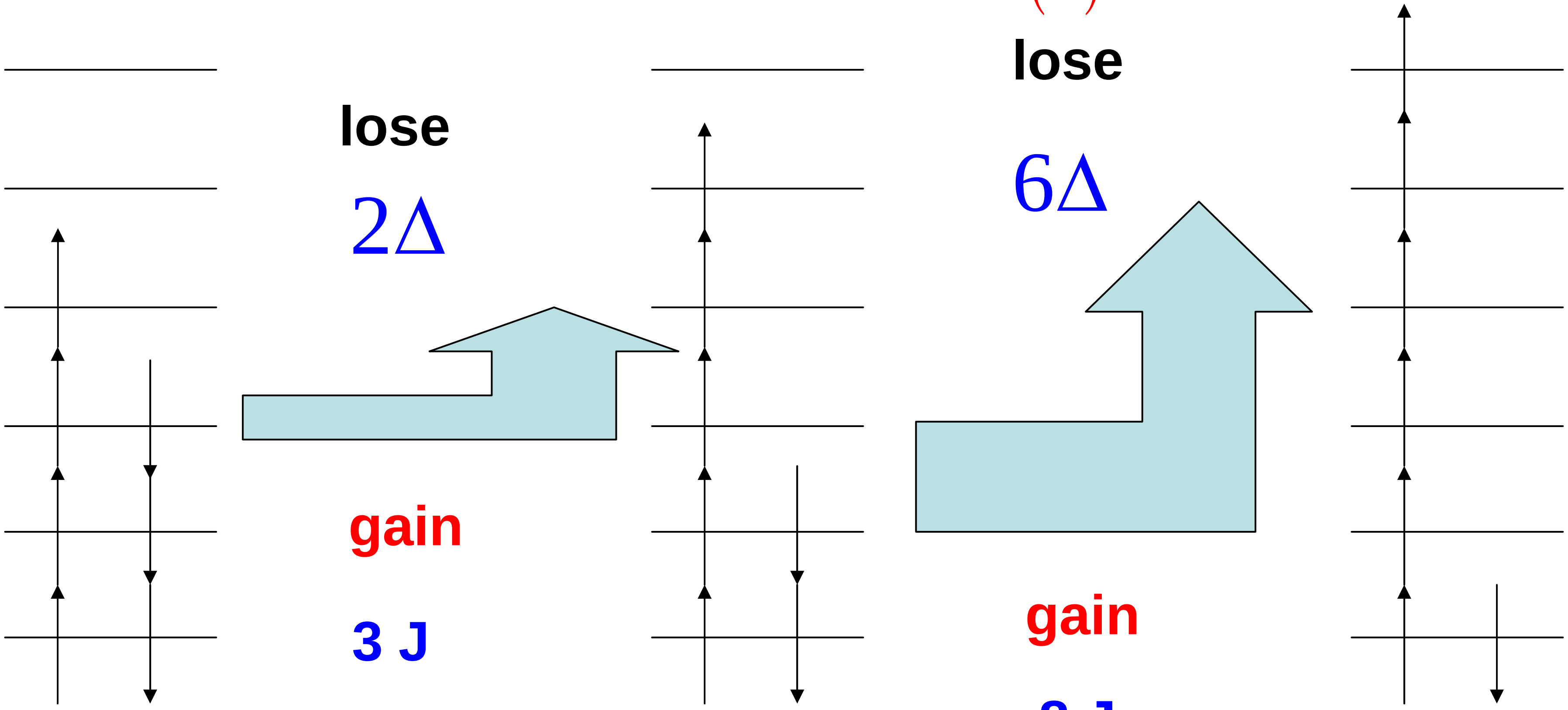
S=1

S=2

Mesoscopic Stoner Instability

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2$$

half-integer spin $\left(\vec{S}\right)^2 = S(S+1)$



S=1/2

lose

2Δ

gain

3 J

S=3/2

lose

6Δ

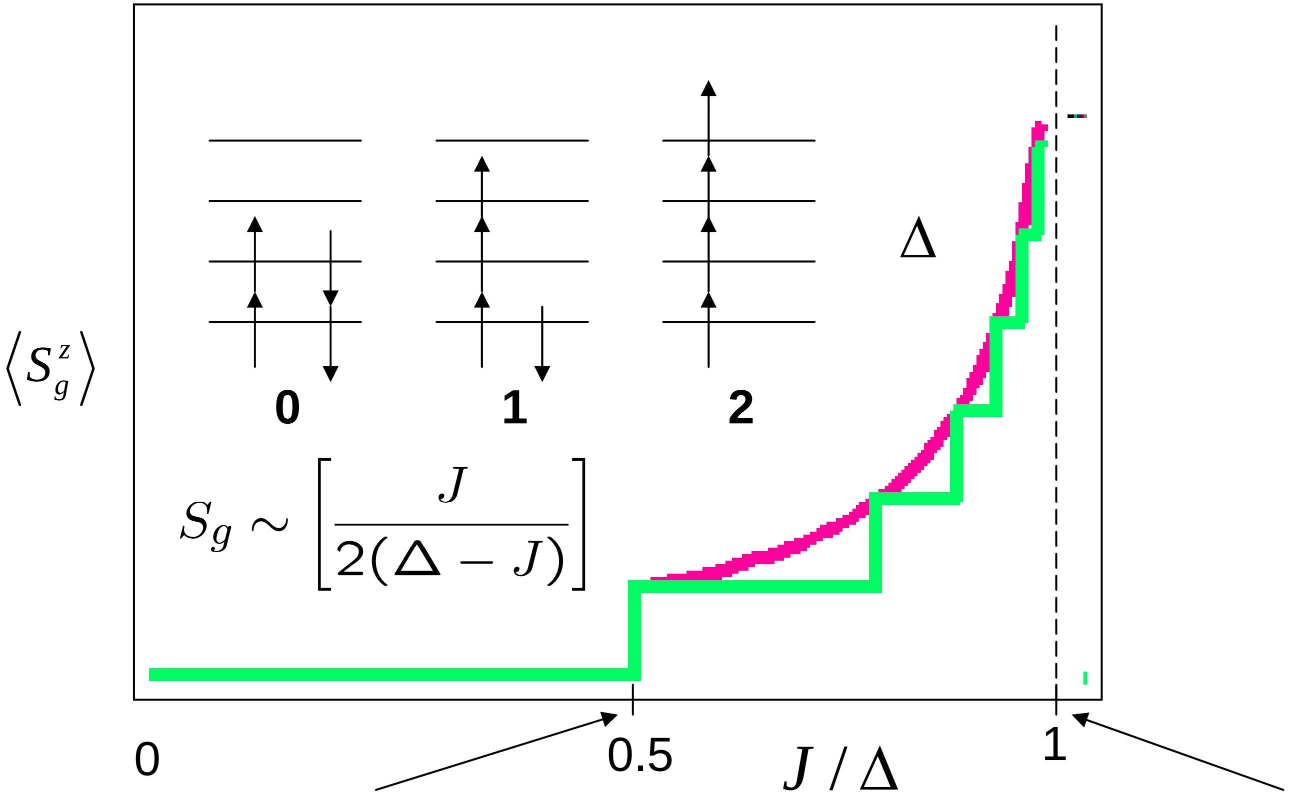
gain

8 J

S=5/2

Mesoscopic Stoner Instability

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2$$



$$S_g \sim \left[\frac{J}{2(\Delta - J)}\right]$$

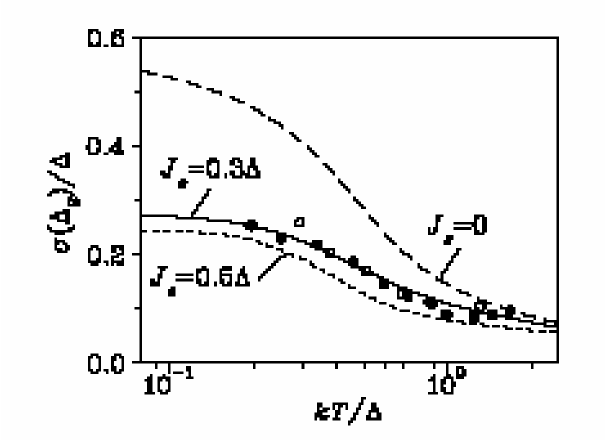
Mesoscopic Instability

Thermodynamic Instability

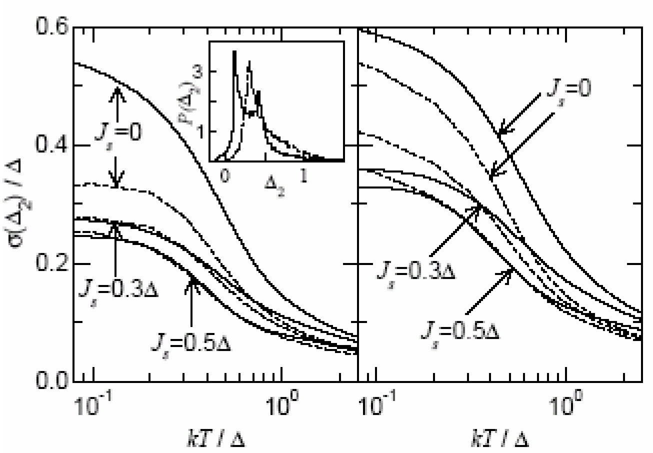
NB: For Ising model mesoscopic and thermodynamic instability points coincide!

