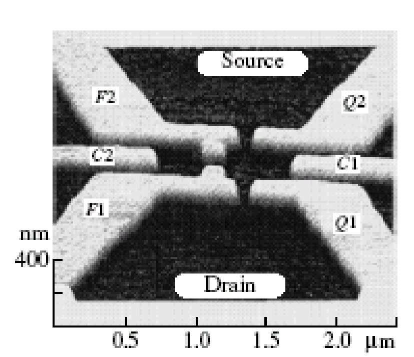
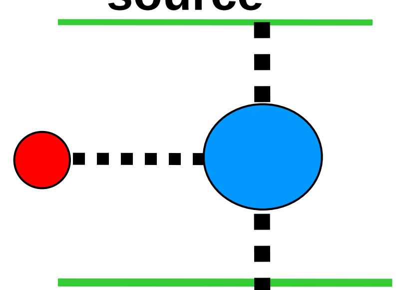
Institut für Theoretische Physik und Astrophysik

Universität Würzburg

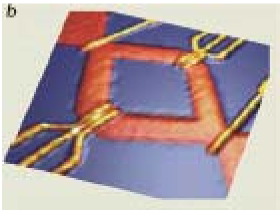
M.N.Kiselev

Dynamical symmetries in nanophysics
(Part II)





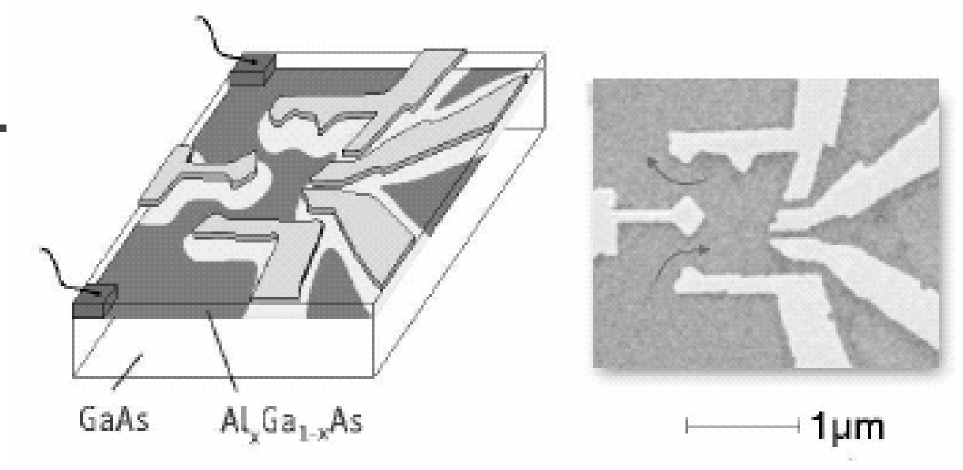
Outline

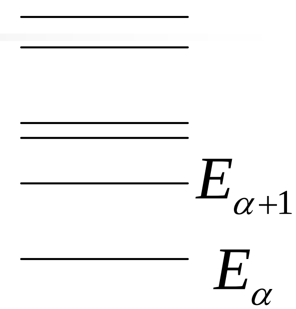


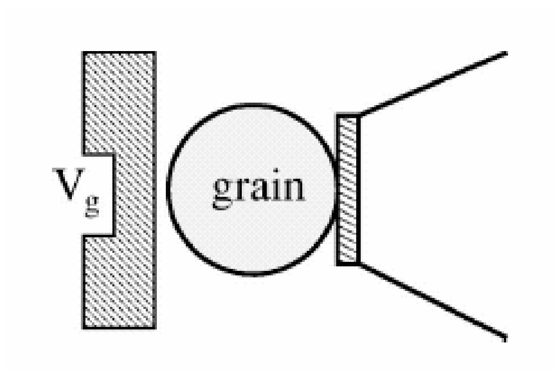
- Quantum Dots with a few electrons
- Equilibrium and Non-Equilibrium Kondo effect
- Coherence and de-coherence
- Hidden dynamical symmetries
- Quantum dots with many electrons
- Stoner instability in isolated dots
- Perspectives

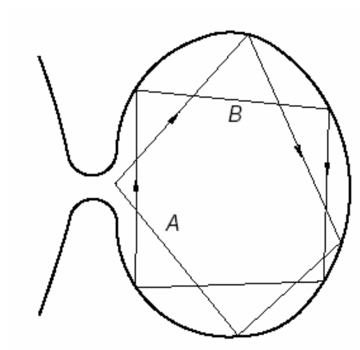
Ref. MK and R.Oppermann, PRL 2000
MK, K.Kikoin and L.W.Molenkamp JETP Lett 2003
MK, K.Kikoin and L.W.Molenkamp, PRB 2003
MK, AHP 2003
K.Kikoin, MK and Y.Avishai, Kuwler 2004, cond-mat/0309606
Y.Avishai, K.Kikoin and MK, NOVA 2005, cond-mat/0407063
MK, D.N.Aristov and K.Kikoin, PRB 2005
MK, Y.Gefen, cond-mat/0504751

