Institut für Theoretische Physik und Astrophysik

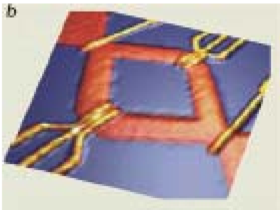
Universität Würzburg

M.Kiselev

Magnetic correlations in mesoscopic and nano - systems

Center for NanoScience

Outline



- Quantum Dot Devices
- Quantum dots with a few electrons
- Kondo effect in Quantum Dots
- Quantum dots with many electrons
- Magnetic instability in isolated dots
- Mesoscopic Stoner magnetism
- Quantum Dot arrays and nano-crystals
- Perspectives

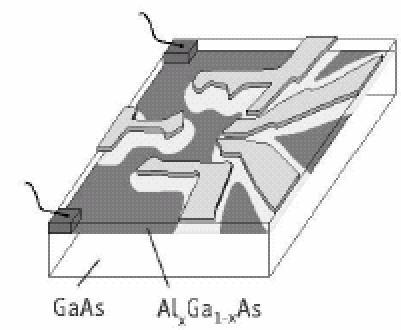
Collaborators:

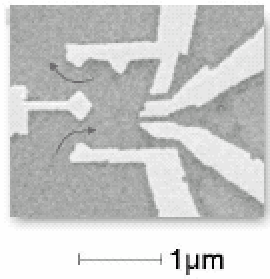
Y.Avishai (Beer Sheva), Y.Gefen (Weizmann), K.Kikoin (Beer Sheva),
L.W.Molenkamp (Würzburg), R.Oppermann (Würzburg)
J.Richert (Strasbourg), V.Vinokur (Argonne), M.Wegewijs (Aachen)

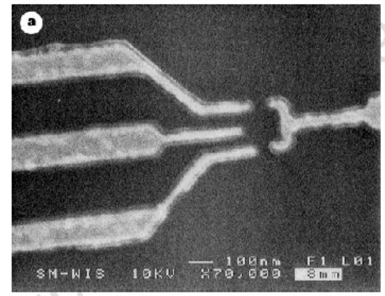
PhD students: H.Feldmann, M.Bechmann (Würzburg)

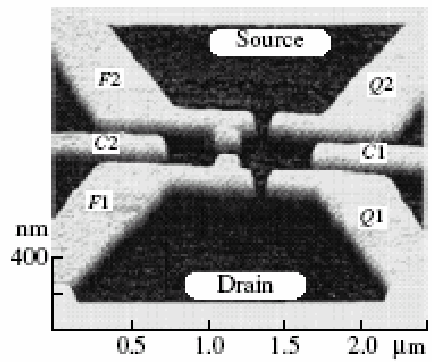
Support: AvH & SFB-410 @ WU, **SFB-630 @ LMU, LSF @ WIS, DOE @ ANL**

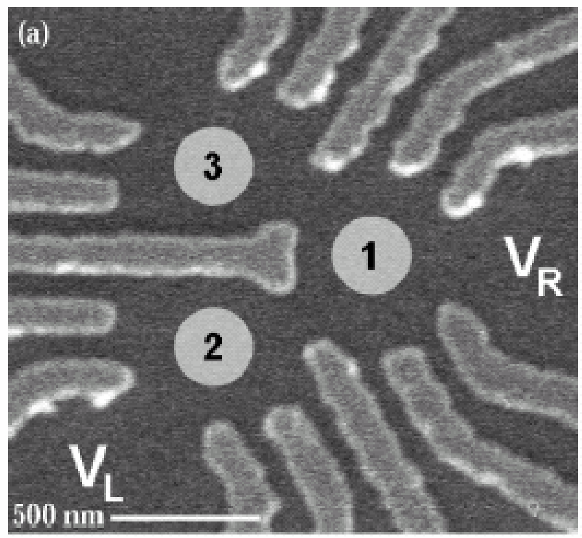
Quantum dots: from simple to complex

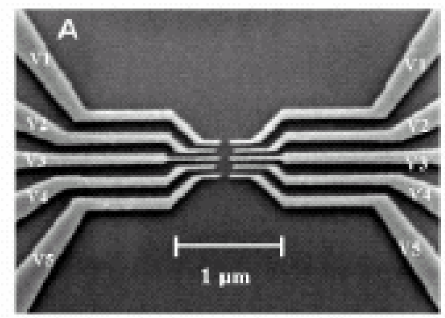


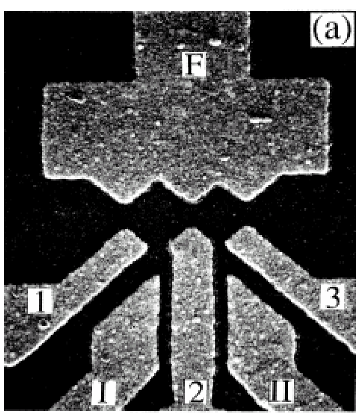








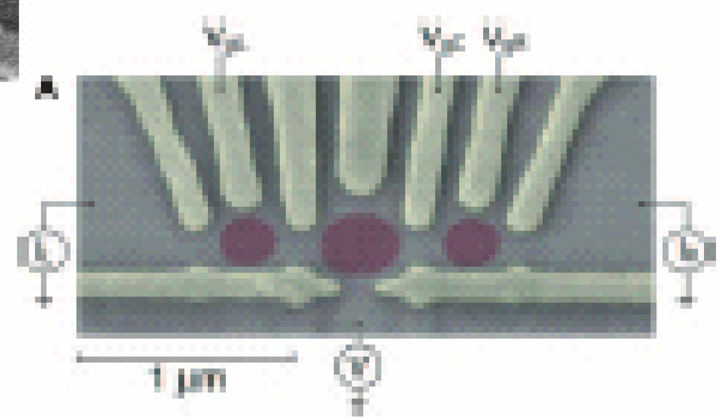




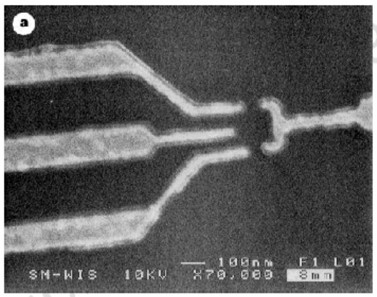
D.Goldhaber-Gordon et al (1998)

L.W.Molenkamp et al (1995)

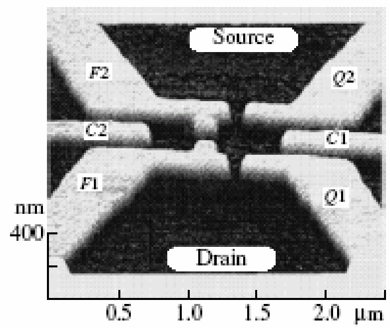
C.Marcus et al (2003)