Spin Exchange. Master Equation (classical) Approach

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2$$

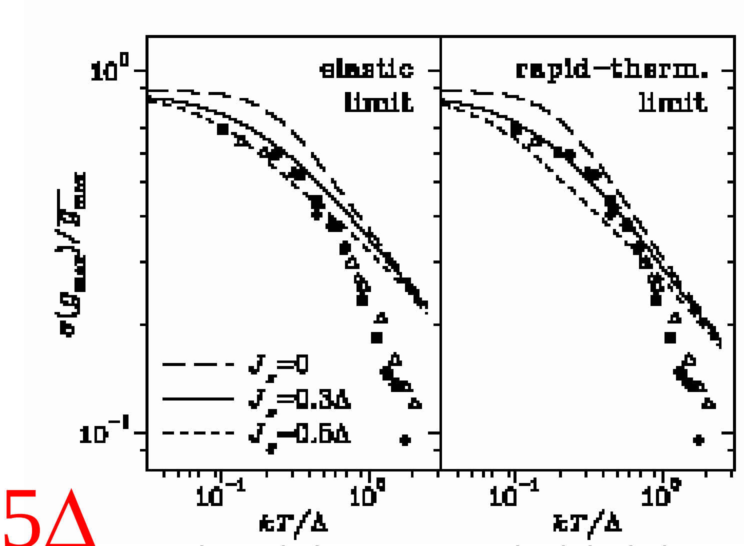


The width of peak-spacing distribution

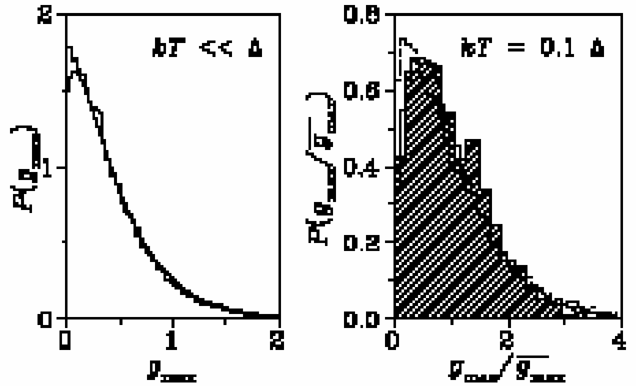


The width of peak-spacing distribution in the presence and absence of the orbital magnetic field.

$$J \geq 0.5\Delta$$



The ratio between standard deviation and the average value of peak height



Peak-height distributions

Y.Alhassid and T.Rupp (2003, 2004)

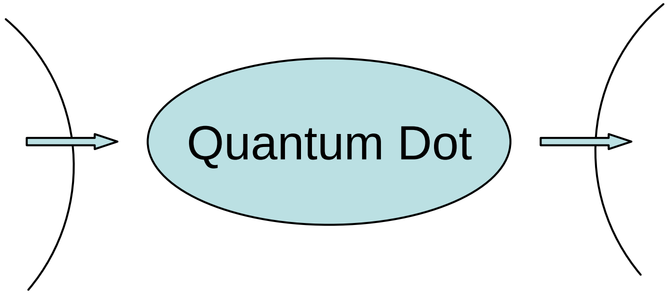
Zero-mode interaction in the Charge and Spin channels