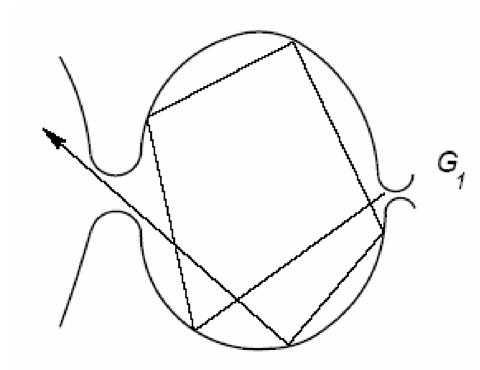
Quantum Dots with many electrons







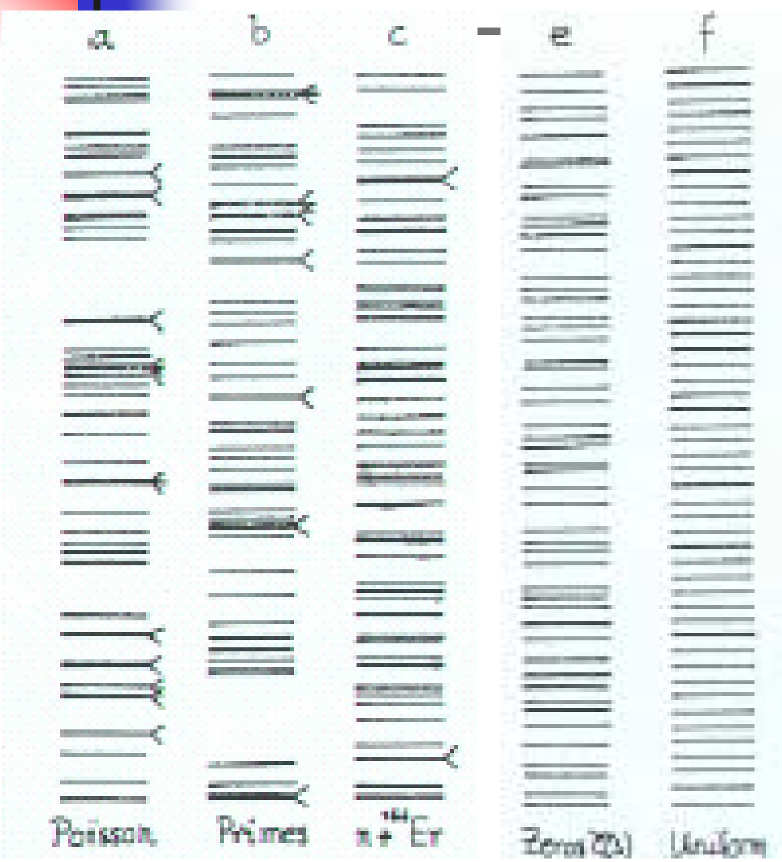




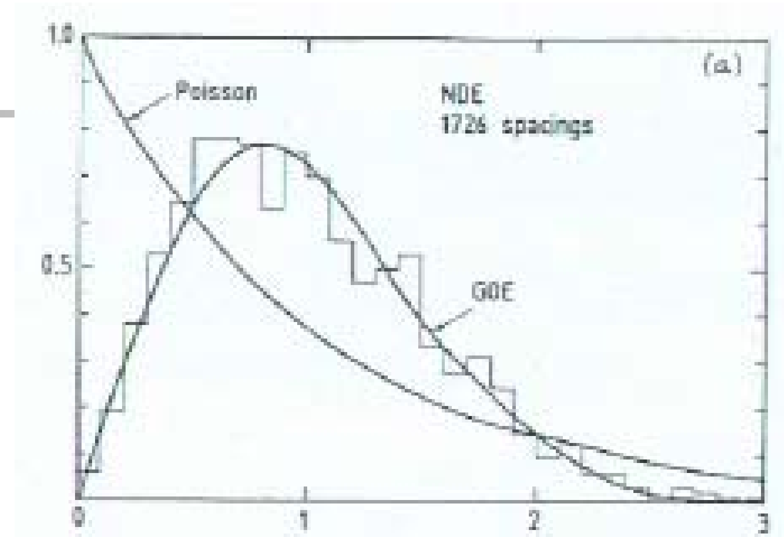
Spin blockade or Spin anti-blockade?

Metallic quantum dots: many-electron system

Random Matrix Theory



GOE GUE



$$P\left(\left\{E_\mu\right\}\right) \propto \exp\left(\frac{\beta}{2}\sum_{\nu\neq\mu}\ln\left[\frac{|E_\mu - E_\nu|}{\delta}\right]\right)$$

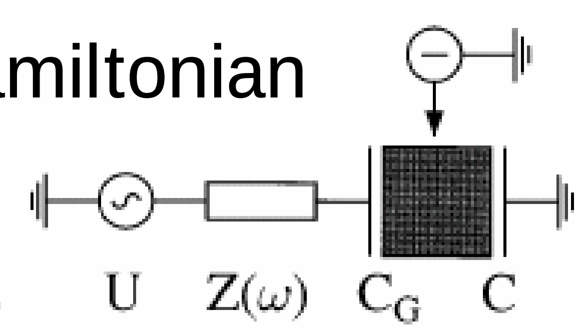
$\beta = 1$ Orthogonal (GOE)

$\beta = 2$ Unitary (GUE)

$\beta = 4$ Simplectic (GSE)

Wigner-Dyson statistics

Metallic Quantum Dot: Universal Hamiltonian



Zero-mode interaction

Electron-electron interactions in isolated metallic grains

Mean-level spacing $\Delta = \langle E_{\alpha+1} - E_\alpha \rangle$ (kinetic energy)

Thouless energy $E_T \sim D \cdot L^{-2}$ diffusive regime

$E_T \sim v_F L^{-1}$ ballistic regime

$E_{\alpha+1}$

E_α

$g = E_T / \Delta \gg 1$ **metallic grain**

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2 - \lambda_{BCS} \hat{T}^+ \hat{T}$$

GUE

charge spin superconducting

$$E_c = \frac{e^2}{2C}$$

Coulomb blockade

Scaling:

Short-range interaction $E_c \sim |J| \sim \Delta$

Coulomb interaction $E_c = r_s (k_F L)^{d-1} \Delta \gg |J|$

Kurland, Aleiner, Altshuler (2000)
Aleiner, Brouwer, Glazman (2002)

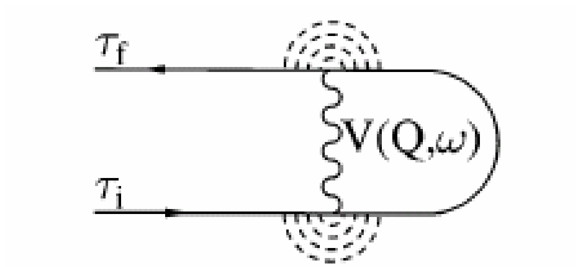
What is a zero-mode interaction?