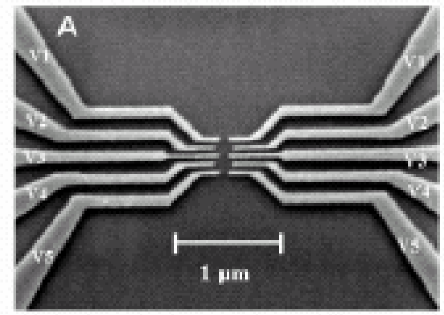
Coupled Quantum Dots



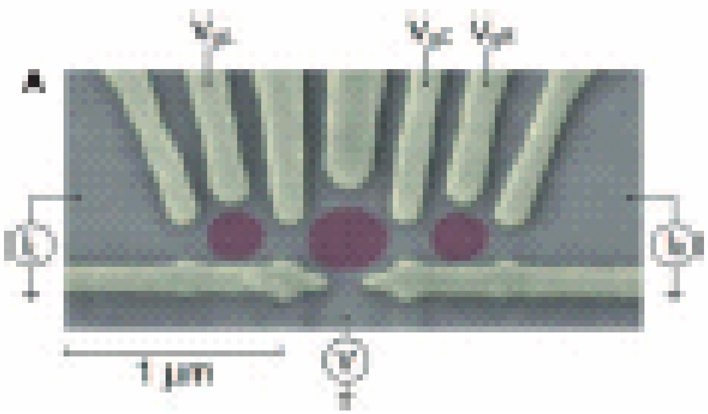
Single QD



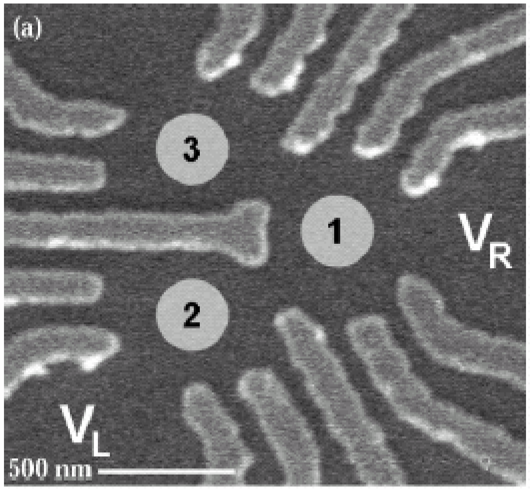
Double parallel QD



Double serial QD

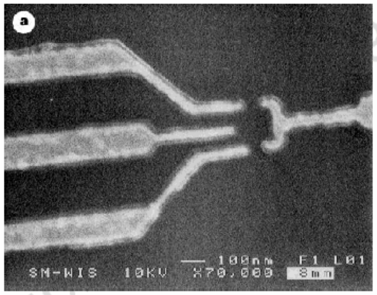


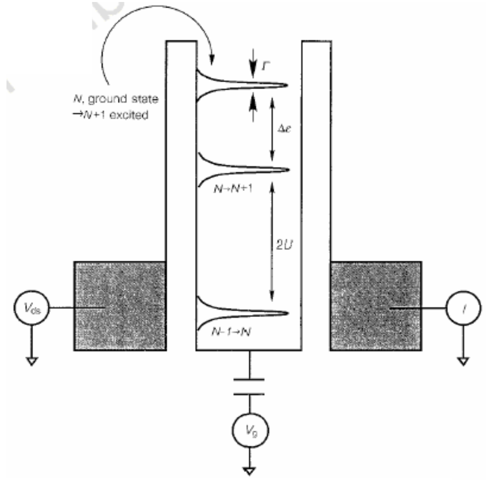
Triple serial QD

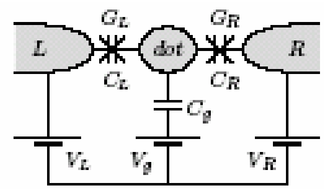


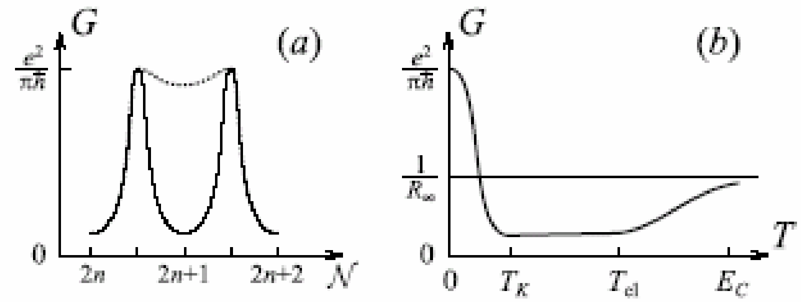
Triangular QD

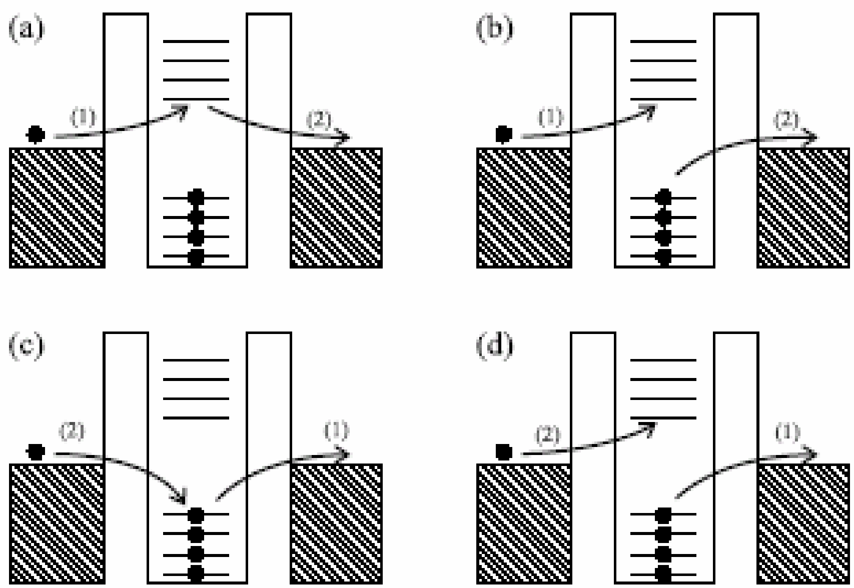
Quantum Dots with few electrons







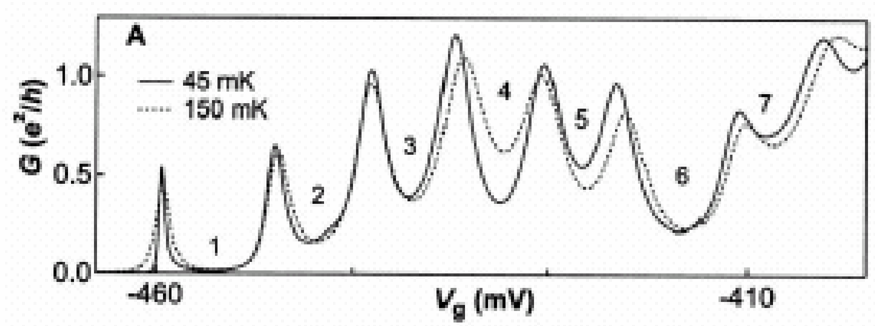




Electron-like

co-tunneling

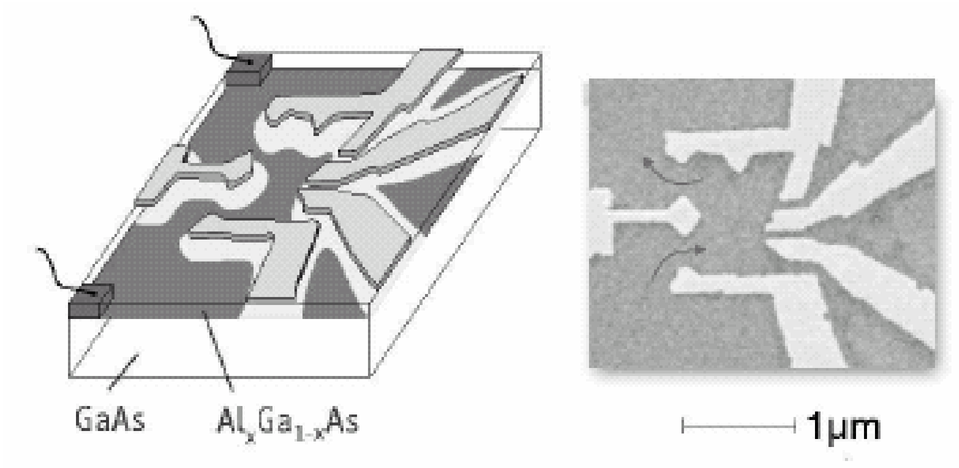
Hole-like

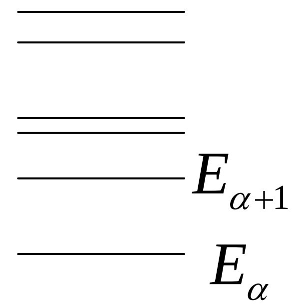


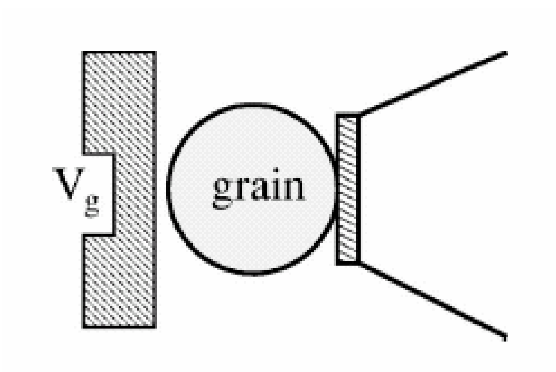
Elastic

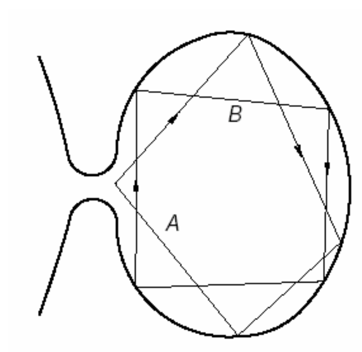
Inelastic

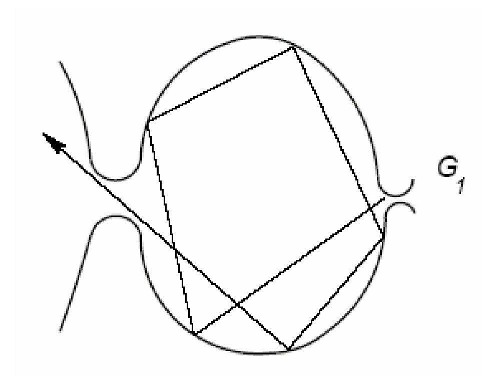
Quantum Dots with many electrons





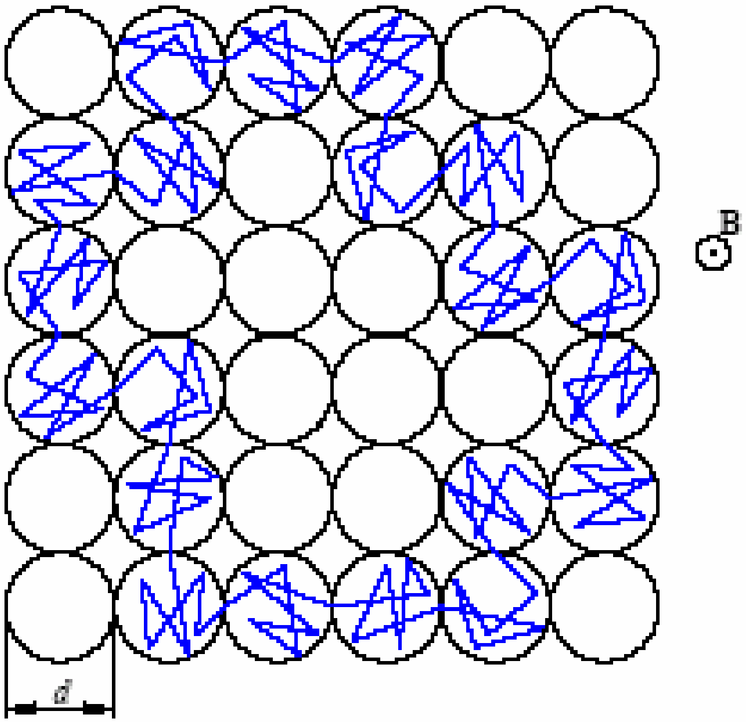


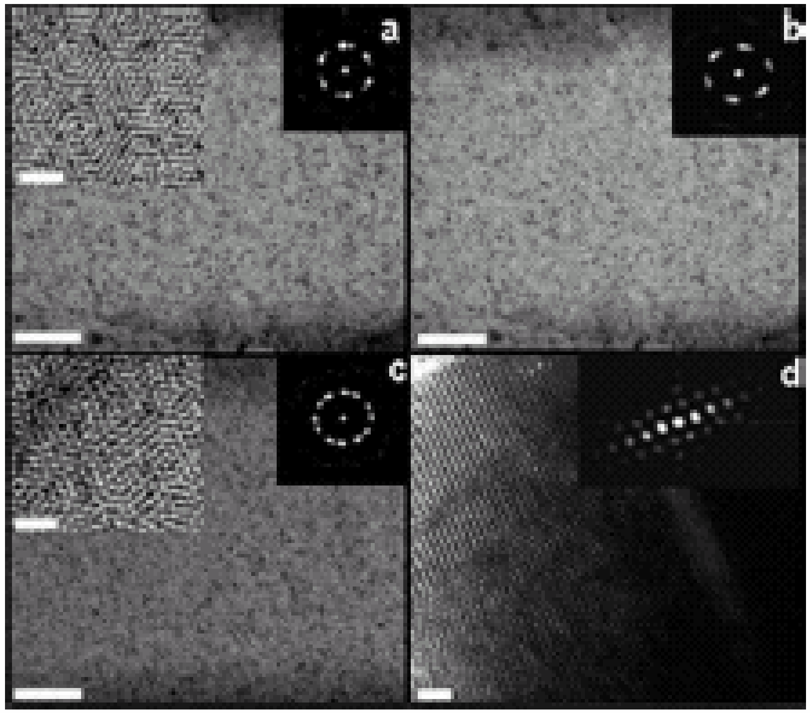




Spin blockade or Spin anti-blockade?

From Quantum Dots to Nano-Crystals



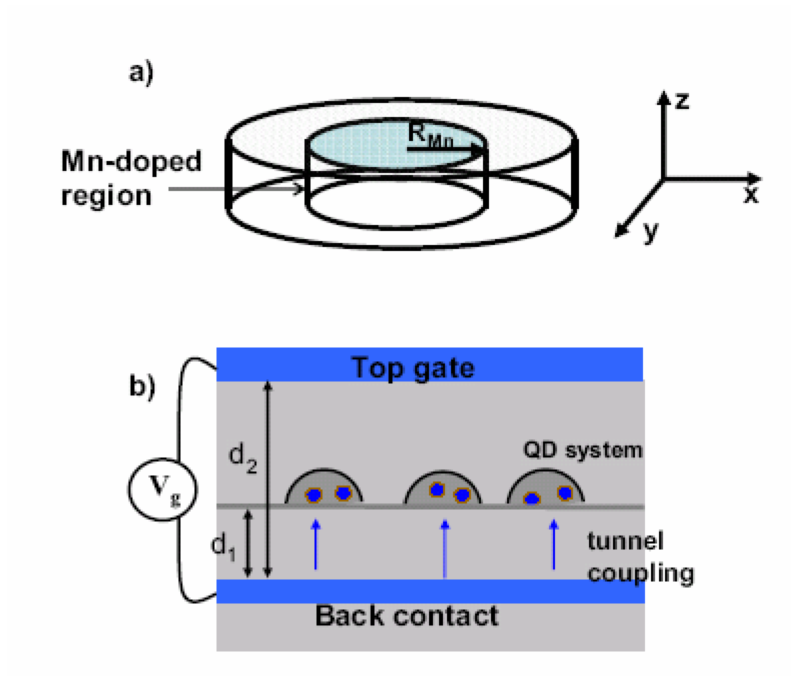


$d \approx 6nm$

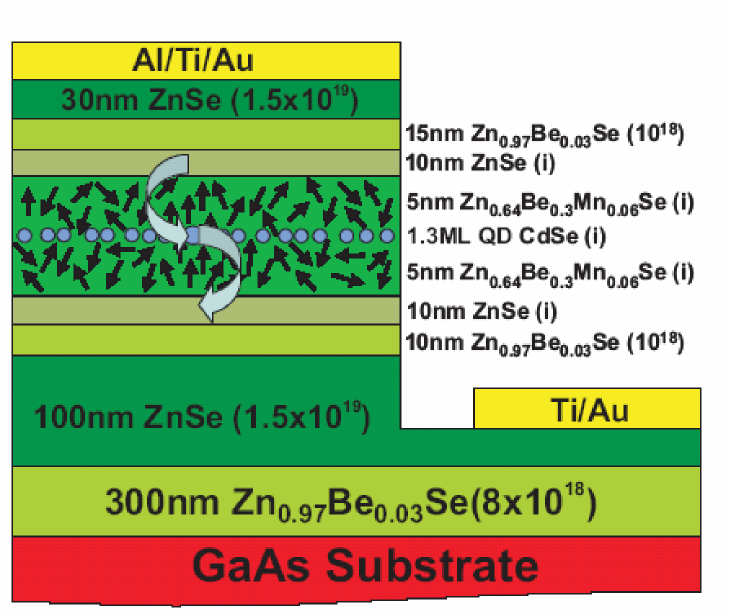
Material: Au in Si_3N_4 substrate

T.B.Tran et al (2005)