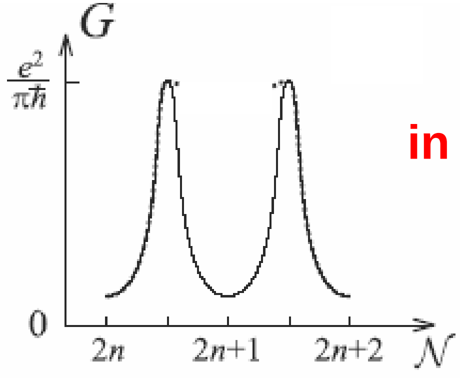
$$H_0 = \sum_{\alpha} E_\alpha n_\alpha \qquad H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2$$

$$H_{tun} = \sum_{\mathbf{k},\sigma,\alpha} [V_{\mathbf{k}\alpha} c^\dagger_{\mathbf{k},\sigma} d_{\sigma\alpha} + h.c.]$$

Non-perturbative calculation of the electron Green's function

$$\mathcal{G}_{\alpha\sigma}(\tau_i, \tau_f) = -\langle T_\tau \Psi_{\alpha\sigma}(\tau_f) \bar{\Psi}_{\alpha\sigma}(\tau_i) \rangle$$





in Coulomb valleys

TDoS

$$\nu(\epsilon) = -\frac{1}{\pi} \cosh\left(\frac{\epsilon}{2T}\right) \int_{-\infty}^{\infty} \mathcal{G}\left(\frac{1}{2T} + it\right) e^{i\epsilon t} dt$$

Conductance

$$g_T = \frac{e}{\hbar} \int d\epsilon \nu(\epsilon) \Gamma(\epsilon) \left(-\frac{\partial f_F}{\partial \epsilon}\right) \qquad \Gamma(\epsilon) = 2\pi\rho(\epsilon)|V|^2$$

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Zero-mode interaction in the Charge channel: U(1) symmetry

Basic ingredients of theory

$$H_{\text{int}} = E_c(\hat{n}-N)^2 \qquad \mathcal{L} = \sum_\alpha \bar{\Psi}_\alpha[\partial_\tau + \mu]\Psi_\alpha - H_0 - H_{int}$$

$$\exp\left(-\int_0^\beta d\tau H_C\right) = \int D[\phi]\exp\left(-\int_0^\beta d\tau\left[\frac{\phi^2}{4E_c} + i\phi\left[\sum_\alpha n_\alpha - N_0\right]\right]\right)$$

$$\mathcal{L} = \sum_\alpha \bar{\Psi}_\alpha \quad [\partial_\tau - E_\alpha + i\phi(\tau) + \mu] \quad \Psi_\alpha - \frac{\phi^2}{4E_c}$$

$$[\partial_\tau - E_\alpha + i\phi_0 + \mu]$$

$$\boxed{W_c^{-1}(\tau)W_c(\tau)} \qquad \boxed{W_c^{-1}(\tau)W_c(\tau)}$$

Gauge transformation $\tilde{\Psi}_\alpha = W_c(\tau)\Psi_\alpha$ Initial condition $W_c(0) = 1$

Necessary to keep anti-periodic BC for Ψ_α Boundary condition $W_c(\beta) = 1$

Gauge equation

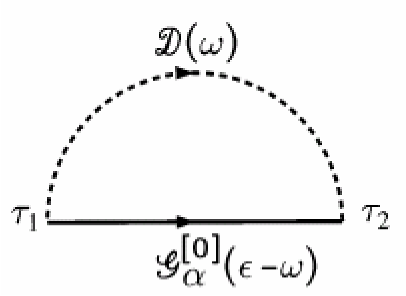
$$\boxed{\partial_\tau W_c^{-1} + i\phi(\tau)W_c^{-1} = iW_c^{-1}\phi_0}$$

Zero-mode interaction in the Charge channel: U(1) symmetry

$$\partial_\tau W_c^{-1} + i\phi(\tau) W_c^{-1} = i W_c^{-1}\phi_0$$

Solution

$$W_c(\tau) = \exp\left(i \int_0^\tau \left[\phi(\tau') - \phi_0 \right] d\tau' \right) \qquad \beta\phi_0 = \int_0^\beta \phi(\tau) d\tau$$



$$\mathcal{G}_{\alpha,\sigma}(\tau_i - \tau_f) = \mathcal{G}^{[0]}_{\alpha,\sigma}(\tau_i - \tau_f, \tilde{\mu}) e^{-S_C(\tau_i - \tau_f)}$$

non-interacting + disorder

interaction

Kamenev, Gefen (1996)
Efetov, Tschersich (2002, 2003)
Sedlmayer, Yurkevich, Lerner (2005)

$$S_C(0) = S_C(\beta) = 0$$

$$S_C(\tau) = T \sum_{n \neq 0} \frac{2E_c}{\omega_n^2} (1 - \cos(\omega_n \tau)) = E_c \left(|\tau| - \frac{\tau^2}{\beta} \right)$$

Gauge invariance

Abelian theory results in Gaussian gauge factor

Zero-mode interaction in the Spin channel: SU(2) symmetry

Isotropic exchange $H_{\text{int}} = -J(\vec{S})^2$ $S^{\gamma} = \frac{1}{2}\sum_{\alpha,\sigma,\sigma'} \Psi^{+}_{\alpha\sigma}\sigma^{\gamma}_{\sigma\sigma'}\Psi_{\alpha\sigma'}$

Gauge transformation $\tilde{\Psi} = W_s(\tau)\Psi$

Initial condition $W_s(0) = 1$

Boundary condition $W_s(\beta) = 1$

$$\boxed{\partial_\tau W_s^{-1} + \vec{\sigma}\vec{\Phi} W_s^{-1} = W_s^{-1}\vec{\sigma}\vec{\Phi}_0}$$

Formal solution $W_s(\tau) = \left[\mathcal{T}\left\{\exp\left(\int_0^{\tau} \vec{\Phi}\vec{\sigma}d\tau'\right)\right\}\exp\left(-\tau\vec{\Phi}_0\vec{\sigma}\right)\right]_{\mu\nu}$

$$\left[\exp\left(-\beta\vec{\Phi}_0\vec{\sigma}\right)\right]_{\alpha\gamma} = \left[\mathcal{T}\left\{\exp\left(-\int_0^{\beta}\vec{\Phi}(\tau)\vec{\sigma}d\tau\right)\right\}\right]_{\alpha\gamma}$$

Non-Abelian gauge theory

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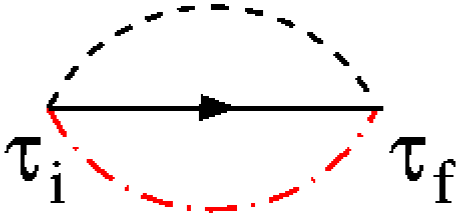
Zero-mode interaction in the Spin channel: SU(2) symmetry

Q: Is it possible to represent all effects of spin interaction as a gauge factor?

