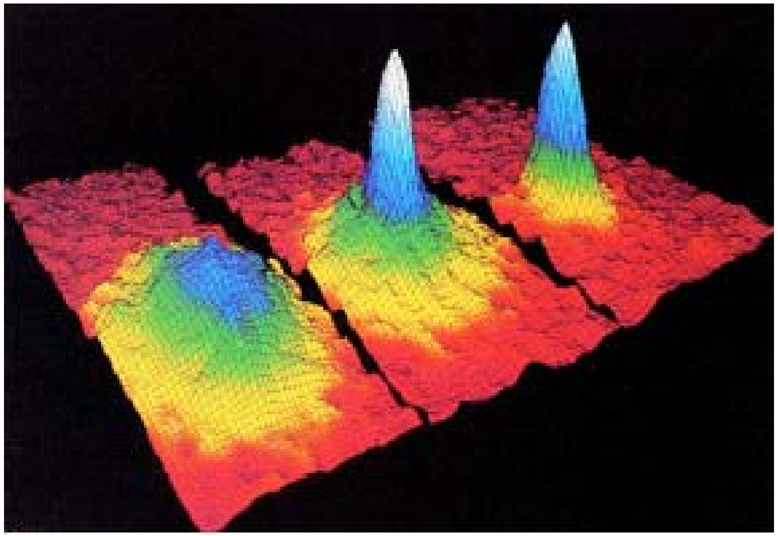
Institut für Theoretische Physik und Astrophysik