Electron-electron interaction

$$H_{\text{int}} = \frac{1}{2}\sum_Q V(Q)\rho(Q)\rho(-Q)$$

TDoS

$$\delta\nu(\epsilon) = -\frac{\nu_0}{\pi} T \,\mathrm{Im} \sum_{\omega_n > \epsilon_m} \sum_Q \frac{2\pi i V(\mathbf{Q},\omega_n)}{(D\mathbf{Q}^2 + |\omega_n| + \gamma_{in})^2}\Big|_{i\epsilon_m \to \epsilon + i\delta}$$



$$V(\mathbf{Q},\omega_n) = \frac{V_0(\mathbf{Q})}{1 + V_0(\mathbf{Q})\Pi(\mathbf{Q},\omega_n)}$$

Bare Coulomb Interaction

Screened Coulomb interaction

Polarization Operator

$$\vec{Q} = \frac{2\pi}{L}\vec{n}$$

$$\Pi(\mathbf{Q},\omega_n) = \nu_0 \frac{D\mathbf{Q}^2}{D\mathbf{Q}^2 + |\omega_n|}$$

Q=0 contribution

$$H_{\text{int}} = \frac{1}{2} V(0)\left[\hat{n} - N\right]^2$$

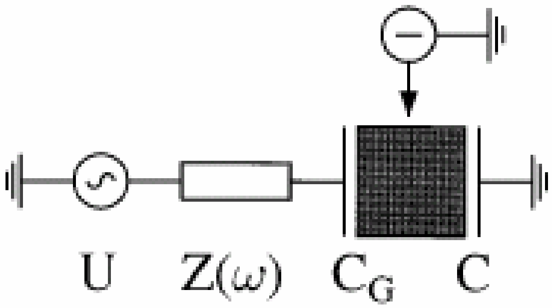
$$V(0) = \frac{e^2}{C}$$

Nazarov (1989)
Levitov, Shitov (1996)
Kamenev, Gefen (1996)

Zero-mode interaction requires a non-perturbative treatment at low temperatures!

Zero-bias anomaly in zero-dimensional systems

$$H_{\text{int}} = E_c(\hat{n} - N)^2$$



"Orthodox" theory of the Coulomb Blockade

R.I.Shekhter (1974)
Ben-Jacob, Gefen (1985)
Mullen, Gefen,Ben-Jacob (1988)
Averin, Likharev (1991)

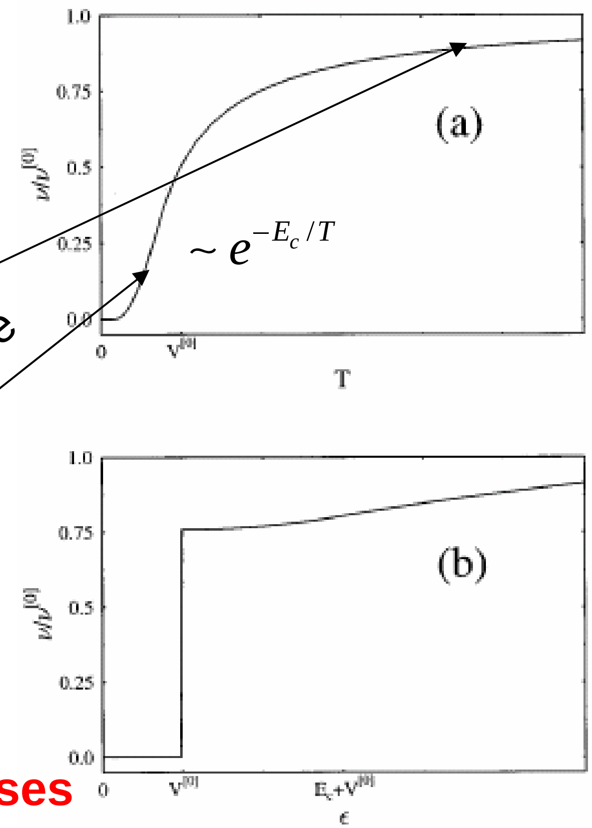
ZBA

perturbative

non-perturbative

$$\nu(\epsilon)/\nu^{[0]}(\epsilon) = 1 - \frac{V}{4T}\text{sech}^2\left(\frac{\epsilon}{2T}\right)$$

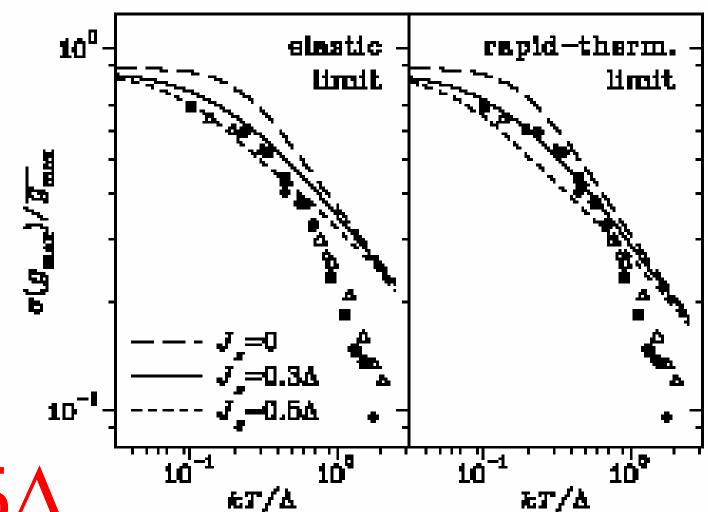
$$\nu(\epsilon)/\nu^{[0]}(0) = \cosh\left(\frac{\epsilon}{T}\right)\exp\left(-\frac{E_c}{T}\right)$$



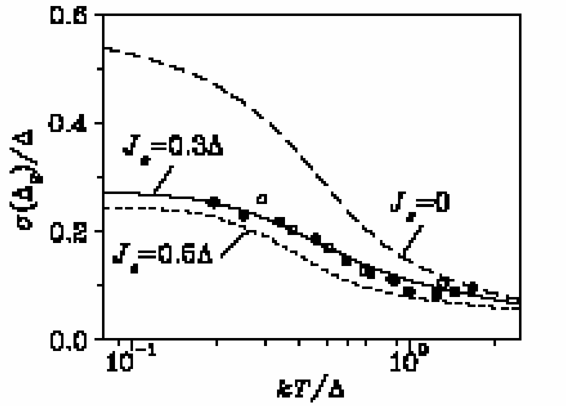
ZBA and Coulomb blockade are two limiting cases of the same theory