A: Yes



$$\boxed{\mathcal{G}_{\alpha,\sigma}(\tau) = \mathcal{G}^{[0]}_{\alpha,\sigma}(\tau,\mu)e^{-S_C(\tau)}e^{-S_s(\tau)}}$$

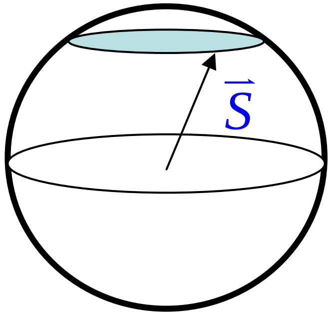
Kiselev, Gefen (2005)

Q: Does the gauge factor depend on the interaction and temperature only?

A: No. It is sensitive to Stoner Instability!

$$S_s = F(J, \Delta, |\tau| - \frac{\tau^2}{\beta})$$

From isotropic to anisotropic spin model



Isotropic

$\varepsilon = 1$

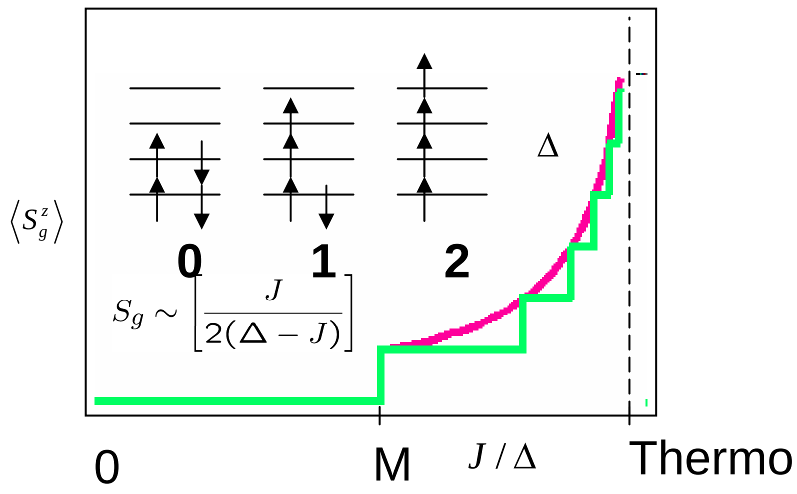
$$H_S = -J(\vec{S})^2$$

$$S^{\gamma} = \frac{1}{2}\sum_{\alpha,\sigma,\sigma'} \Psi^{+}_{\alpha\sigma}\sigma^{\gamma}_{\sigma\sigma'}\Psi_{\alpha\sigma'}$$

$\varepsilon = J_{\perp} / J_{\parallel} \ll 1$

Easy axis model

$$H_S = -J\left[\left(\sum_{\alpha} S^z_{\alpha}\right)^2 + \epsilon\left\{\left(\sum_{\alpha} S^x_{\alpha}\right)^2 + \left(\sum_{\alpha} S^y_{\alpha}\right)^2\right\}\right]$$



$\propto \varepsilon$

Rotation symmetry is reduced to SO(2)

S^z is conserved

$\vec{S}$ is not conserved

Transverse spin fluctuations in anisotropic spin system

Gauge transformation

$$W(\tau) = W_C W_I = e^{i\theta(\tau)} \begin{pmatrix} e^{\eta(\tau)} & 0 \\ 0 & e^{-\eta(\tau)} \end{pmatrix}$$

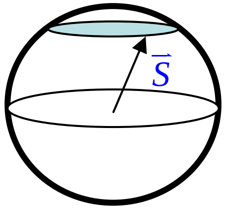
Charge Ising

Technicalities: Hubbard-Stratonovich transformation for charge and spin
“exact” solution for Coulomb + Ising interactions
perturbation theory in transverse spin fluctuations
winding numbers for proper accounting periodicity in τ

Charge channel:
Efetov, Tschersich (2002, 2003)
Sedlmayer, Yurkevich, Lerner (2005)

$$\mathcal{D}^{+-}(\tau_1,\tau_2) \to \boxed{\langle \Phi^+(\tau_1)\Phi^-(\tau_2)\rangle = \frac{J}{2}\delta(\tau_1-\tau_2) + \frac{\epsilon J^2}{2\beta(\Delta-\epsilon J)} + O(\epsilon^2)}$$

white-noise correlations



random initial conditions

Green‘s Function

$$\mathcal{G}_{\alpha,\sigma}(\tau) = \mathcal{G}^{[0]}_{\alpha,\sigma}(\tau,\mu)\, e^{-S_C(\tau)}\, e^{-S_{\parallel}(\tau)} \boxed{F_{\perp}(\epsilon, J, \Delta, \tau)}$$

Basic inequalities

$$E_c \gg T > \Delta > J > \varepsilon J$$

Strong coupling regime for the charge sector

Metallic regime for QD (winding numbers treatment)

Non-magnetic regime (above Stoner Instability point)

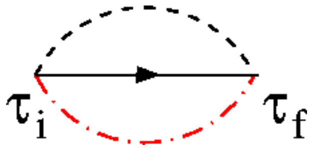
Easy axis anisotropy of spin interaction

Two-parametric expansion $$A = \varepsilon \frac{J}{T} \qquad B = \frac{\varepsilon^2 J^2}{T(\Delta - \varepsilon J)}$$