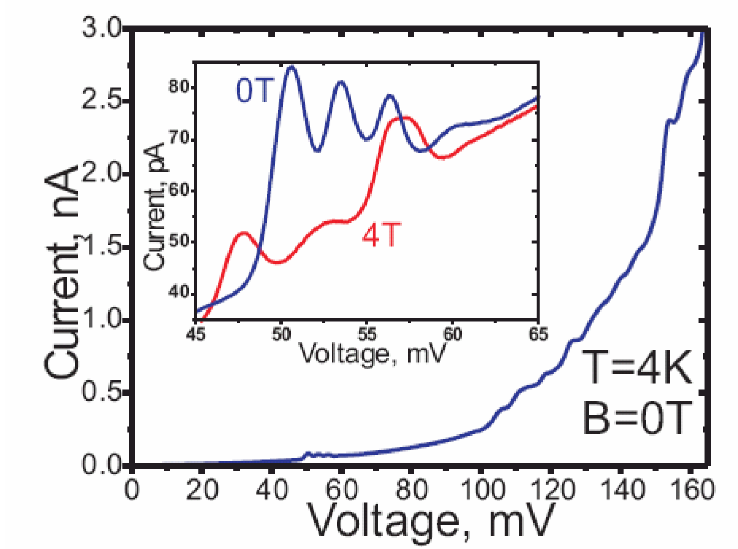
Self-assembled quantum dots

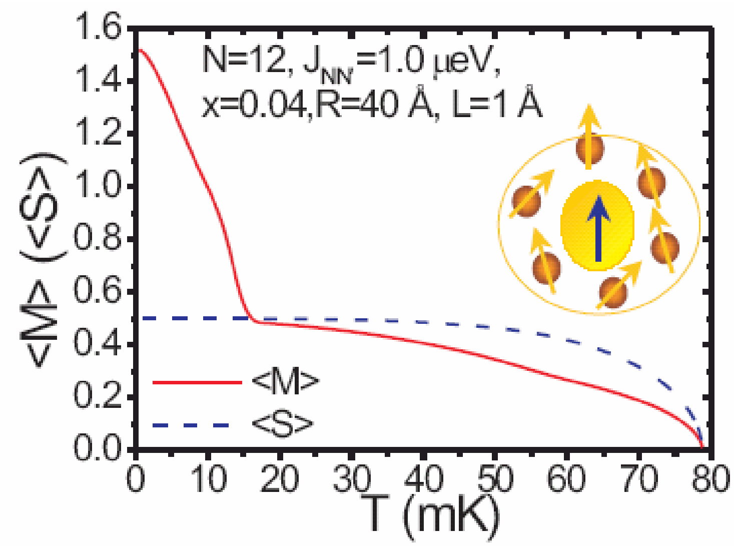


Govorov (2005)



Molenkamp et al (2005)





Magnetic correlations in Quantum Dots

Kondo effect in Quantum Dots

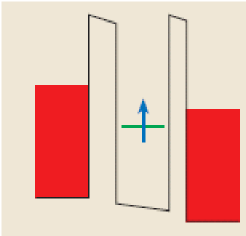
Magnetic correlations between dots

Magnetic instability in isolated dot

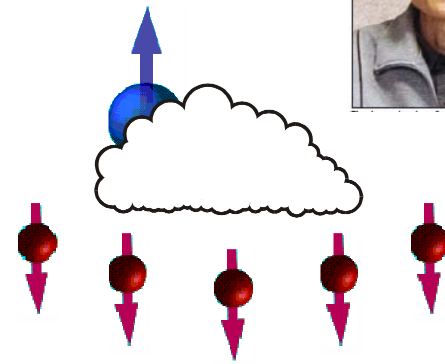
Magnetism in quantum dot arrays

Quantum criticality in granular media

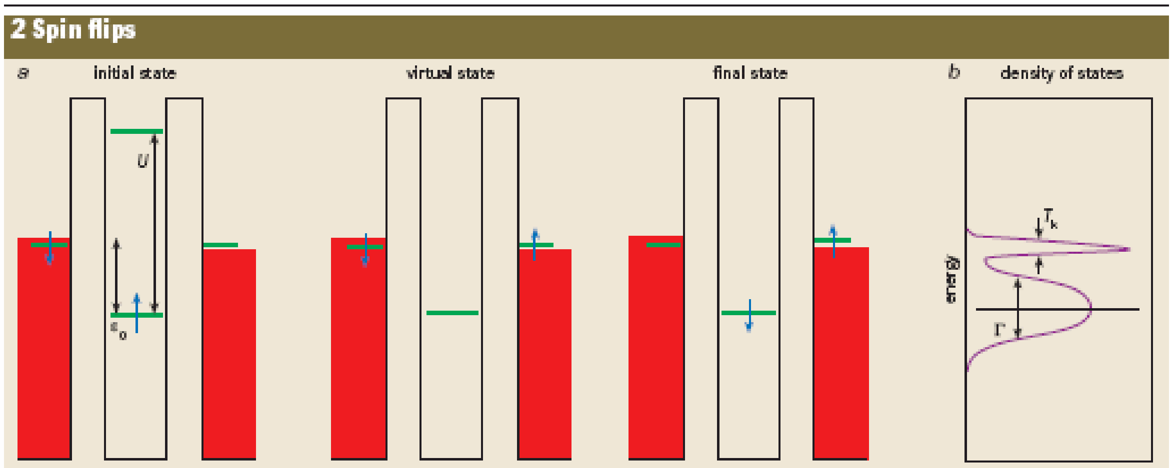
From Quantum dots to Quantum Spin Chains



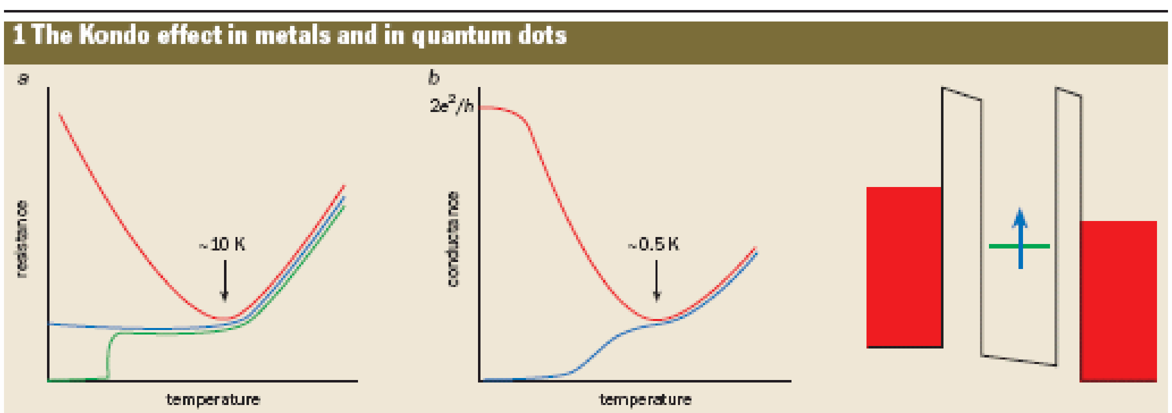
Kondo Effect in Quantum Dots

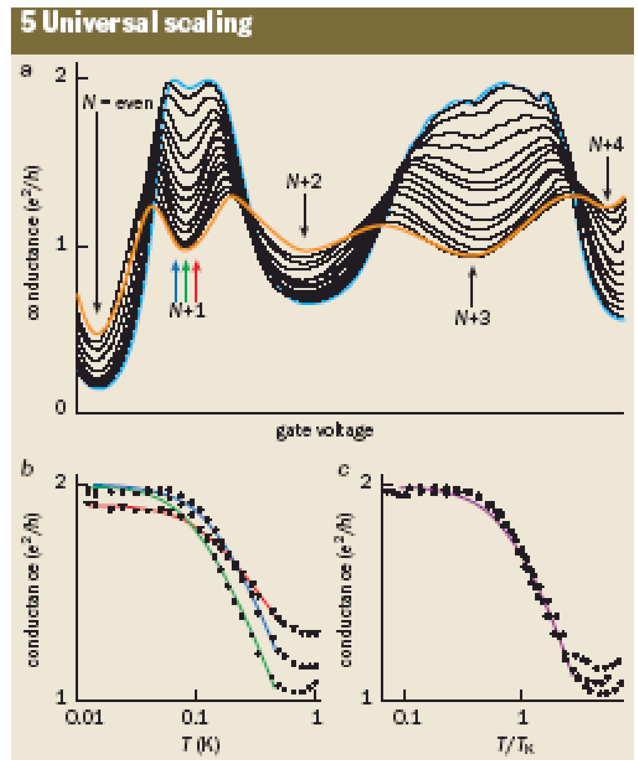


Kondo-cloud

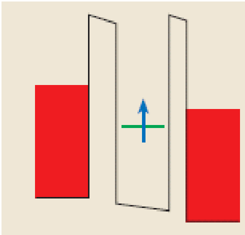


$$G/G_0 \propto \ln^{-2}\left(T/T_K\right)$$

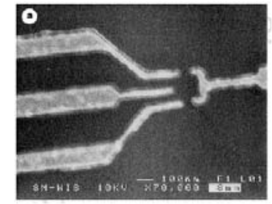


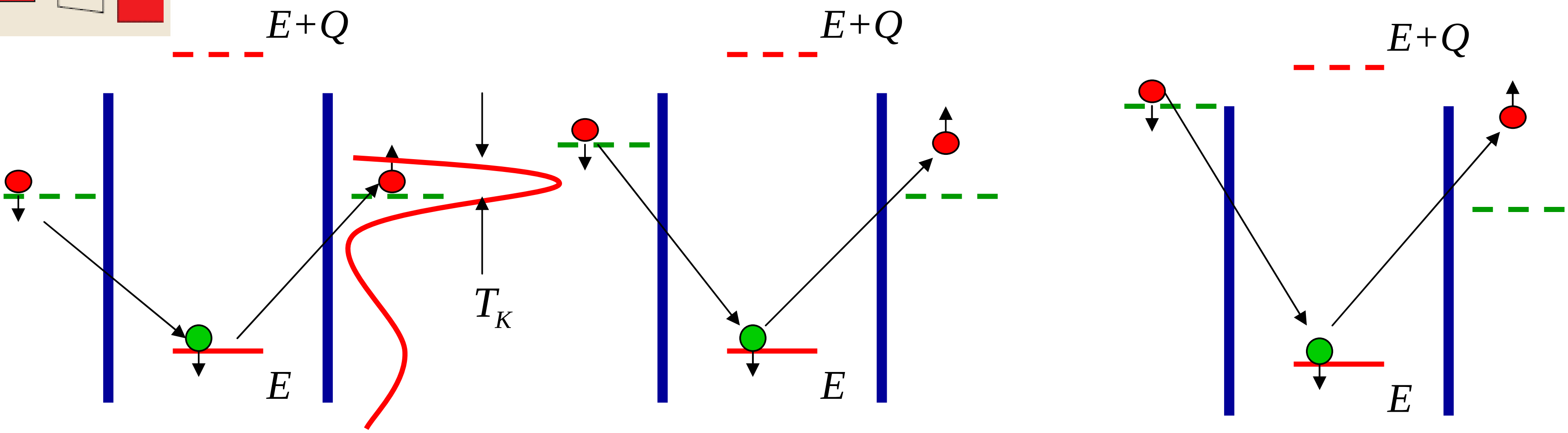


L.Kouwenhoven and L.Glazman (2001)



Non-equilibrium Kondo effect



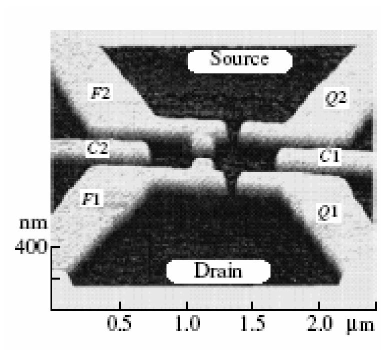


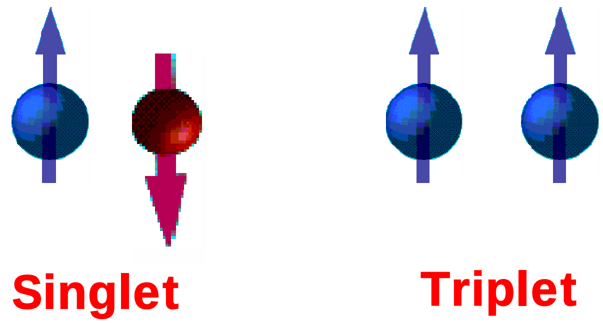
Zero-bias (equilibrium)	Small bias (quasi-equilibrium)	Large bias (out of equilibrium)
T_K	$eV \ll T_K$	$eV \gg T_K$

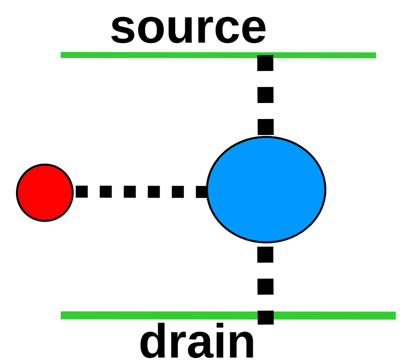
Effects of decoherence $\Gamma_{rel} \sim eV$ $\Gamma_{rel} \sim eV / \ln^2 \left(eV / T_K \right)$