Spin Exchange. Master Equation (classical) Approach

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2$$

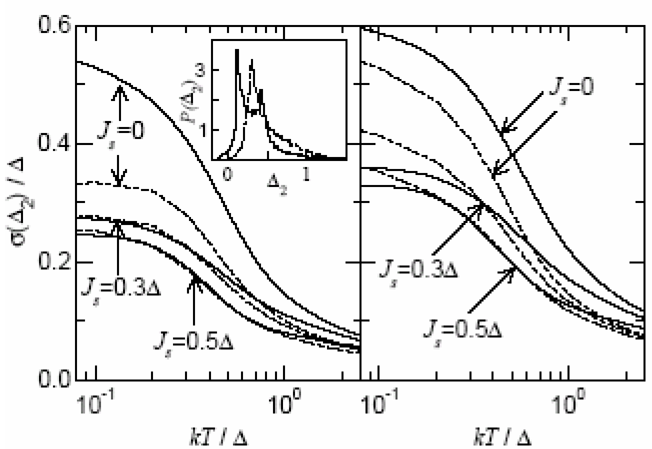


The ratio between standard deviation and the average value of peak height

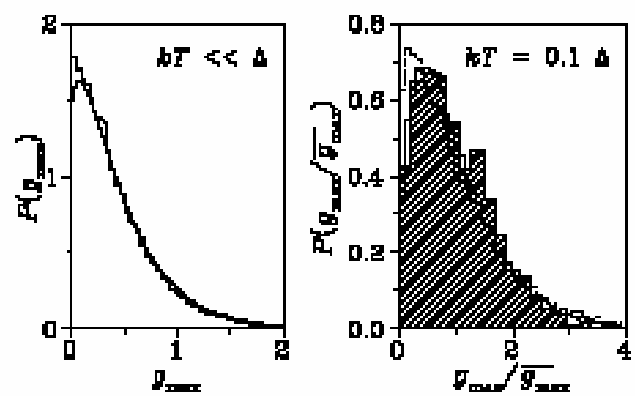


The width of peak-spacing distribution

$$J \geq 0.5\Delta$$



The width of peak-spacing distribution in the presence and absence of the orbital magnetic field.

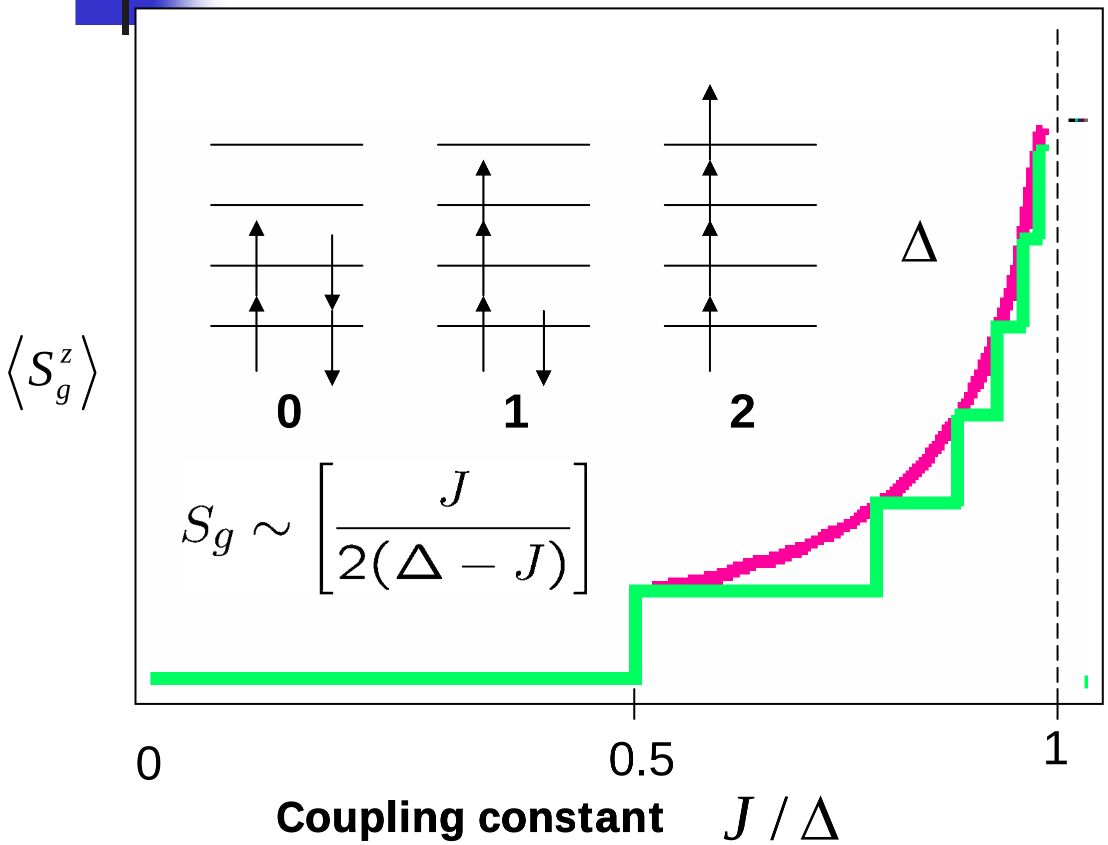


Peak-height distributions

Y.Alhassid and T.Rupp (2003, 2004)

Mesoscopic Stoner Instability

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J\left[(S^z)^2 + \varepsilon\left\{(S^x)^2 + (S^y)^2\right\}\right]$$



Isotropic exchange $\varepsilon = 1$

$$J_c^{even} = \Delta/2$$

$$J_c^{odd} = 2\Delta/3$$

Ising anisotropy $\varepsilon = 0$

$$J_c^{even} = J_c^{odd} = \Delta$$

Easy axis anisotropy

$$\varepsilon = J_\perp / J_\parallel < 1$$

$$J_c^{even} = \Delta/(1+\epsilon) \qquad J_c^{odd} = \Delta/(1+\epsilon/2)$$

Kurland, Aleiner, Altshuler (2000)

Kiselev, Gefen (2005)

Charge and Spin Interactions

amiltonian

$$H = \sum_{\alpha,\sigma} \varepsilon_\alpha \Psi^\dagger_{\alpha,\sigma} \Psi_{\alpha,\sigma} + H_C + H_S$$

Charge

$$H_C = E_c(\hat{n} - N)^2$$

$$\hat{n} = \sum_{\alpha,\sigma} \Psi^+_{\alpha\sigma} \Psi_{\alpha\sigma}$$

Commutative algebra

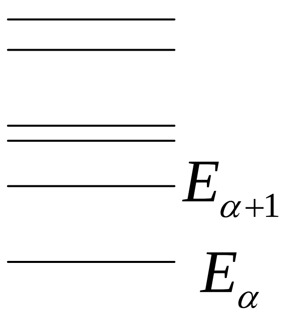
Dynamical symmetry U(1)