Transverse spin fluctuations in anisotropic spin system.
Diagrammatic expansion in $\varepsilon = J_\perp / J_\parallel < 1$

$$\langle \Phi^+(\tau_1)\Phi^-(\tau_2)\rangle = \frac{J}{2}\delta\left(\tau_1 - \tau_2\right) + \frac{\epsilon J^2}{2\beta(\Delta - \epsilon J)}$$

Coulomb + Ising gauge factors

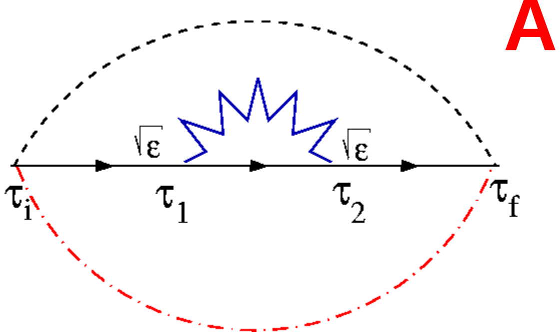


$$\mathcal{G}_{\alpha,\sigma}(\tau_i - \tau_f) = \mathcal{G}^{[0]}_{\alpha,\sigma}(\tau_i - \tau_f, \tilde{\mu}) e^{-S_\parallel(\tau_i - \tau_f)}$$

$$S_\parallel = \left[E_c - \frac{J}{4}\right]\left(\tau - \frac{\tau^2}{\beta}\right)$$

ε corrections

A



$$\delta S_\parallel = -\epsilon J \left(\tau - \frac{\tau^2}{\beta}\right)$$

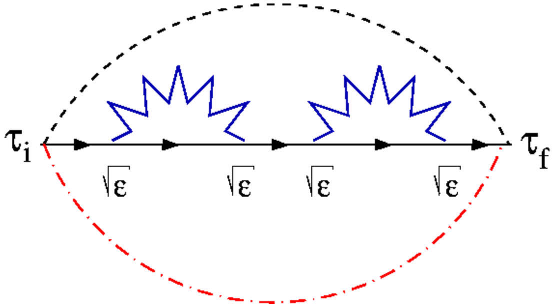
$$2JS_z^2 \to \frac{J}{2}(1 + 2\epsilon) \to 2JS(S+1)$$

Depends on Δ

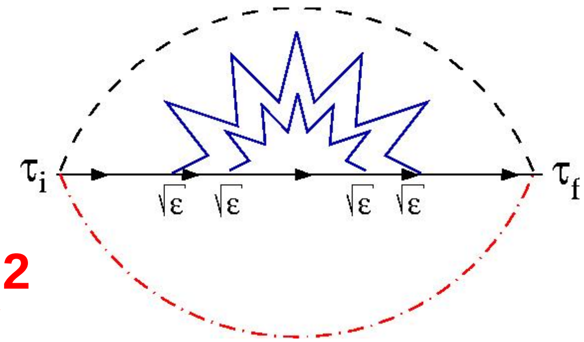
Gauge Factor is Gaussian!

ε^2 (non-Gaussian) corrections

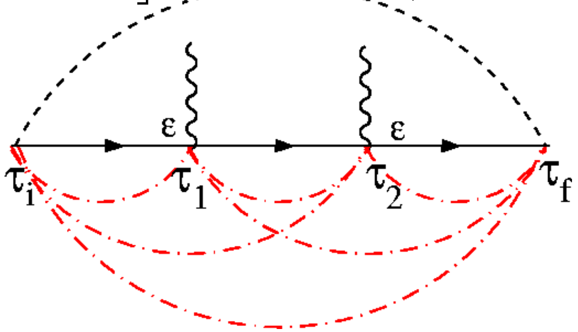
$$\delta S_\perp \sim \epsilon^2 J_{eff}^2 \left[\tau - \frac{\tau^2}{\beta}\right]^2$$