There is no strong coupling (Kondo) regime at low T in out of equilibrium

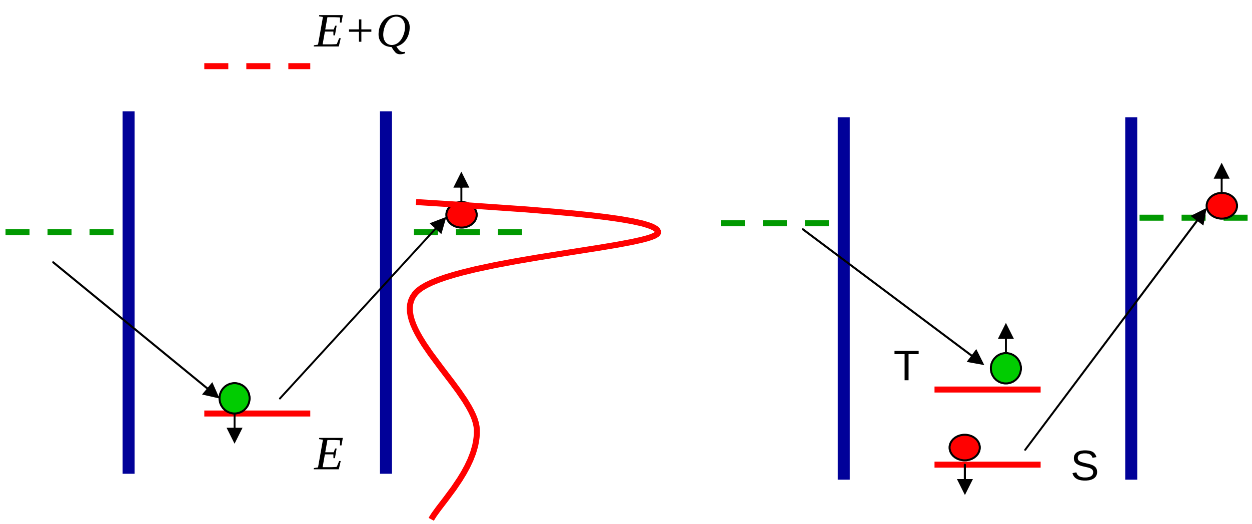
From Single Quantum Dot to Double Quantum Dot

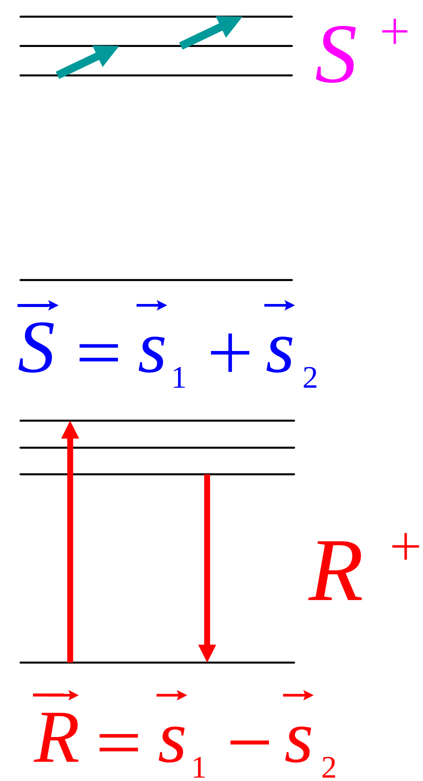






- Kondo co-tunneling through QD: N=1
- Kondo co-tunneling through DQD: N=2





Kondo Hamiltonian

$H = J\,(\mathbf{S}\ \mathbf{s})$

$S=1/2$

Generalized Kondo Hamiltonian

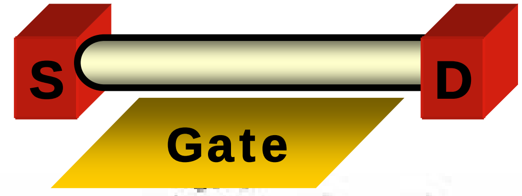
$H = J_1\,(\mathbf{S}\ \mathbf{s}) + J_2\,(\mathbf{R}\ \mathbf{s})$

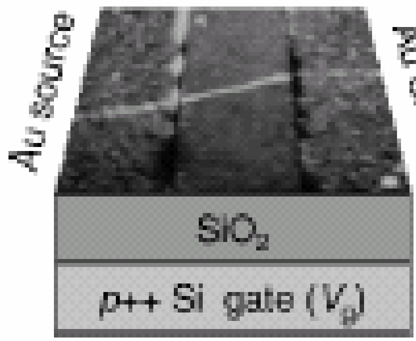
$S=1$ (triplet) plus $S=0$ (singlet)

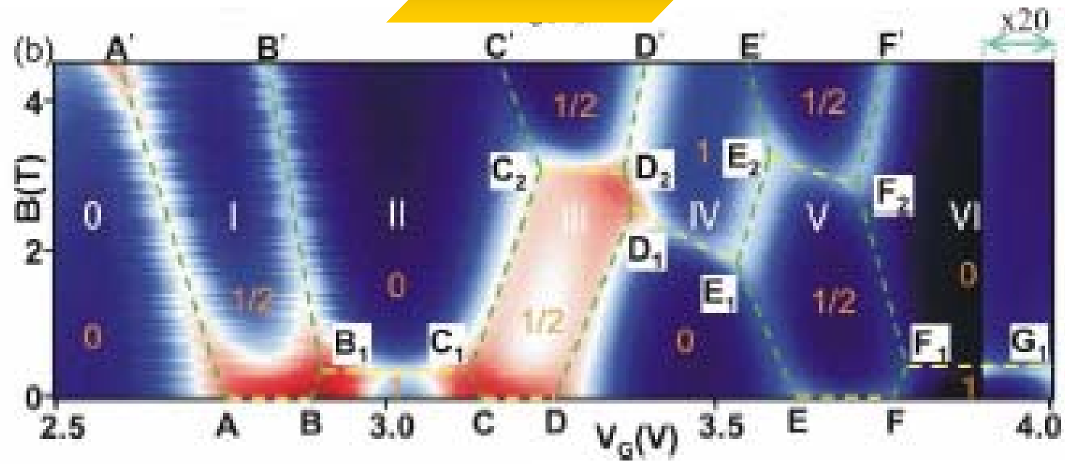
Non-universal Kondo temperature

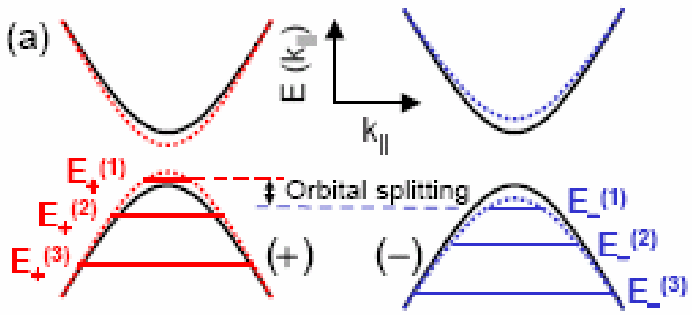
$$\Delta_{TS} \sim T_K(\Delta_{TS})$$

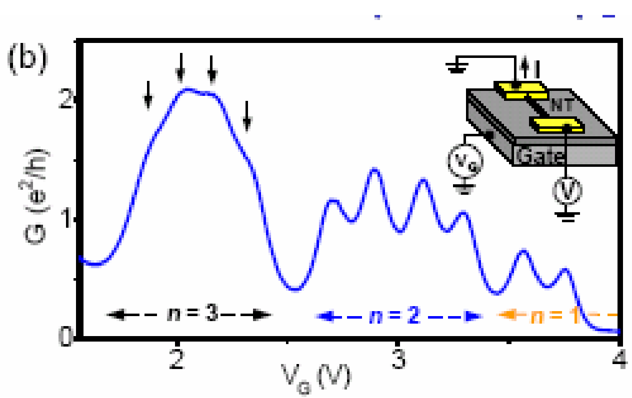
Experiment in carbon nanotubes

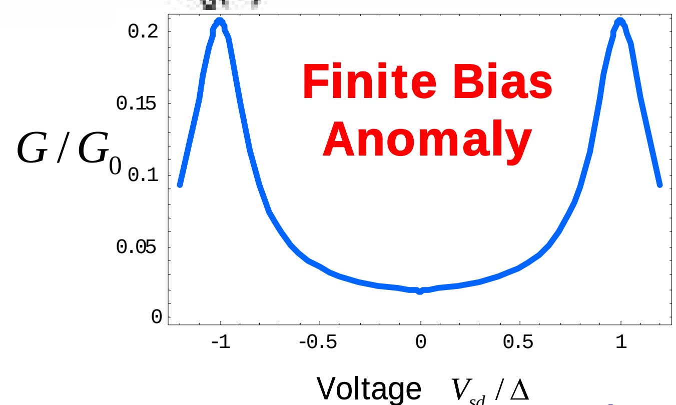




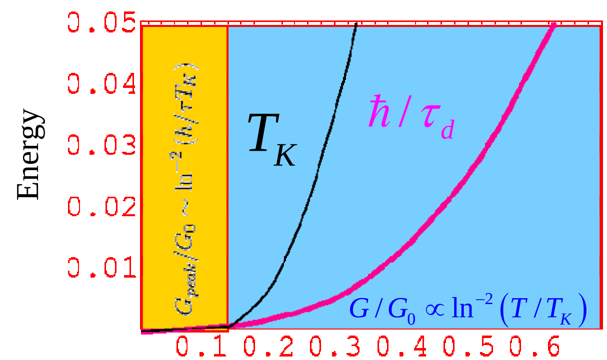








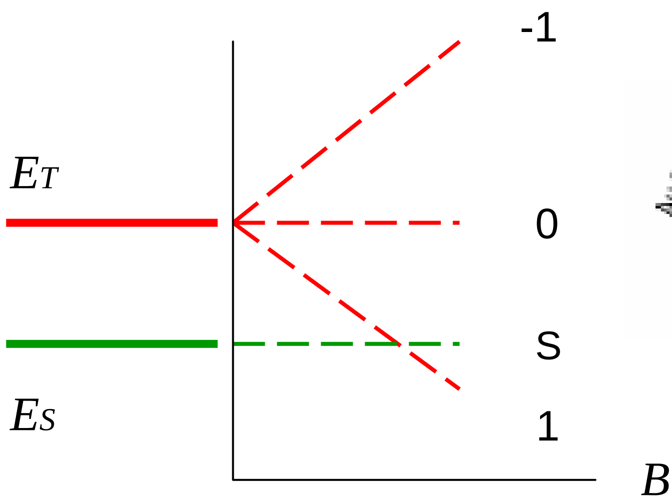
MNK (2004)

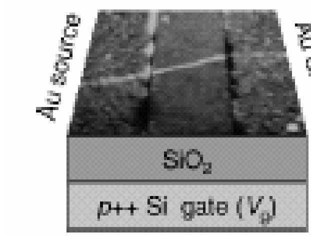


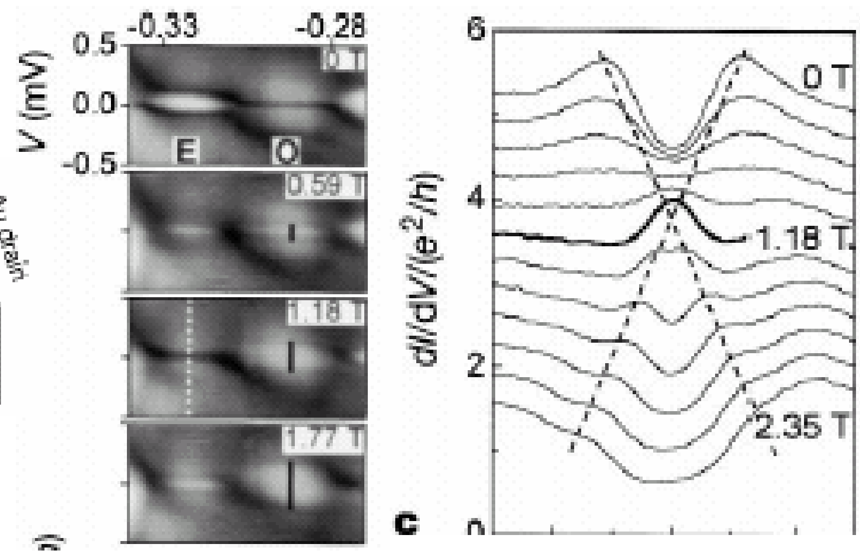
P.Jarillo-Herrero, cond-mat/0411440

S/T transition: Magnetic field induced Kondo effect

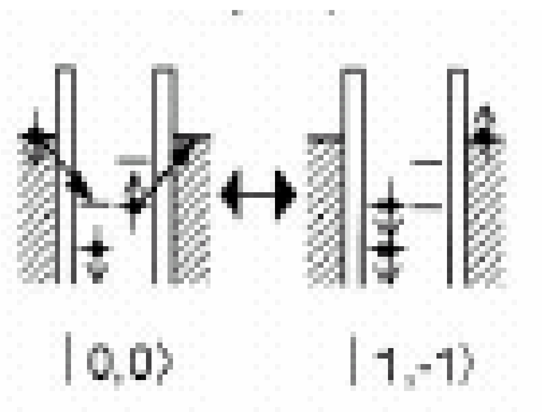
Symmetry reduction from SO(4) to SU(2)







$$H_{Kondo} = J\left(\vec{R}\bullet\vec{s}\right)$$



Kondo effect due to dynamical symmetry of DQD

M. Pustilnik, Y. Avishai & K.Kikoin (2000)

D. Kobden et al (2000)

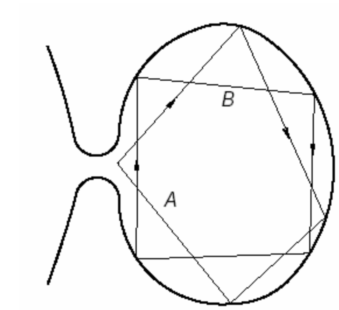
Messages

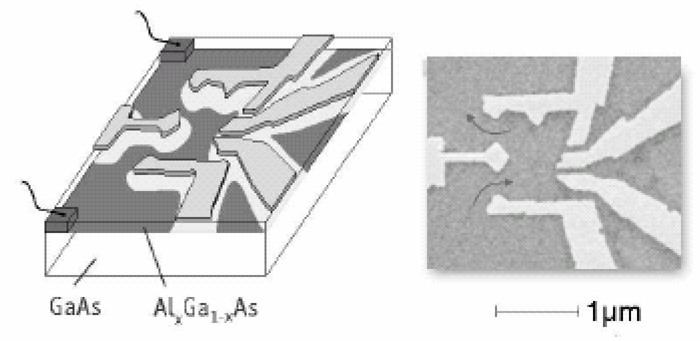
Kondo tunneling in QD with few electrons is more rich effect than Kondo scattering in metals