Spin

$$H_s = -J(\vec{S})^2$$

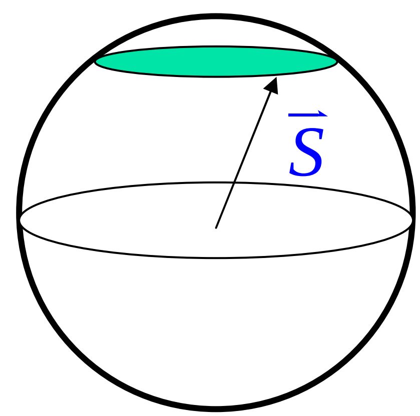
$$\hat{\vec{S}} = \sum_{\alpha,\sigma,\sigma'} \Psi^+_{\alpha\sigma} \vec{\sigma}_{\sigma\sigma'} \Psi_{\alpha\sigma'}$$

$$\left[S_j, S_k\right] = i\varepsilon_{jkl} S_l$$

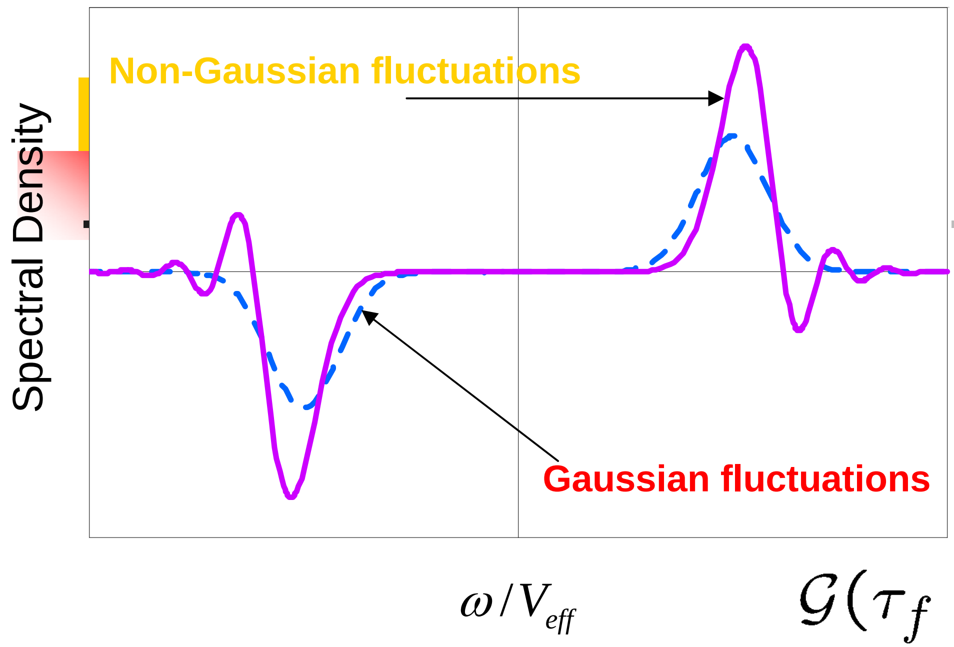


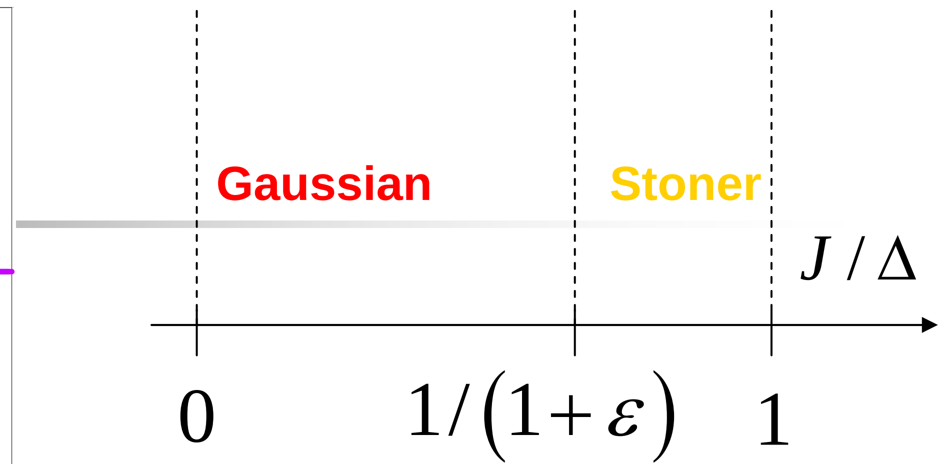
Non-commutative algebra

Dynamical symmetry SU(2)