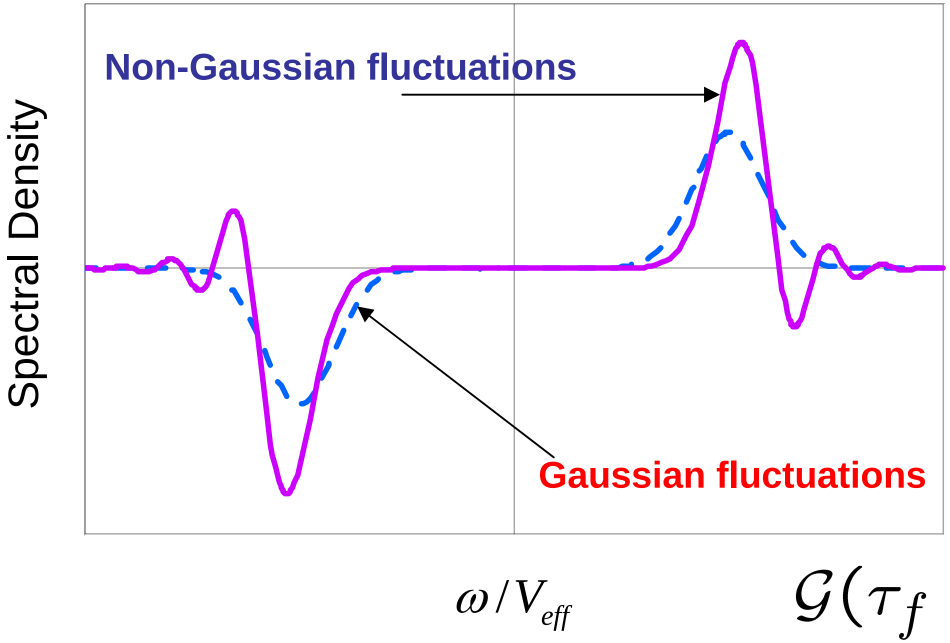
A^2

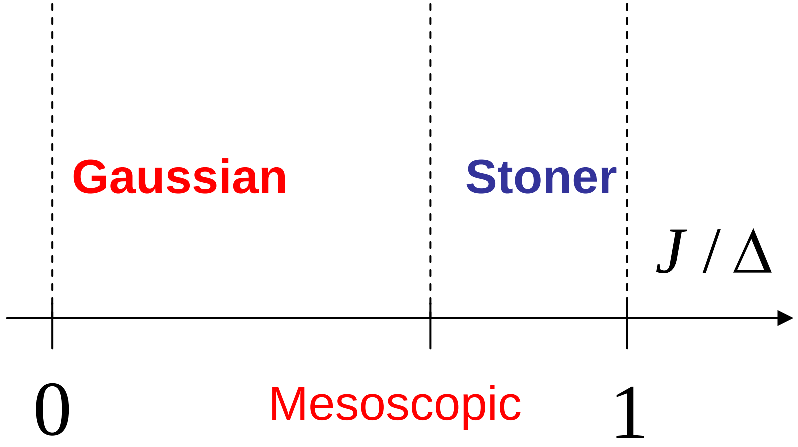


B

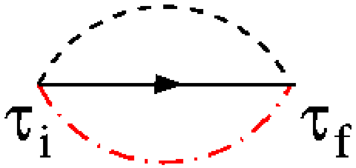


Tunneling Density of States

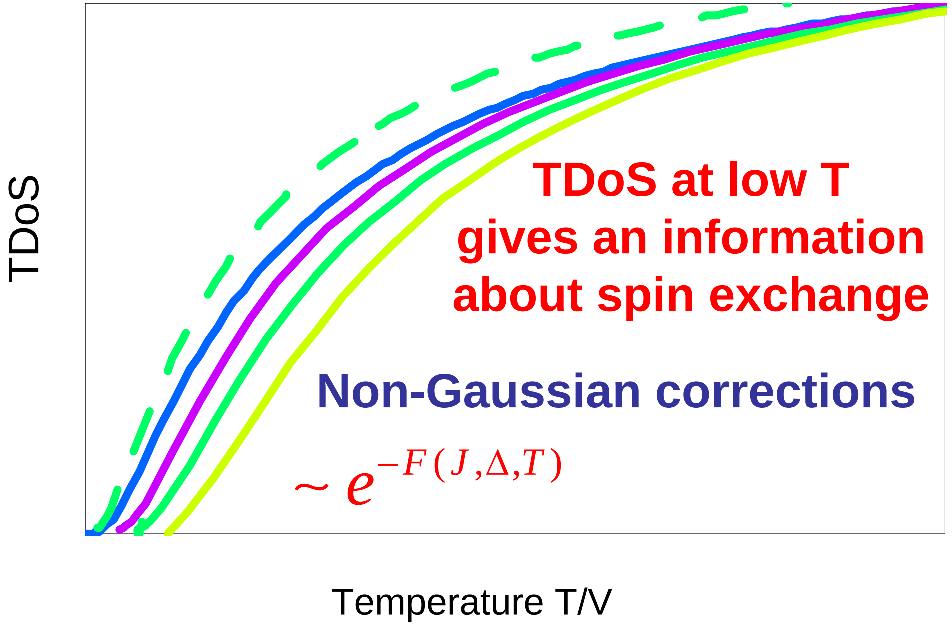




$$\mathcal{G}(\tau_f - \tau_i) =$$



Zero-mode interaction



TDoS at low T gives an information about spin exchange

Non-Gaussian corrections

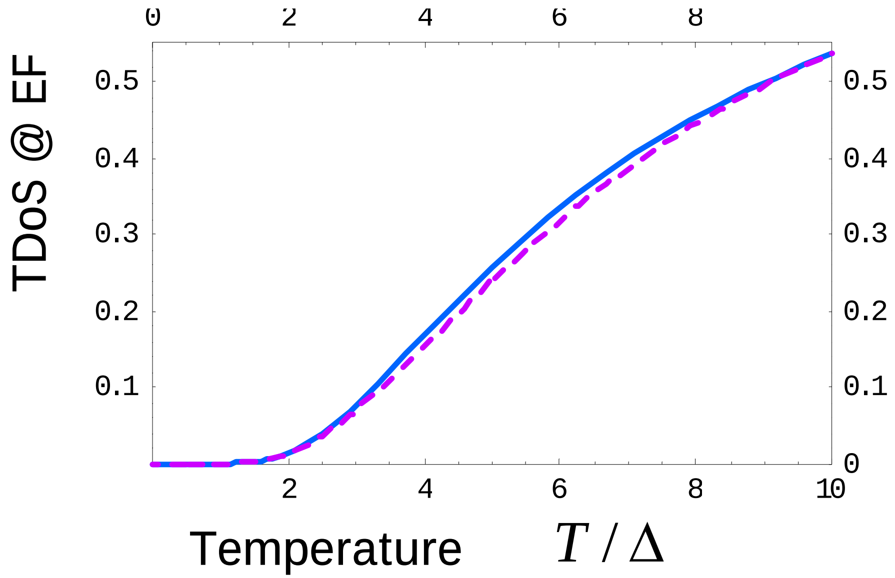
$$\sim e^{-F(J,\Delta,T)}$$

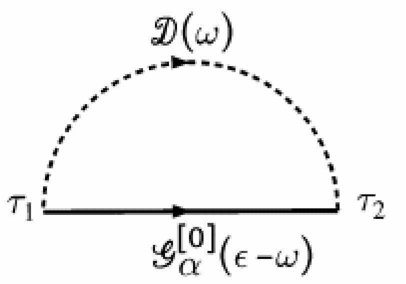
$$\nu(\epsilon) = -\frac{1}{2}\int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\left[\tanh\left(\frac{\epsilon-\omega}{2T}\right) + \right.$$

$$\left. + \coth\left(\frac{\omega}{2T}\right)\right] B(\omega)\nu^{[0]}(\epsilon - \omega)$$