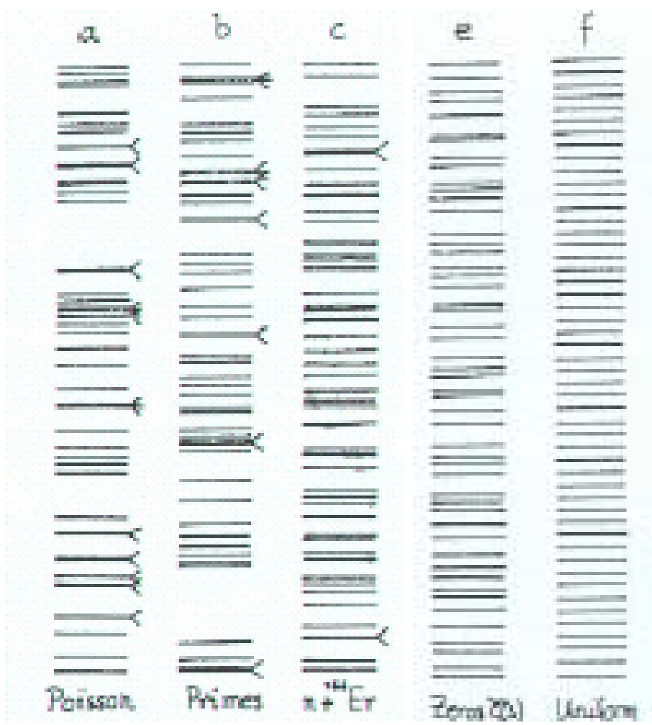
The effects of dynamical symmetries are directly observable in transport experiments

The magnetic (RKKY) correlations between dots is a controllable parameter

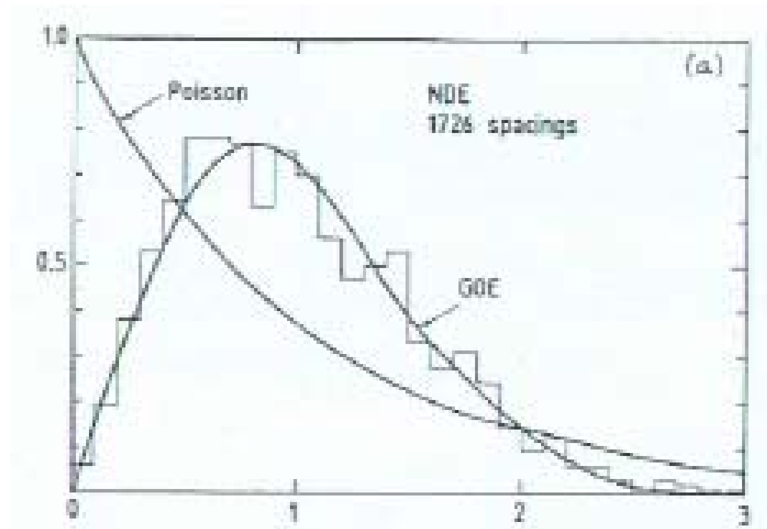
Quantum Dots as artificial atoms







GOE GUE



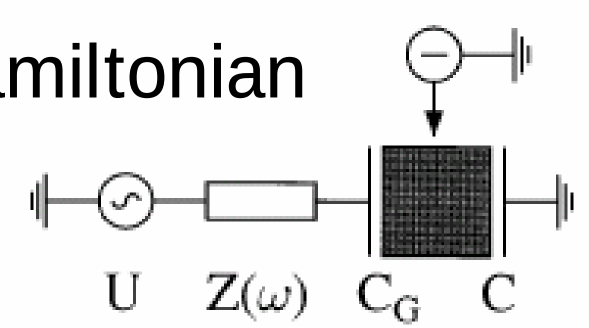
$$P\left(\left\{E_\mu\right\}\right) \propto \exp\left(\frac{\beta}{2}\sum_{\nu\neq\mu}\ln\left[\frac{|E_\mu - E_\nu|}{\delta}\right]\right)$$

$\beta = 1$ Orthogonal (GOE)

$\beta = 2$ Unitary (GUE)

$\beta = 4$ Simplectic (GSE)

Metallic Quantum Dot: Universal Hamiltonian



Zero-mode interaction

Electron-electron interactions in isolated metallic grains

Mean-level spacing $\Delta = \langle E_{\alpha+1} - E_\alpha \rangle$ (kinetic energy)

Thouless energy $E_T \sim D \cdot L^{-2}$ diffusive regime

$E_T \sim v_F L^{-1}$ ballistic regime

$E_{\alpha+1}$

E_α

$g = E_T / \Delta \gg 1$ **metallic grain**

GUE

$$H_{\mathrm{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2 - \lambda_{BCS} \hat{T}^+ \hat{T}$$

charge spin superconducting

$$E_c = \frac{e^2}{2C}$$

Coulomb blockade

Scaling:

Short-range interaction $E_c \sim |J| \sim \Delta$

Coulomb interaction $E_c = r_s (k_F L)^{d-1} \Delta \gg |J|$

Kurland, Aleiner, Altshuler (2000)
Aleiner, Brouwer, Glazman (2002)

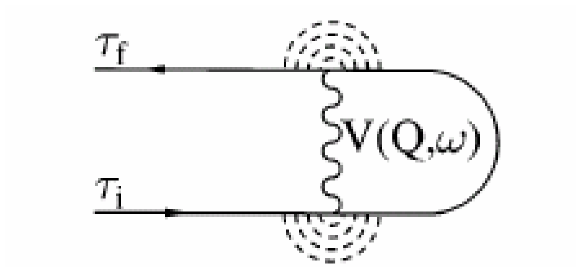
What is a zero-mode interaction?

Electron-electron interaction

$$H_{\text{int}} = \frac{1}{2}\sum_{Q} V(Q)\rho(Q)\rho(-Q)$$

TDoS

$$\delta\nu(\epsilon) = -\frac{\nu_0}{\pi} T \,\mathrm{Im} \sum_{\omega_n > \epsilon_m} \sum_{\mathbf{Q}} \frac{2\pi i V(\mathbf{Q},\omega_n)}{(D\mathbf{Q}^2 + |\omega_n| + \gamma_{in})^2}\Big|_{i\epsilon_m \to \epsilon + i\delta}$$



$$V(\mathbf{Q},\omega_n) = \frac{V_0(\mathbf{Q})}{1 + V_0(\mathbf{Q})\Pi(\mathbf{Q},\omega_n)}$$

Bare Coulomb Interaction

Screened Coulomb interaction

Polarization Operator

$$\vec{Q} = \frac{2\pi}{L}\vec{n}$$

$$\Pi(\mathbf{Q},\omega_n) = \nu_0 \frac{D\mathbf{Q}^2}{D\mathbf{Q}^2 + |\omega_n|}$$

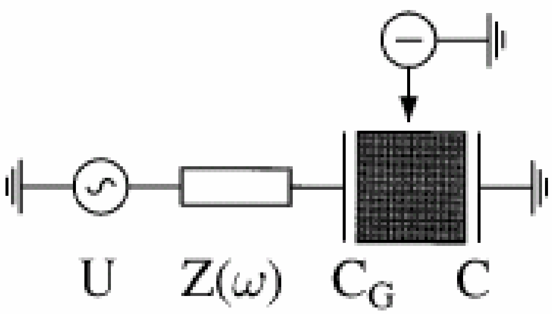
Q=0 contribution

$$H_{\text{int}} = \frac{1}{2} V(0)\left[\hat{n} - N\right]^2 \qquad V(0) = \frac{e^2}{C}$$

Zero-mode interaction requires a non-perturbative treatment at low temperatures!

Nazarov (1989)
Levitov, Shitov (1996)
Kamenev, Gefen (1996)

Zero-bias anomaly in zero-dimensional systems



$$H_{\text{int}} = E_c(\hat{n} - N)^2$$

Hubbard Interaction

“Orthodox” theory of the Coulomb Blockade

ZBA

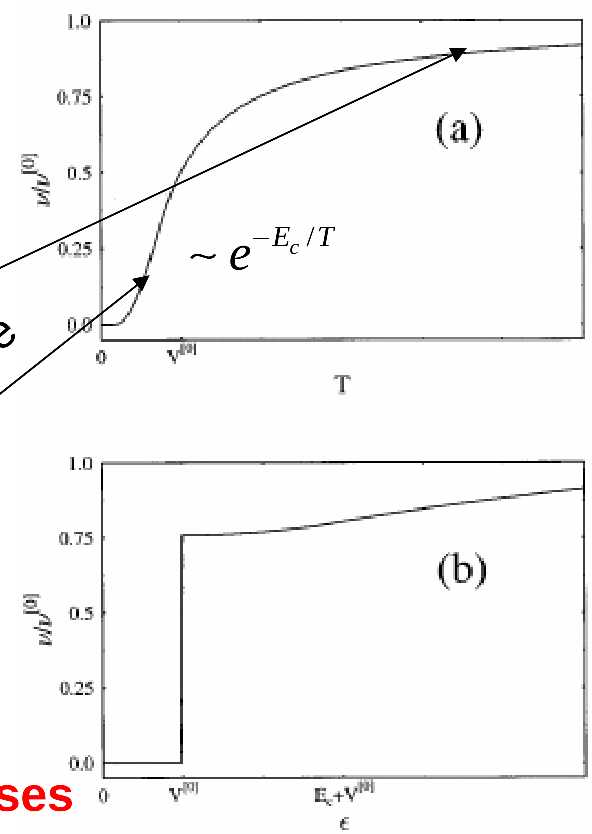
R.I.Shekhter (1974)
Ben-Jacob, Gefen (1985)
Mullen, Gefen,Ben-Jacob (1988)
Averin, Likharev (1991)

perturbative

non-perturbative

$$\nu(\epsilon)/\nu^{[0]}(\epsilon) = 1 - \frac{V}{4T}\text{sech}^2\left(\frac{\epsilon}{2T}\right)$$

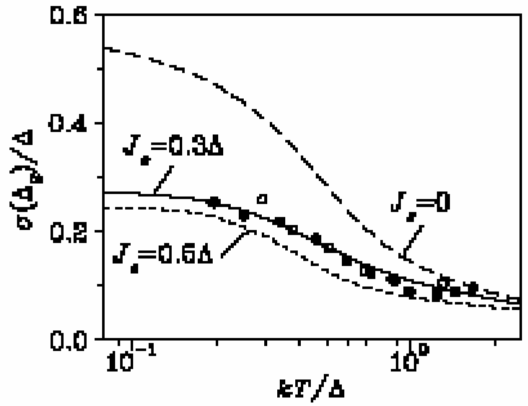
$$\nu(\epsilon)/\nu^{[0]}(0) = \cosh\left(\frac{\epsilon}{T}\right)\exp\left(-\frac{E_c}{T}\right)$$



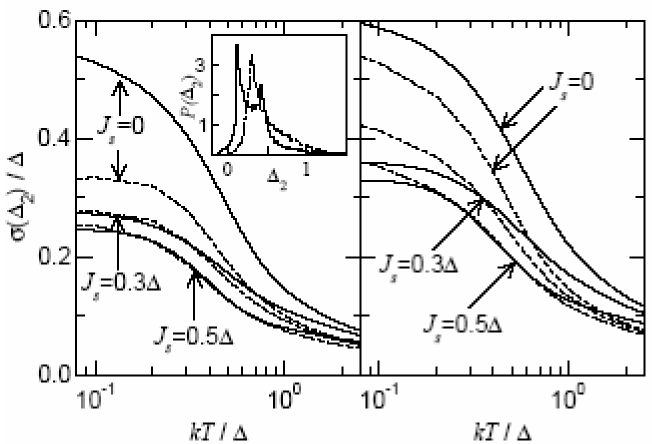
ZBA and Coulomb blockade are two limiting cases of the same theory