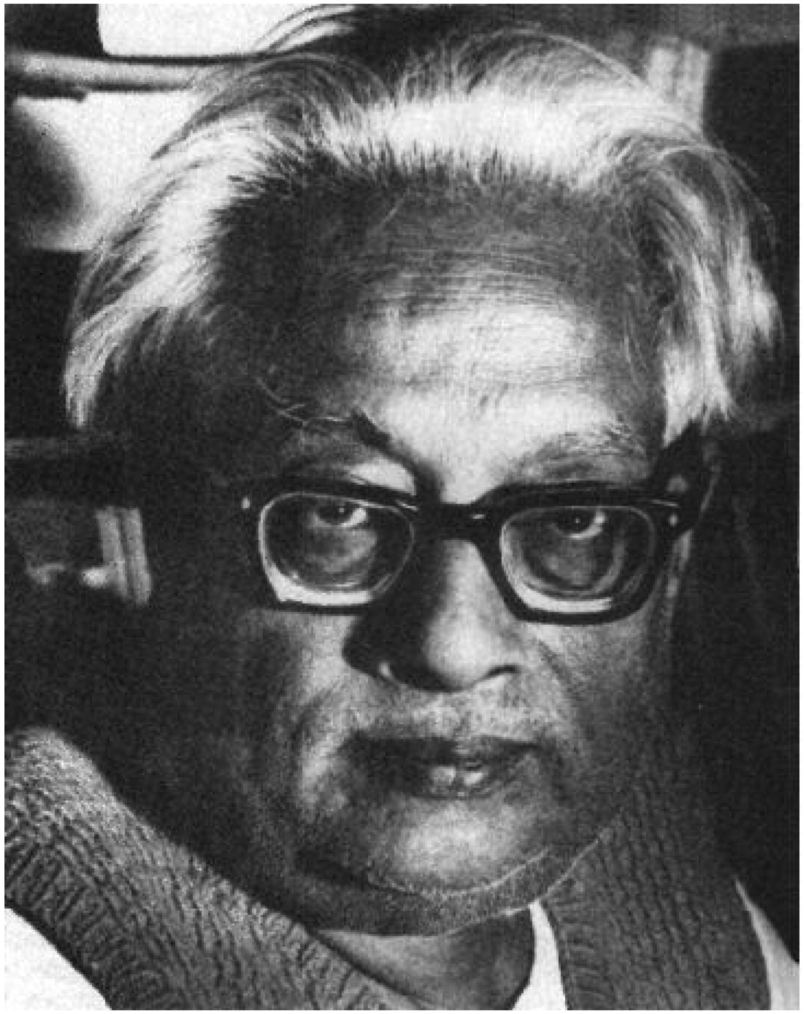
Universität Würzburg

Bose-Einstein Condensation

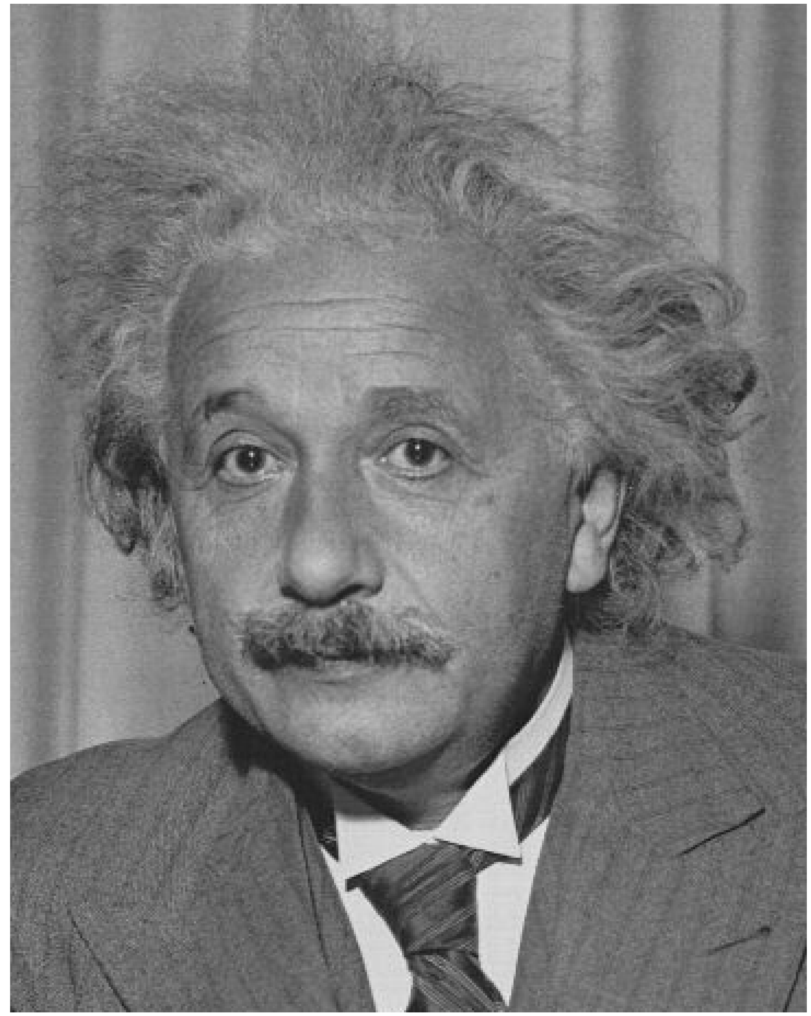
M.N.Kiselev



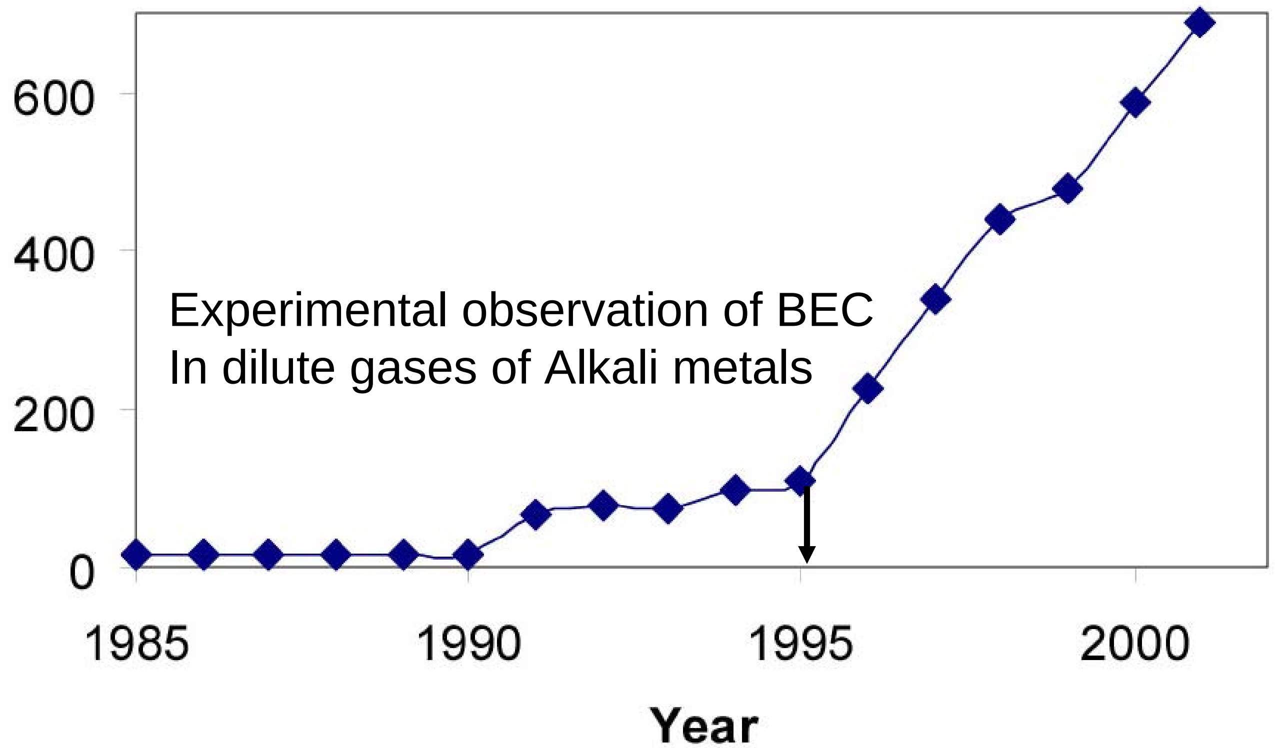
Bose-Einstein condensation *1924



Satyendra Nath Bose



Albert Einstein



Annual number of published papers, which have the words “Bose” and “Einstein” in their title, abstracts or keywords (ISI database)

Nobel Prize in Physics 2001

For the achievement of Bose-Einstein Condensation in dilute gases of Alkali metals…