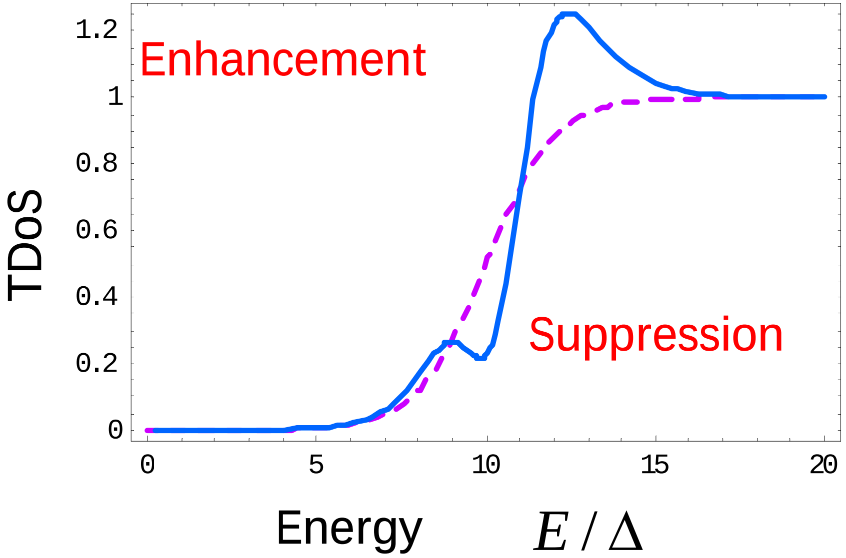
Spectral Density

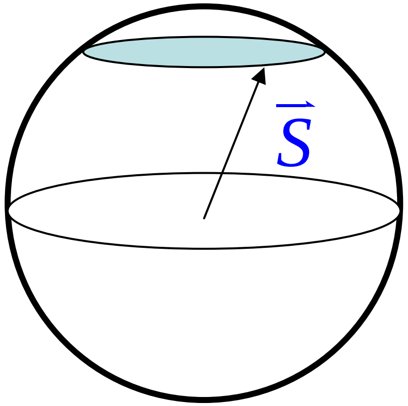
Quantum Dot Spectroscopy $T > \Delta$





Spin and Charge gauge factors

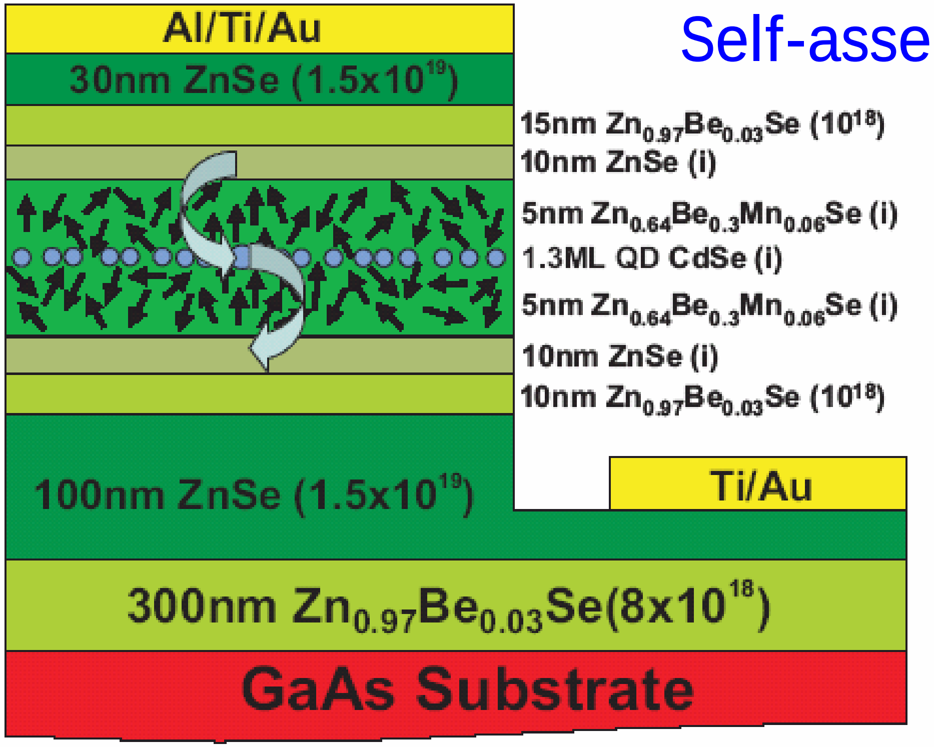




Spin channel affects the charge transport

Kiselev, Gefen (2005)

Self-assembled quantum dots



Al/Ti/Au

30nm ZnSe (1.5x10^{19})

15nm $Zn_{0.97}Be_{0.03}Se$ (10^{18})

10nm ZnSe (i)

5nm $Zn_{0.64}Be_{0.3}Mn_{0.06}Se$ (i)

1.3ML QD CdSe (i)

5nm $Zn_{0.64}Be_{0.3}Mn_{0.06}Se$ (i)

10nm ZnSe (i)

10nm $Zn_{0.97}Be_{0.03}Se$ (10^{18})

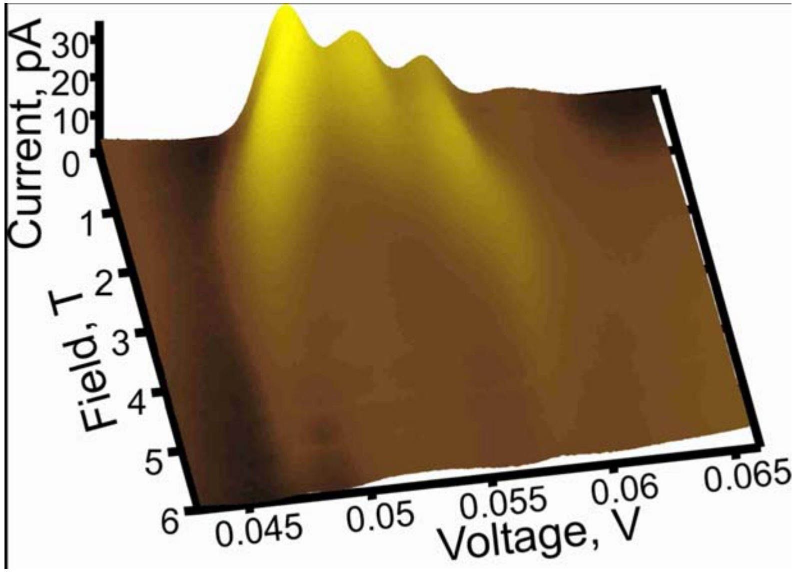
100nm ZnSe (1.5x10^{19})