Spin Exchange. Master Equation (classical) Approach

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2$$

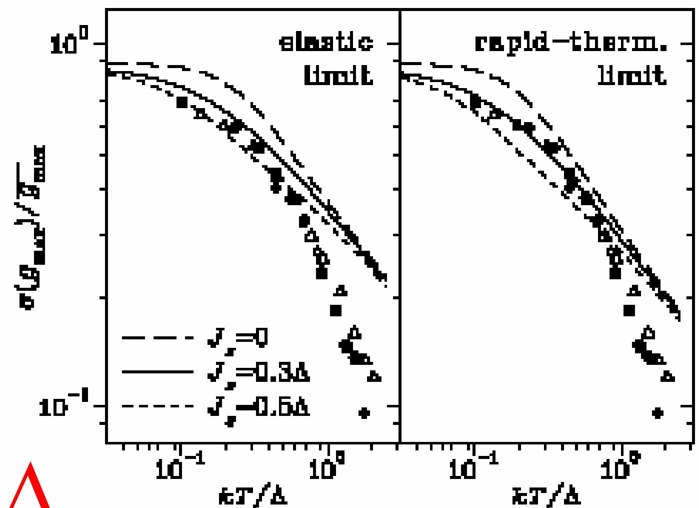


The width of peak-spacing distribution

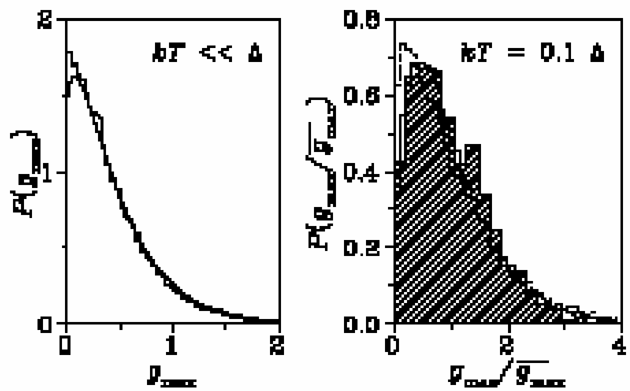


The width of peak-spacing distribution in the presence and absence of the orbital magnetic field.

$$J \geq 0.5\Delta$$



The ratio between standard deviation and the average value of peak height



Peak-height distributions

Y.Alhassid and T.Rupp (2003, 2004)

Charge and Spin Interactions

Hamiltonian

$$H = \sum_{\alpha,\sigma} \varepsilon_\alpha \Psi^\dagger_{\alpha,\sigma} \Psi_{\alpha,\sigma} + H_C + H_S$$

Charge

$$H_C = E_c(\hat{n} - N)^2$$

$$\hat{n} = \sum_{\alpha,\sigma} \Psi^+_{\alpha\sigma} \Psi_{\alpha\sigma}$$

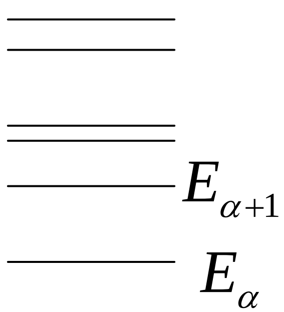
Commutative algebra

Spin

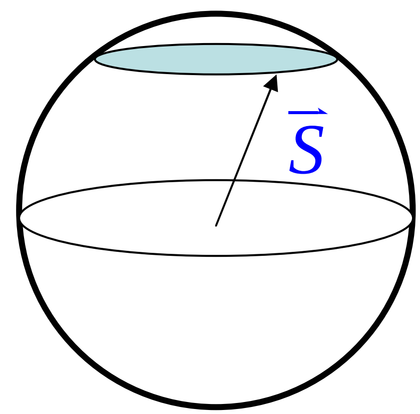
$$H_S = -J(\vec{S})^2$$

$$\hat{\vec{S}} = \sum_{\alpha,\sigma,\sigma'} \Psi^+_{\alpha\sigma} \vec{\sigma}_{\sigma\sigma'} \Psi_{\alpha\sigma'}$$

$$\left[S_j, S_k\right] = i\varepsilon_{jkl} S_l$$

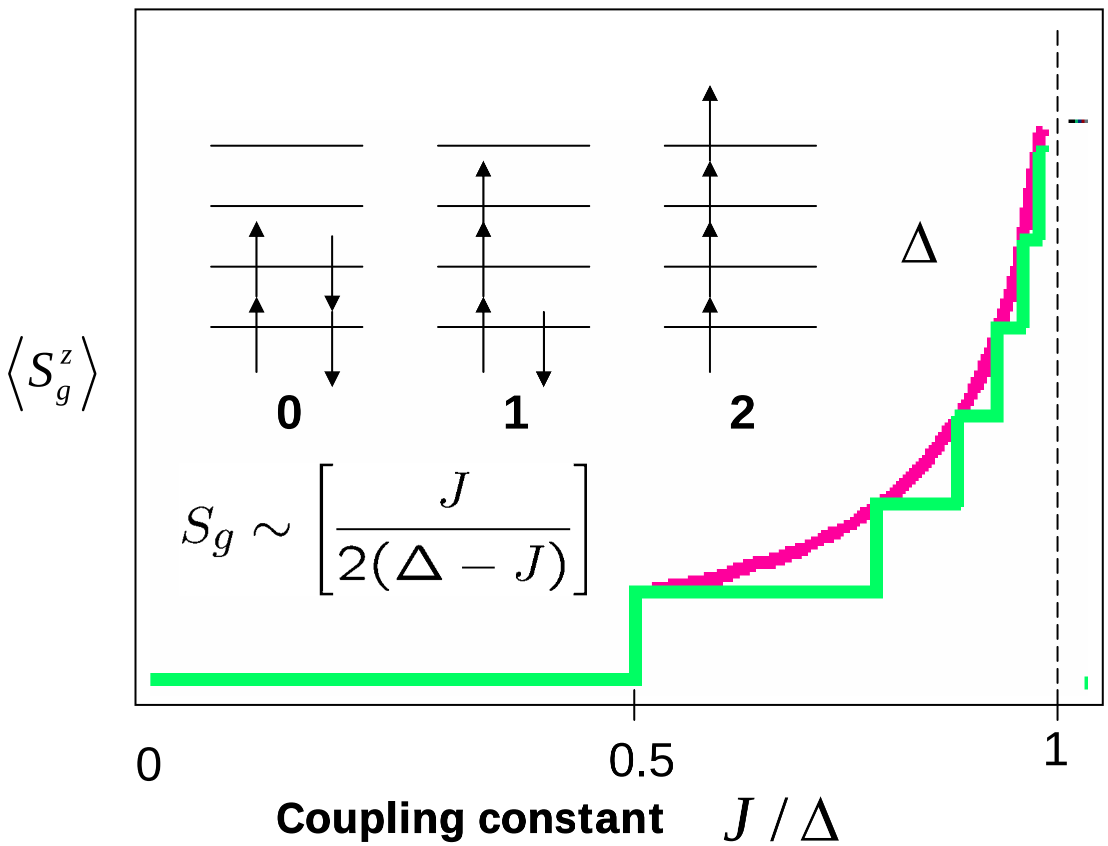


Non-commutative algebra



Mesoscopic Stoner Instability

$$H_{\rm int} = E_c(\hat{n} - N)^2 - J\left[(S^z)^2 + \varepsilon\left\{(S^x)^2 + (S^y)^2\right\}\right]$$



Isotropic exchange $\varepsilon = 1$

$$J_c^{even} = \Delta/2$$

$$J_c^{odd} = 2\Delta/3$$

Ising anisotropy $\varepsilon = 0$

$$J_c^{even} = J_c^{odd} = \Delta$$

Easy axis anisotropy

$$\varepsilon = J_\perp / J_\parallel < 1$$

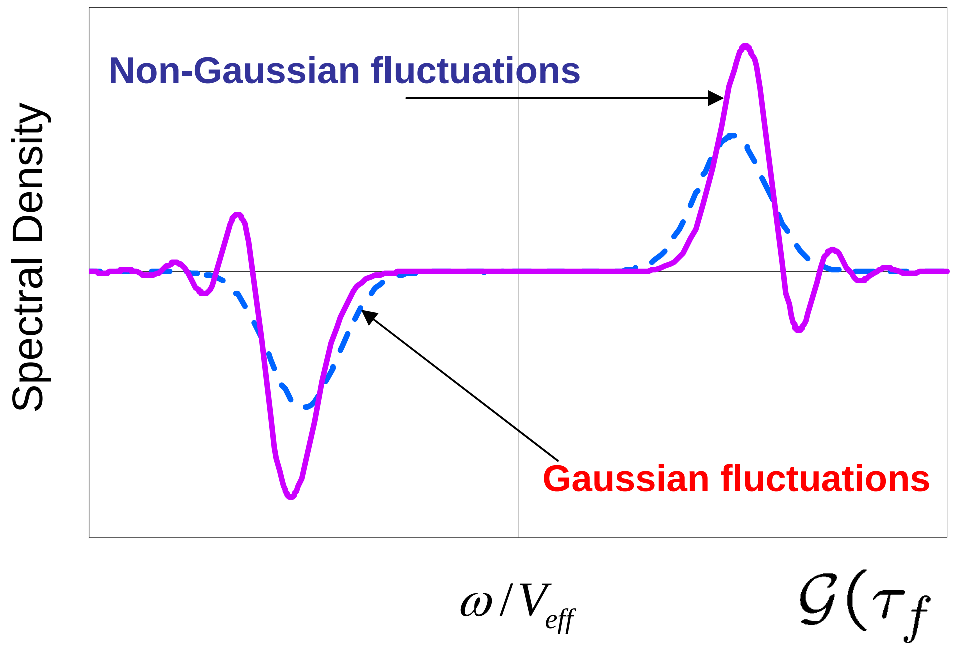
$$J_c^{even} = \Delta/(1+\epsilon)$$

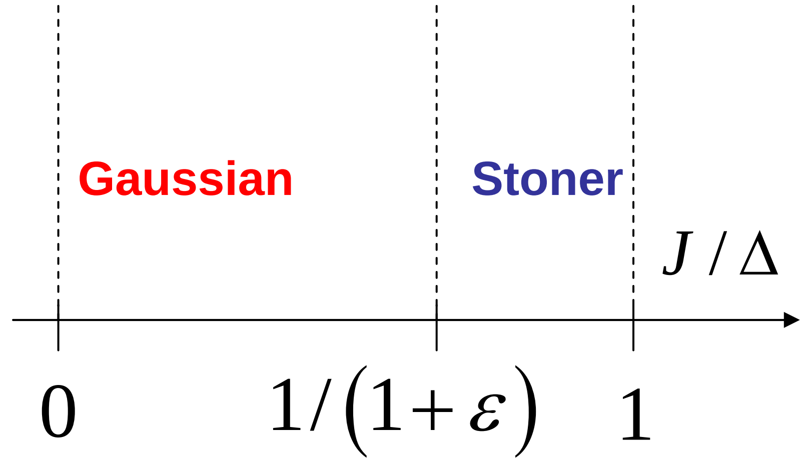
$$J_c^{odd} = \Delta/(1+\epsilon/2)$$

Kurland, Aleiner, Altshuler (2000)

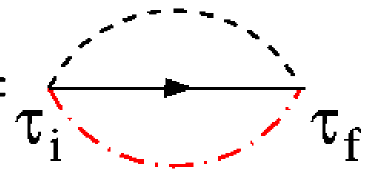
Kiselev, Gefen (2005)