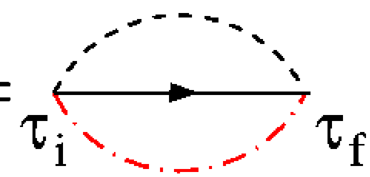


Tunneling Density of States





$$\mathcal{G}(\tau_f - \tau_i) =$$

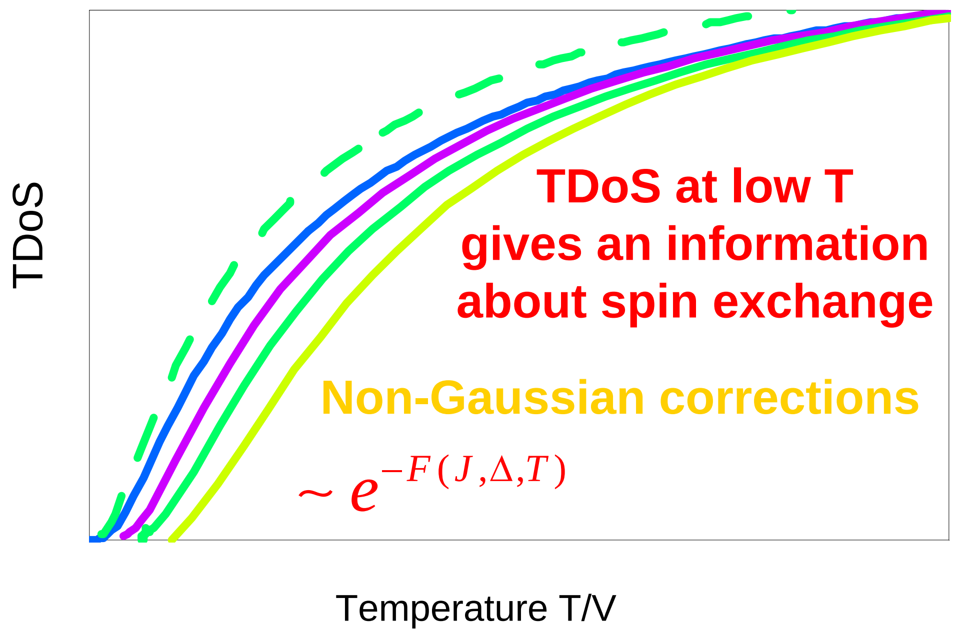


Zero-mode interaction

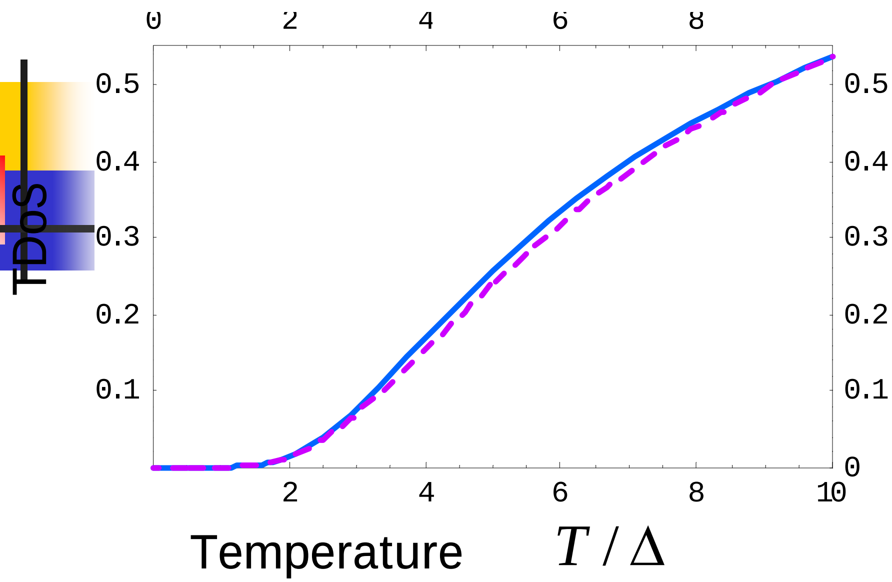
$$\nu(\epsilon) = -\frac{1}{\pi}\cosh\left(\frac{\epsilon}{2T}\right)\int_{-\infty}^{\infty}\mathcal{G}\left(\frac{1}{2T} + it\right)e^{i\epsilon t}dt$$

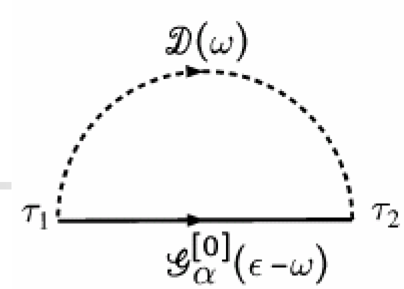
$$\nu(\epsilon) = -\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left[\tanh\left(\frac{\epsilon-\omega}{2T}\right) + \right.$$

$$\left. + \coth\left(\frac{\omega}{2T}\right)\right] B_{\|}(\omega)\nu^{[0]}(\epsilon - \omega)$$