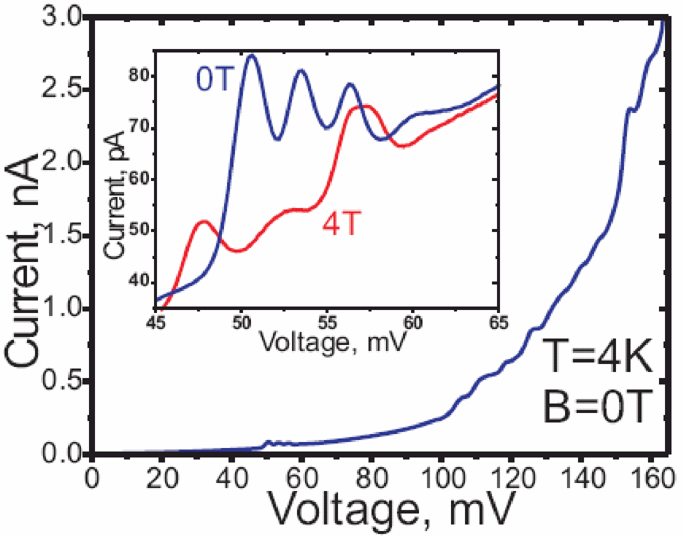
Ti/Au

300nm $Zn_{0.97}Be_{0.03}Se$(8x10^{18})

GaAs Substrate

x=0.04, R=40A





Molenkamp et al (2005)

Spin susceptibilities

$$\chi^{\gamma\gamma'}(\tau_i,\tau_f) = -\langle\bar{\Psi}_\mu(\tau_i)\sigma^{\gamma}_{\mu\nu}\Psi_\nu(\tau_i)\bar{\Psi}_\nu(\tau_f)\sigma^{\gamma'}_{\nu\mu}\Psi_\mu(\tau_f)\rangle$$

Longitudinal Susceptibility

$$\chi^{zz} = \frac{\chi_0}{1-J\chi_0}$$

Stoner Instability

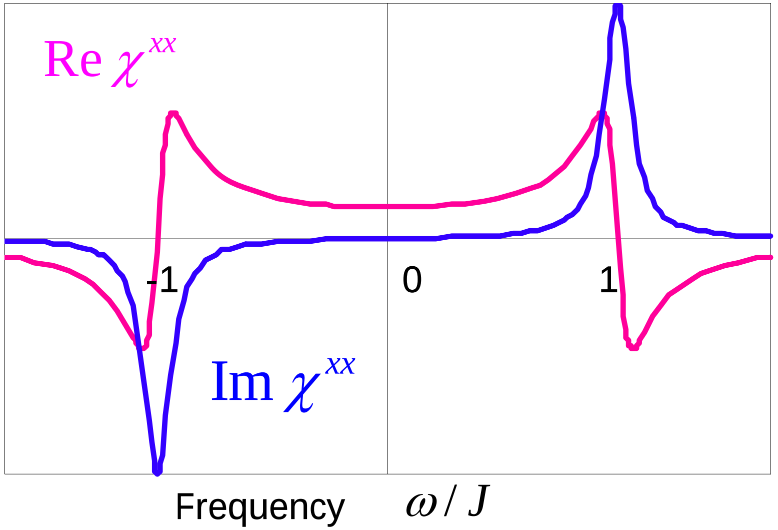
Static longitudinal susceptibility diverges at Stoner Instability point

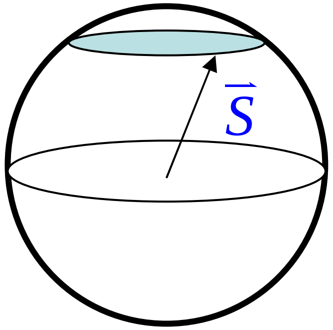
Transverse Susceptibility

Exponentially enhanced!

Gauge factor $F_\perp \leftrightarrow f\left[\chi^{+-}\right]$

$$\chi^{xx}(t) \rightarrow \frac{\chi_0 \varepsilon e^{J/T}}{1-\varepsilon J\chi_0} e^{i(1-\varepsilon)Jt}$$



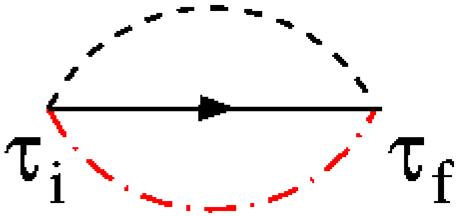


$$\partial_t S^{x,y} = iJ\left(1-\varepsilon\right)S^{x,y} + \ldots$$

Response Functions

Electron Green’s Function

Add electron to the dot
(charge + spin)

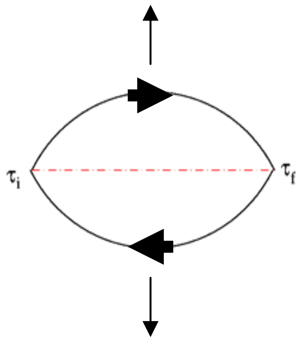


Remove electron from the dot
(charge + spin)

Spin and Charge channels affect transport properties

Spin susceptibility

Charge is conserved,
Spin flips

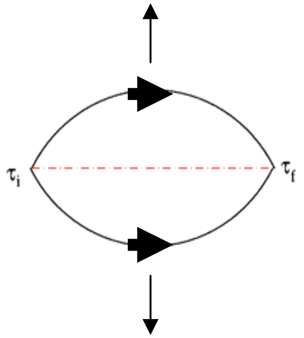


Charge is completely gauged out
Transverse and Longitudinal
fluctuations work

Only Spin channel matters

Superconducting loop

Spin is conserved,
Charge 2e is transferred



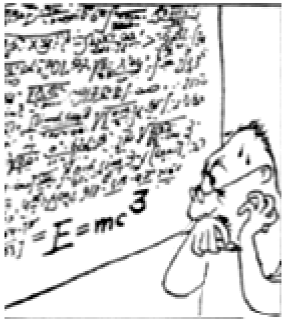
Charge fluctuations
work

Only Charge channel matters

Open questions and perspectives

- Spin and Charge channels in the Coulomb peak regime
- Spin blockade and anti-blockade
- Spin-orbit interaction. Mechanisms of spin relaxation
- System of coupled grains in the vicinity of Stoner Instability Point
- ...

Conclusions



- **Charge and Spin zero-mode interactions strongly affect an electron transport through metallic grain in Coulomb valley regime**

- **Transverse Spin fluctuations become important as one approaches the Stoner Instability Point**

- **Spin fluctuations result in non-monotonic behavior of TDoS and enhance dynamic transverse susceptibility**