Eric A.Cornell (USA), Wolfgang Ketterle (Germany), Carl E. Wieman (USA)

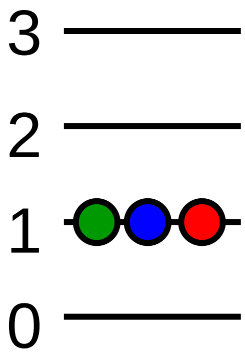
Outlook

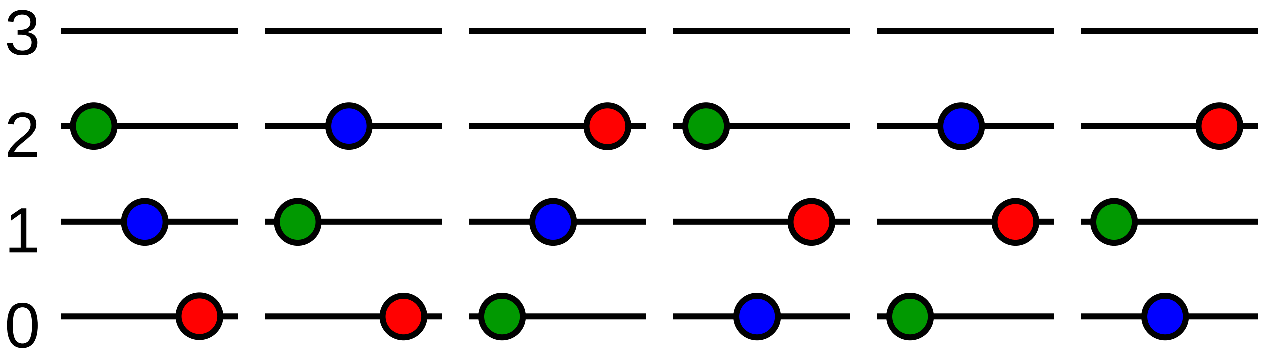
- Symmetry properties of many-particle wave functions: Fermions and Bosons.
- Statistics of Bosons: Bose-Einstein distribution function.
- Ideal Bose gas. Bose-Einstein Condensation.
- Thermodynamics of the Ideal Bose gas.
- Weakly interacting Bose gas.
- Experiments and perspectives.