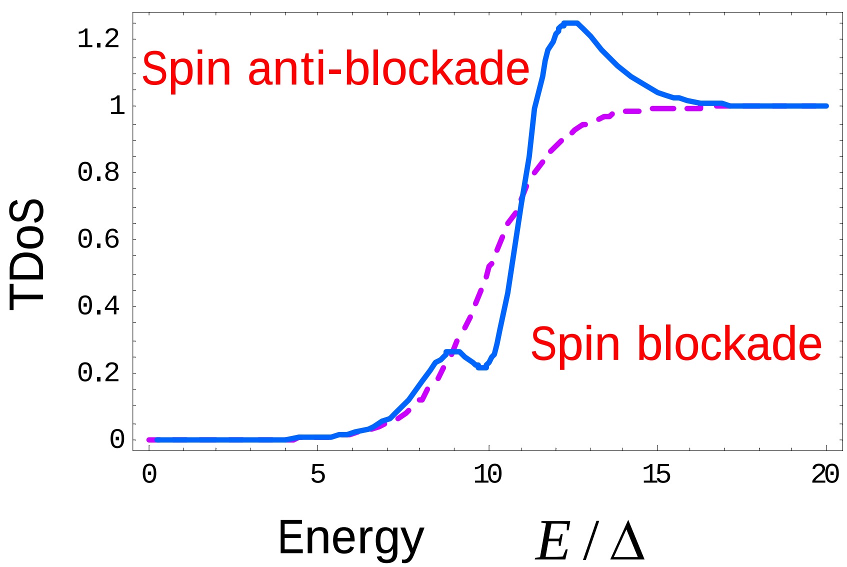


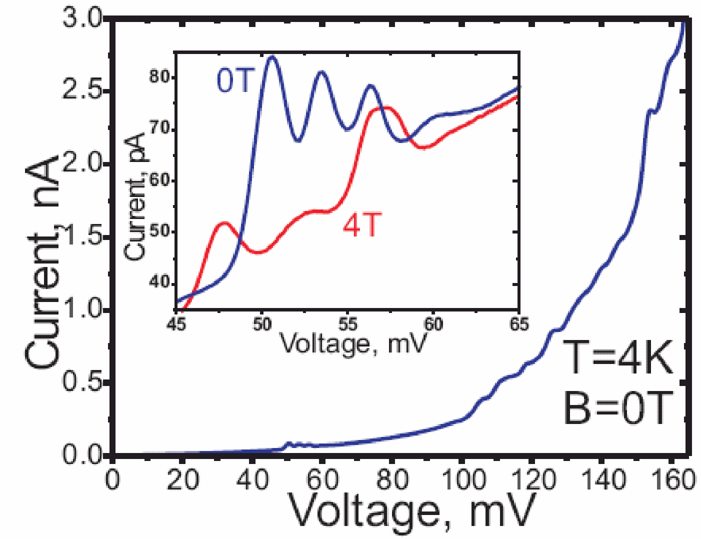
Quantum Dot Spectroscopy $T > \Delta$





Spin and Charge gauge factors



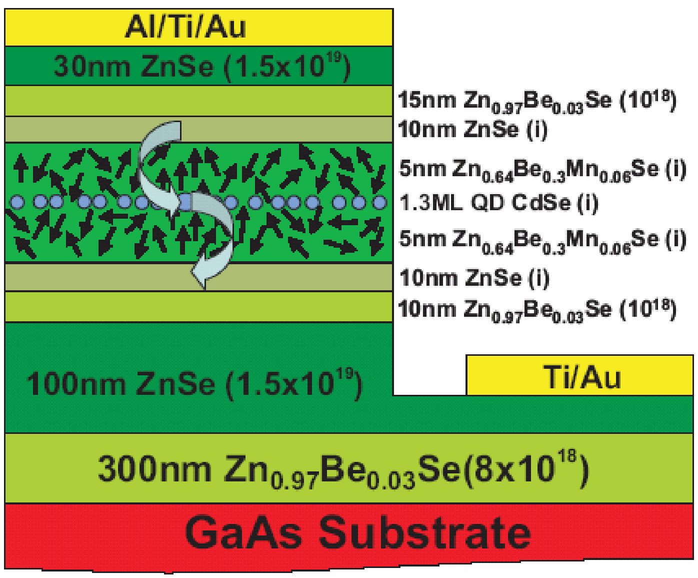


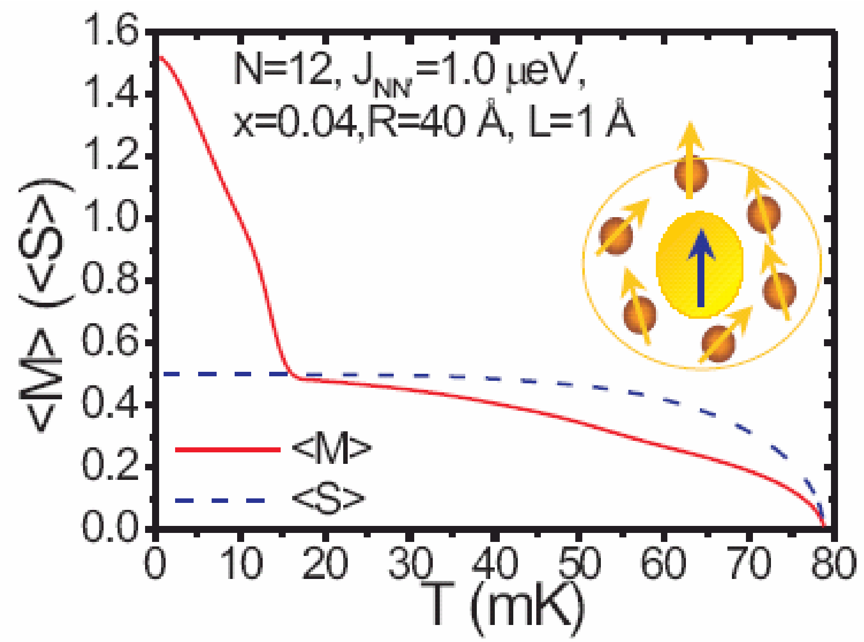
Molenkamp et al (2005)

Spin channel affects the charge transport

Kiselev, Gefen (2005)

Self-assembled quantum dots





Molenkamp et al (2005)

Spin susceptibilities

Longitudinal Susceptibility

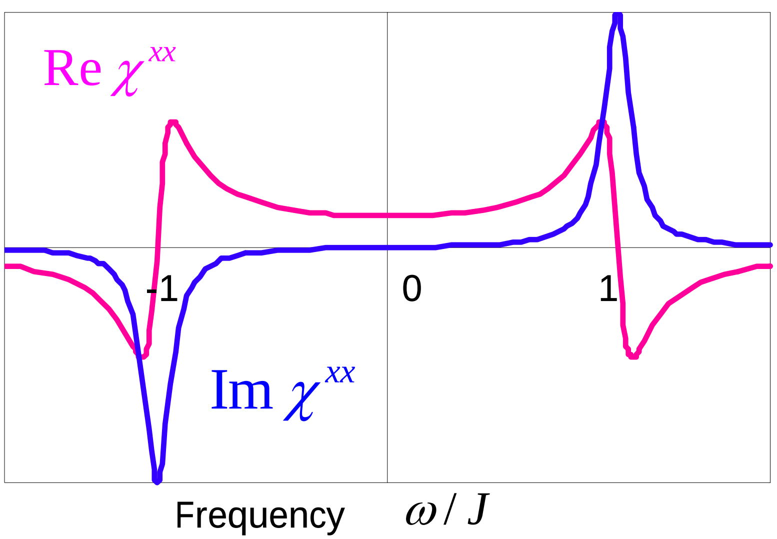
$$\chi^{zz} = \frac{\chi_0}{1 - J\chi_0}$$