Tunneling Density of States



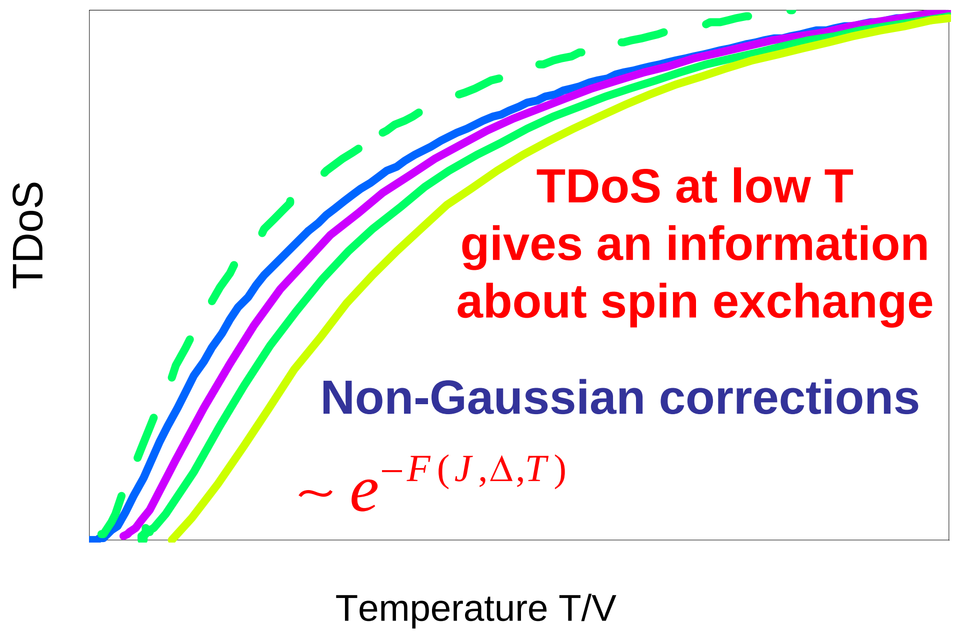


$$\mathcal{G}(\tau_f - \tau_i) =$$



Zero-mode interaction

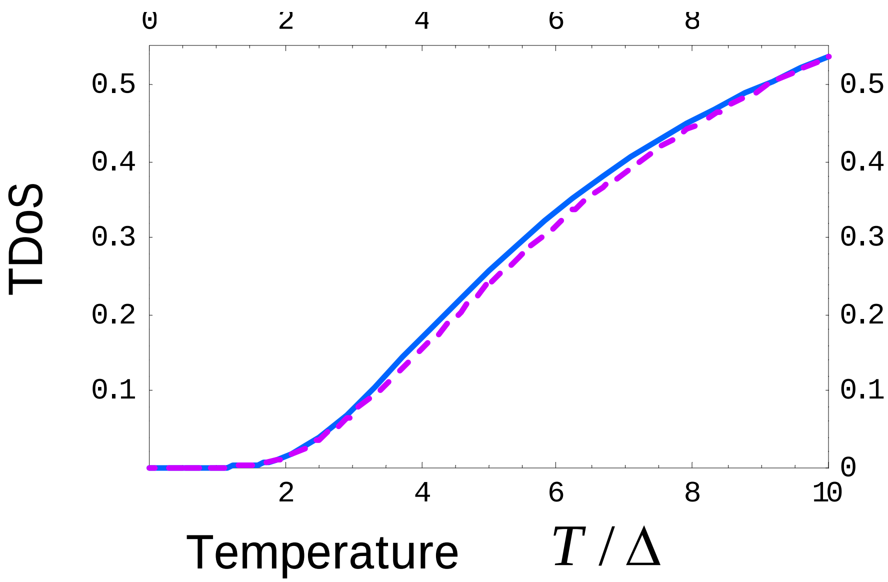
$$\nu(\epsilon) = -\frac{1}{\pi} \cosh\left(\frac{\epsilon}{2T}\right) \int_{-\infty}^{\infty} \mathcal{G}\left(\frac{1}{2T} + it\right) e^{i\epsilon t} dt$$

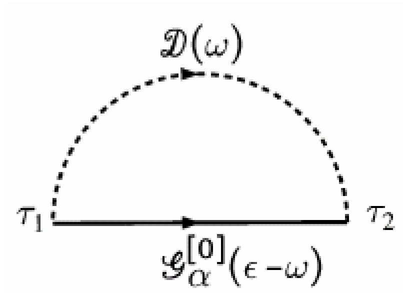


$$\nu(\epsilon) = -\frac{1}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[\tanh\left(\frac{\epsilon - \omega}{2T}\right) + \right.$$

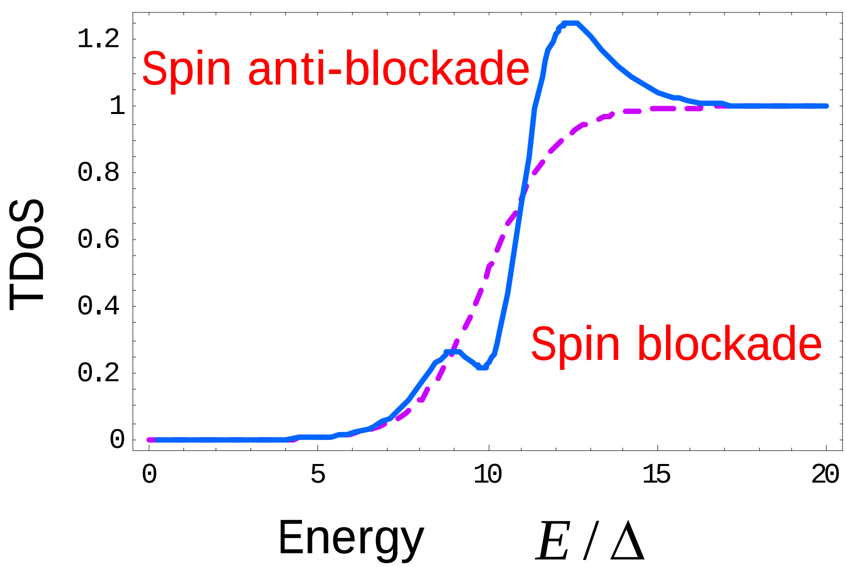
$$\left. + \coth\left(\frac{\omega}{2T}\right)\right] B_{\|}(\omega) \nu^{[0]}(\epsilon - \omega)$$

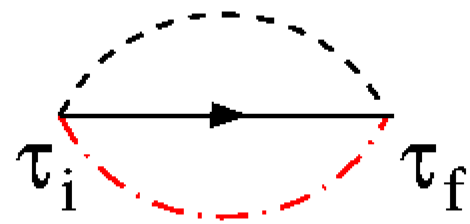
Quantum Dot Spectroscopy $T > \Delta$





Charge gauge factor





Spin gauge factor

Spin channel affects the charge transport

Kiselev, Gefen (2005)

Spin susceptibilities

Longitudinal Susceptibility

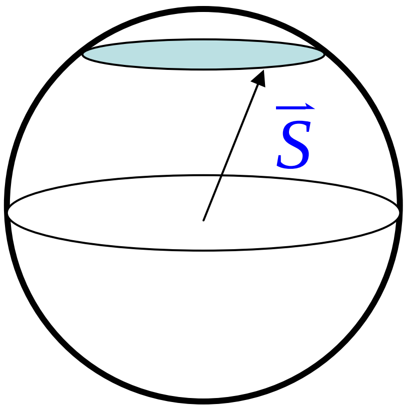
$$\chi^{zz} = \frac{\chi_0}{1 - J\chi_0}$$

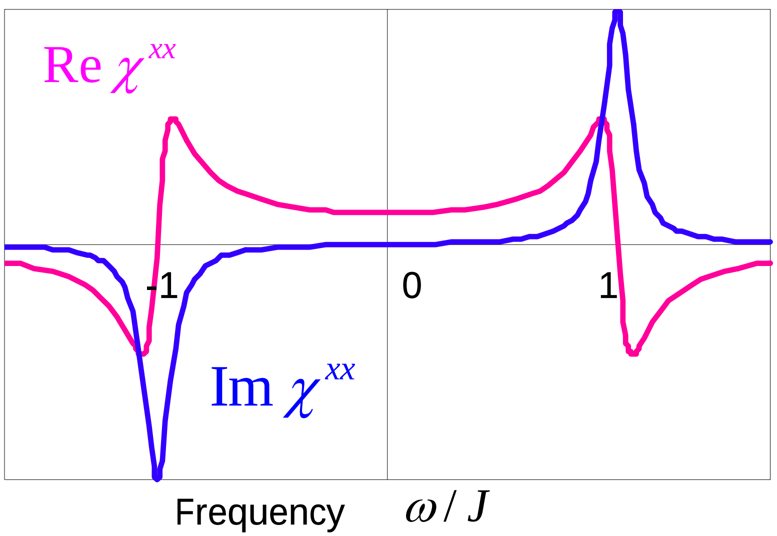
Stoner Instability

Static longitudinal susceptibility diverges at Stoner Instability point

Transverse Susceptibility

$$\partial_t S^{\pm} = iJ(1-\varepsilon)\left[1 - 2S^z\right]S^{\pm}$$





Charging energy does not affect spin correlations

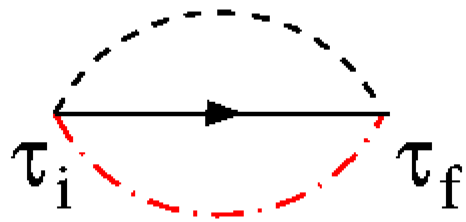
Exponentially enhanced!

$$\chi^{xx}(t) = \frac{\chi_0 \varepsilon e^{J/T}}{1 - \varepsilon J \chi_0} e^{i(1-\varepsilon)Jt}$$

Response Functions

Electron Green's Function

Add electron to the dot (charge + spin)

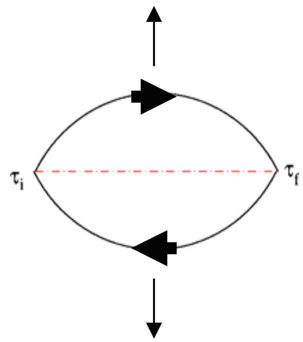


Remove electron from the dot (charge + spin)

Spin and Charge channels affect transport properties

Spin susceptibility

Charge is conserved, Spin flips

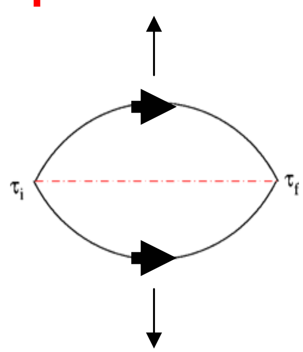


Charge is conserved, Spin flips

Only Spin channel matters

Superconducting loop

Spin is conserved, Charge 2e is transferred

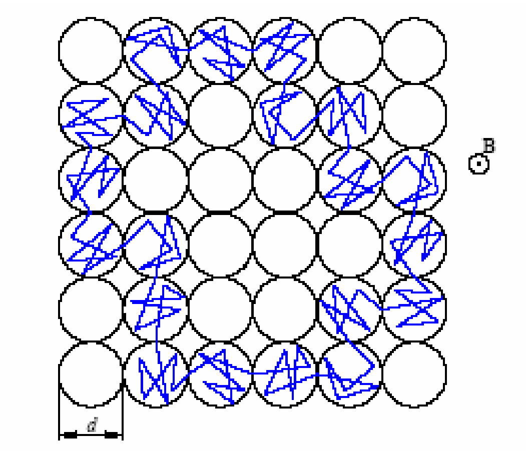


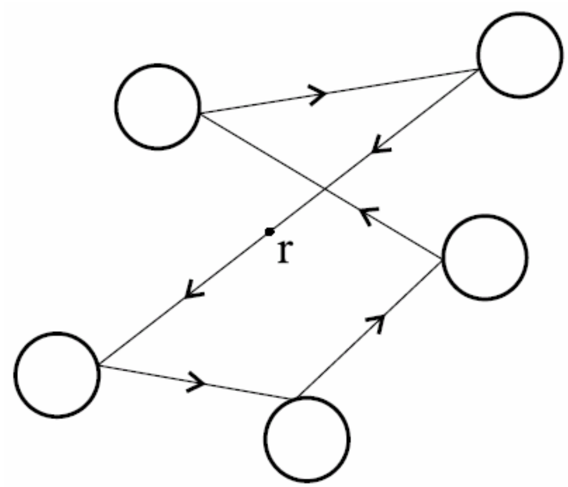
Spin is conserved, Charge 2e is transferred