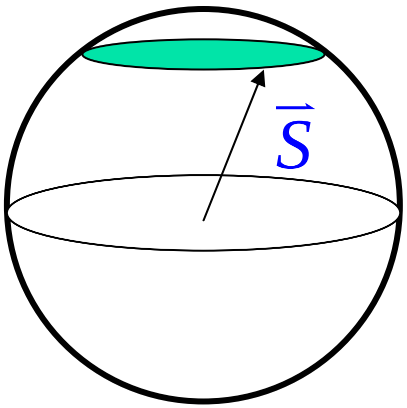
Stoner Instability

Static longitudinal susceptibility diverges at Stoner Instability point

Transverse Susceptibility

$$\partial_t S^{\pm} = iJ(1-\varepsilon)\left[1-2S^z\right]S^{\pm}$$



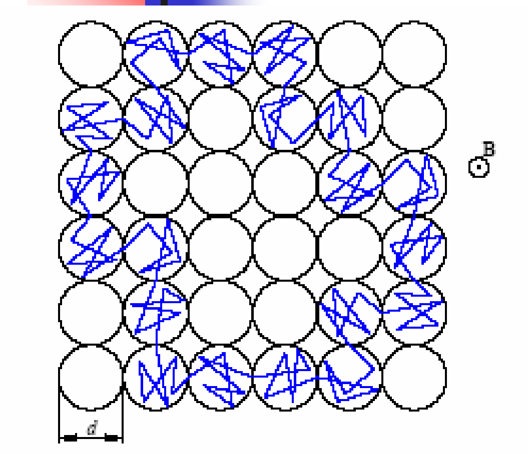


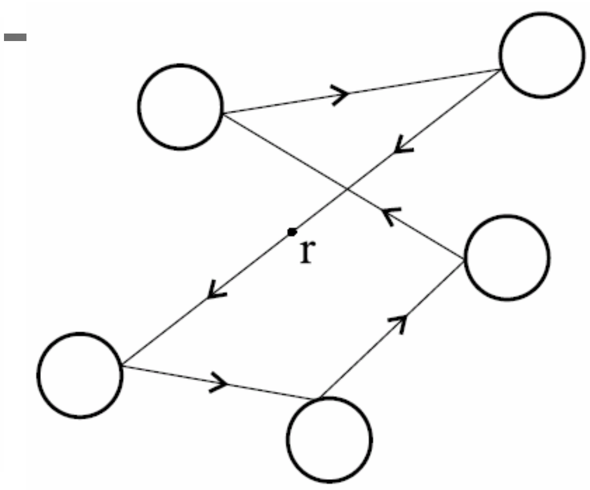
Exponentially enhanced!

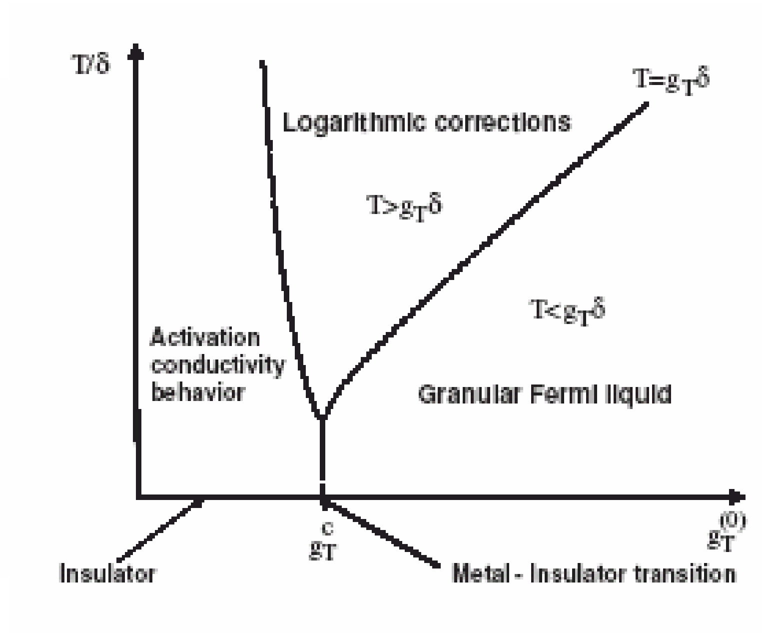
$$\chi^{xx}(t) = \frac{\chi_0 \varepsilon e^{J/T}}{1 - \varepsilon J \chi_0} e^{i(1-\varepsilon)Jt}$$

Magnetic instability in a system of coupled dots

2D arrays



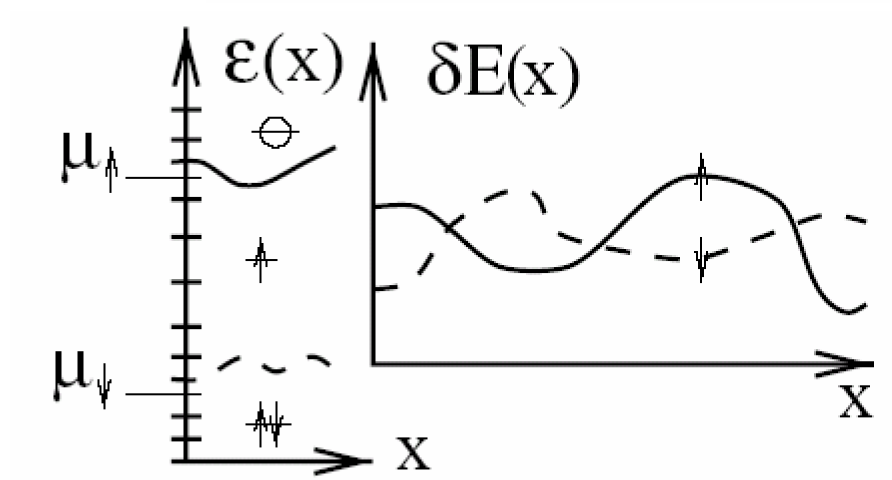




What about spin correlations?

$$\chi_s(T^*) = 2\left(1 - V(k_F) - \frac{V(k_F)}{\pi^2 g}\ln\frac{\tilde{\epsilon}_F}{T^*}\right)^{-1}$$

Quantum spin criticality?



Metallic Quantum Dots: summary

- **Charge and Spin zero-mode interactions strongly affect an electron transport through metallic grain (Quantum Dot) in Coulomb valley regime**

- **Dynamical symmetries associated with Charge Channel lead to Zero-Bias Anomaly**

- **Dynamical symmetries associated with Spin Channel may give rise to a Spin Blockade**

Collaborators:

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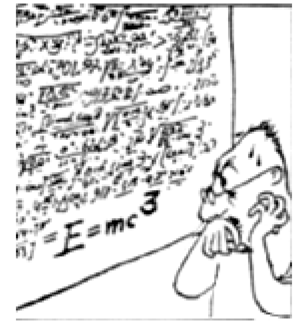
J.Richert (Strasbourg), V.Vinokur (Argonne), M.Wegewijs (Aachen)

PhD students: H.Feldmann, M.Bechmann (Würzburg)

Support: AvH & SFB-410 @ WU, **SFB-631 @ LMU, LSF @ WIS, DOE @ ANL**

Conclusions



• Complex quantum dots possess hidden symmetries responsible for several exotic transport properties of these nano-devices

• Magnetic correlations between electrons in a dot result in many interesting effects (Stoner instability, Kondo effect, Non-Fermi-Liquid behavior etc)

• Dynamical symmetry explains many known properties and predicts new effects in low-D nano-objects