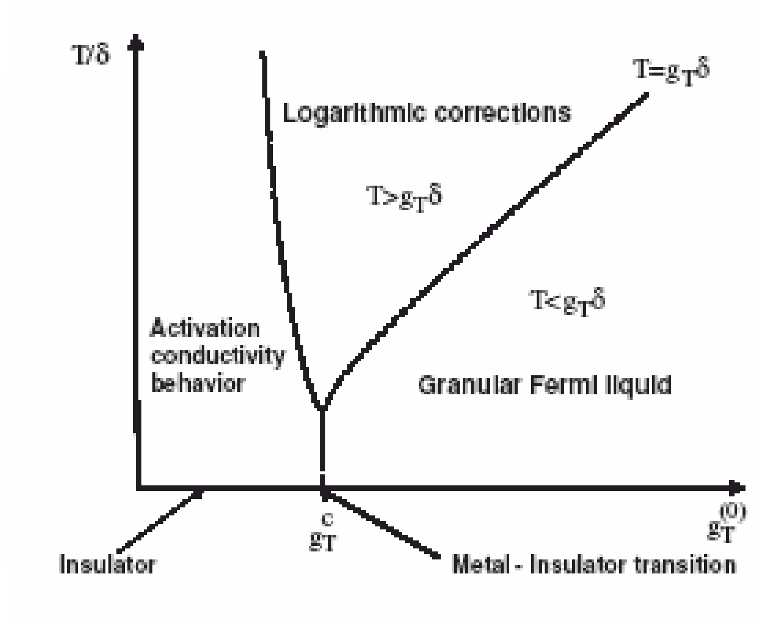
Only Charge channel matters

Magnetic instability in a system of coupled dots

2D arrays



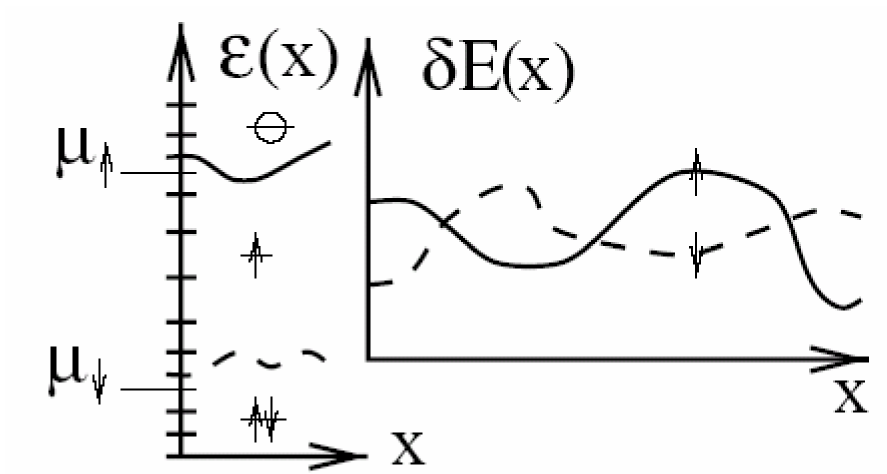




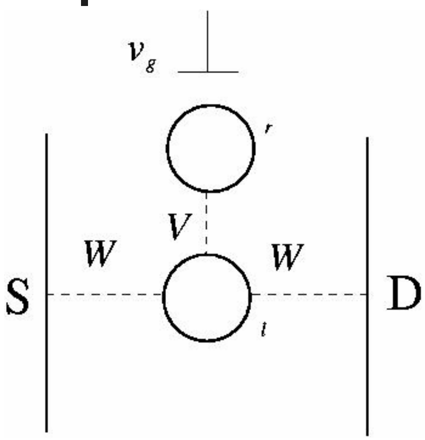
What about spin correlations?

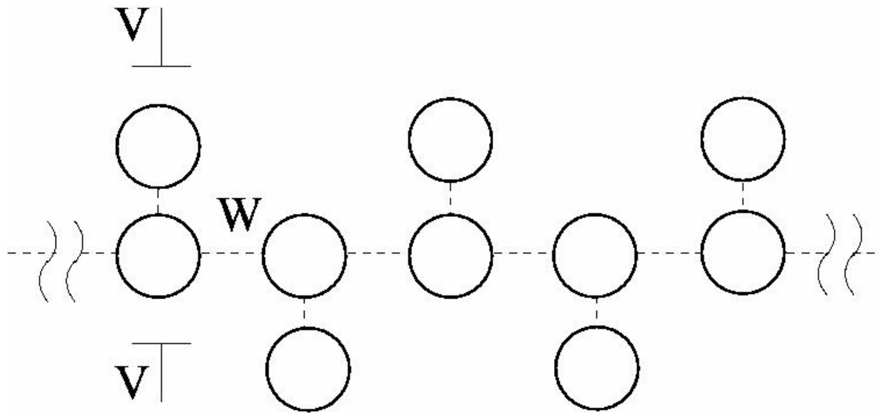
$$\chi_s(T^*) = 2\left(1 - V(k_F) - \frac{V(k_F)}{\pi^2 g}\ln\frac{\tilde{\epsilon}_F}{T^*}\right)^{-1}$$

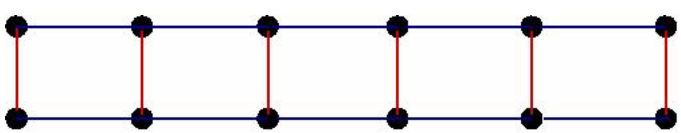
Quantum spin criticality?



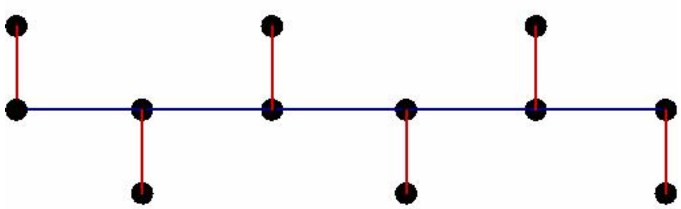
From complex dots to quantum chains



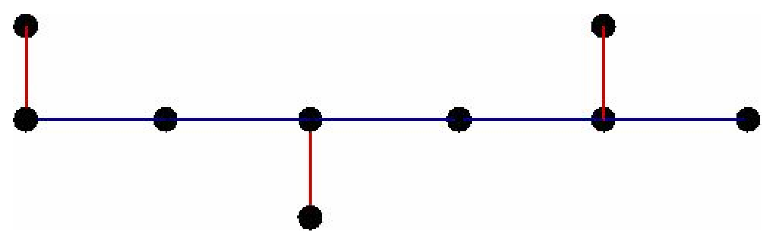




(a)



(b)

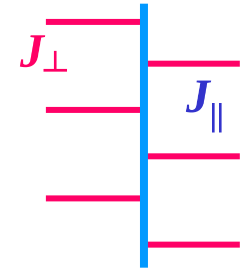


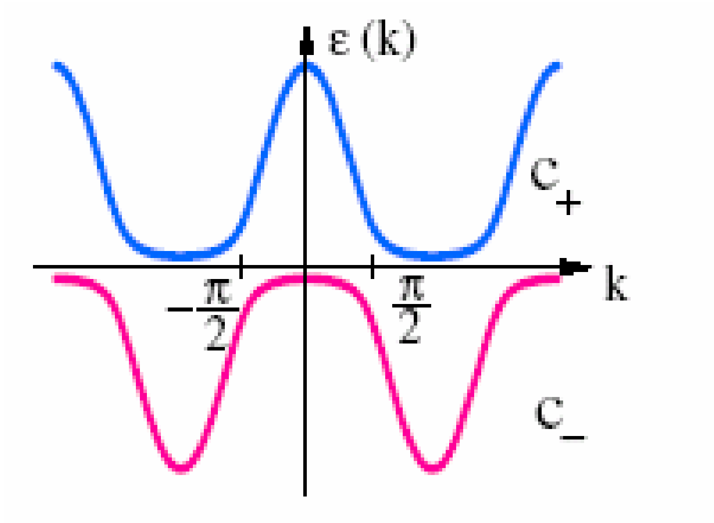
- Haldane Gap
- Charge Density Waves
- Exciton propagation

K.Kikoin, Y.Avishai and MNK, Kuwler (2004)

Haldane Gap in Spiral Staircase Model

Role of dynamical symmetries





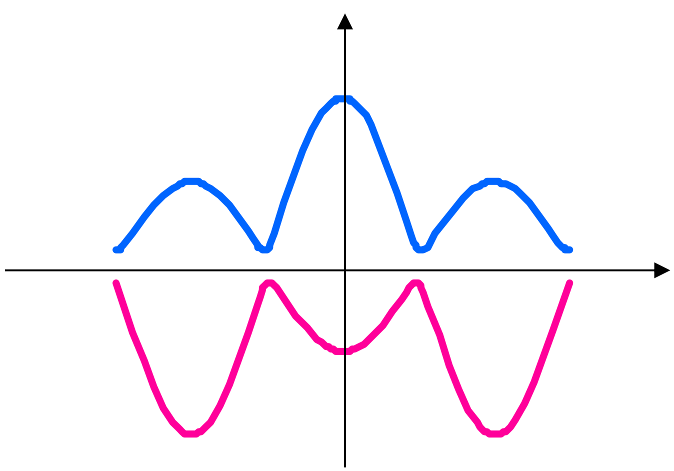
$\theta = \pi$

Two Fermi velocities in uncoupled railings

Two energy scales (gaps) in staircase model

Two stage renormalization in continuum limit

$$\Delta \sim J_{\|}(J_{\perp}/J_{\|})^{2/3}$$



$\theta = \pi / 2$

Single gap regime $\Delta \sim J_{\perp}$

One stage renormalization procedure

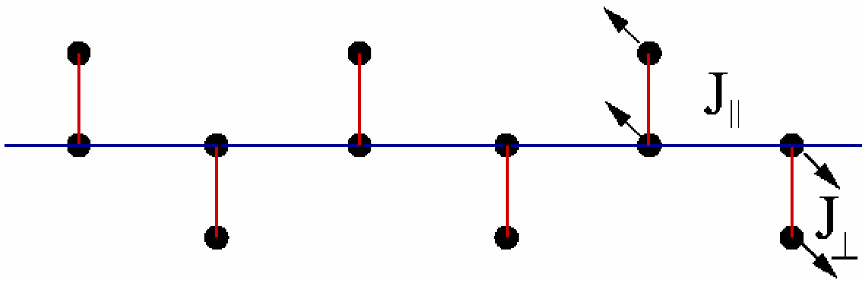
Hidden symmetries: Z_2 and $Z_2 \otimes Z_2$

MK et al (PRB 2005)

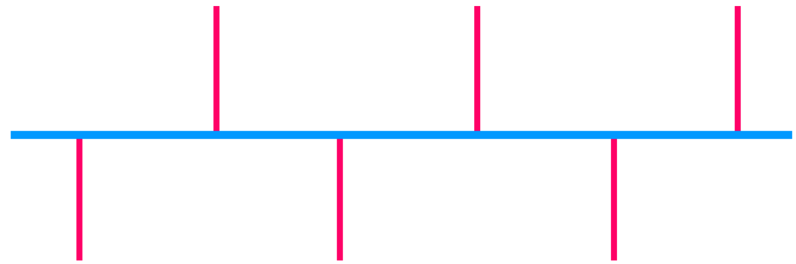
Open questions and perspectives

From “half filling” to “quarter filling”

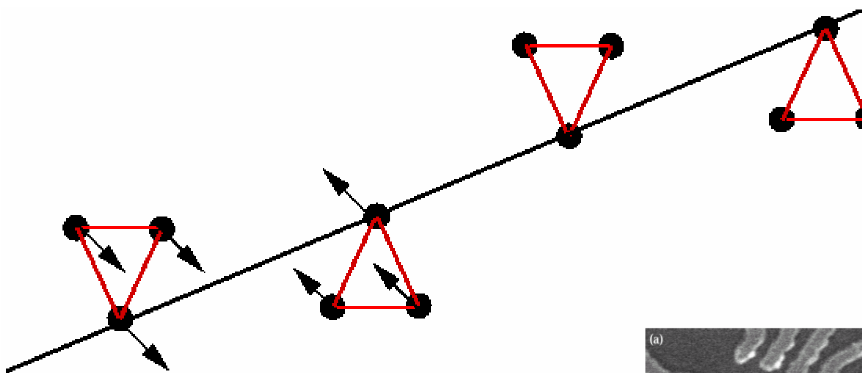
Two electrons per rung $SO(4)$

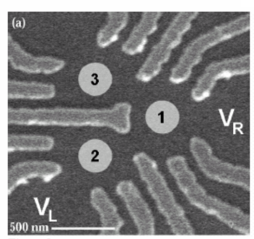


One electron per rung $SU(4)$

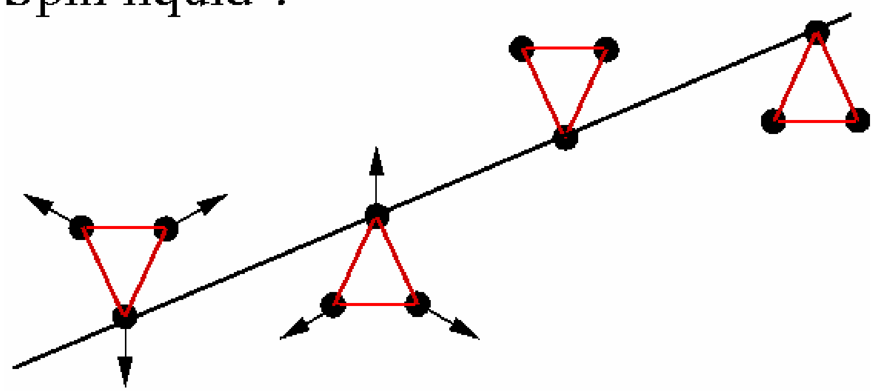


From “double dot” rung to “triple dot” rung

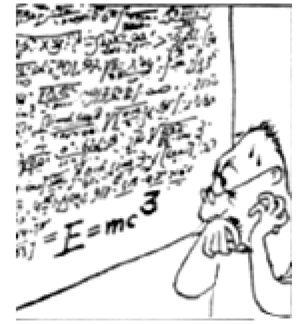




Spin liquid ?



Summary



- Complex quantum dots possess hidden symmetries responsible for several exotic transport properties of these nano-devices
- Magnetic correlations between electrons in a dot result in many interesting effects (Stoner instability, Kondo effect, Non-Fermi-Liquid behavior etc)
- Transport properties of quantum dots is an interesting object both for experimental and theoretical investigations