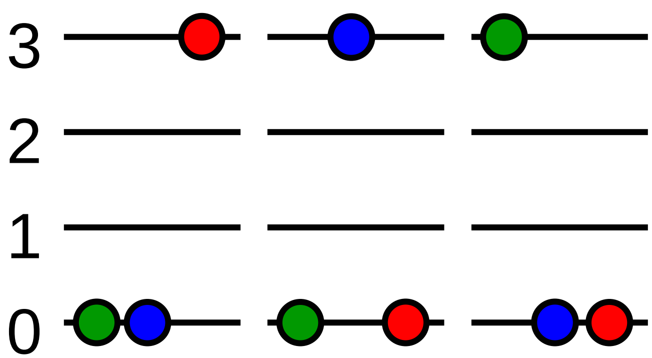
***) I acknowledge the use of materials and slides from W.Ketterle Nobel Lecture 2001 and his talk given at MIT's Teachers Program 24/06/03. Some pictures were lent to me by courtesy of W.Ketterle.**

3 particles, total energy = 3 (Arbitrary units)

Identical,
but classically distinguishable

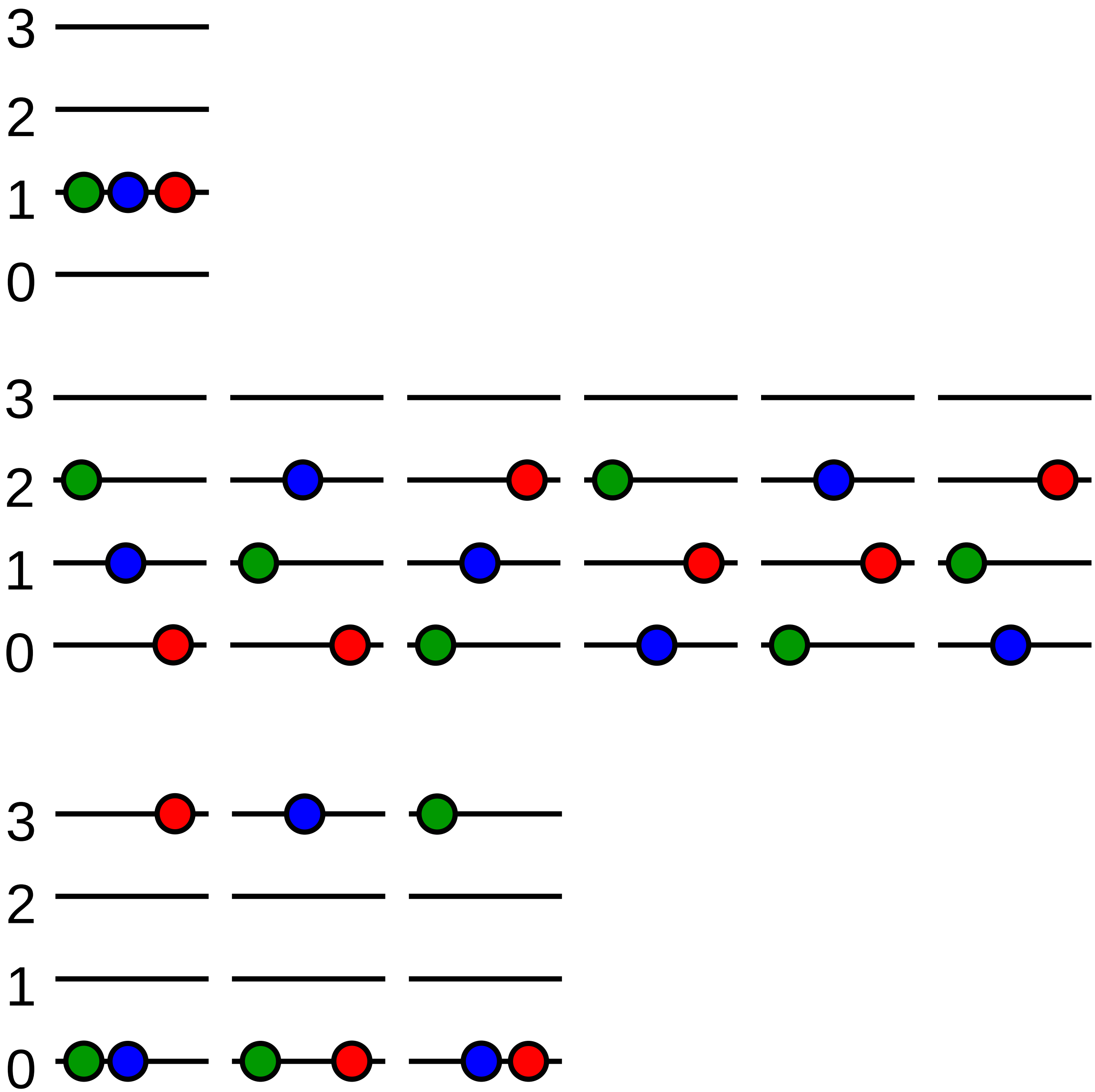




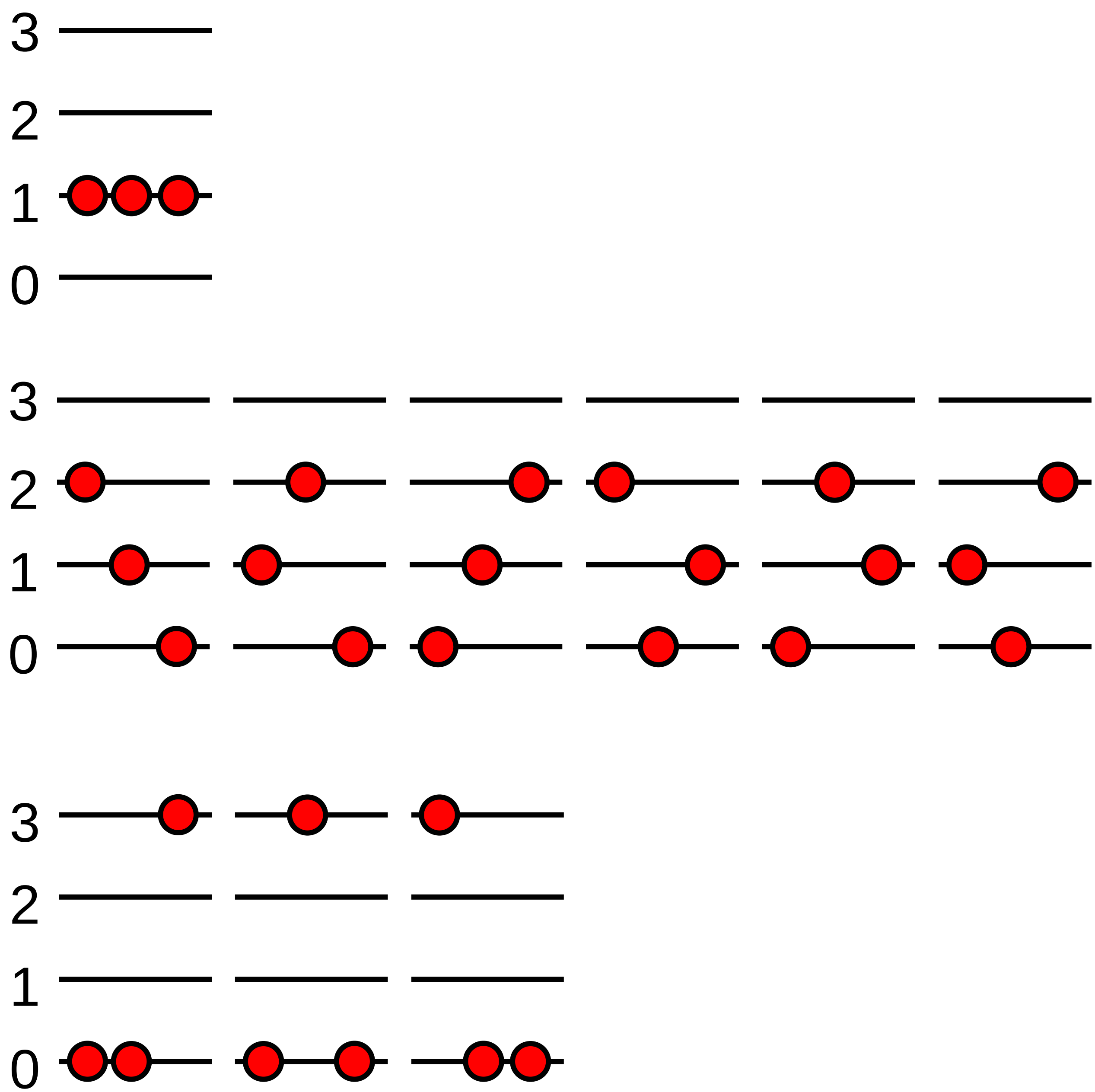


3	10%
6	20%
9	30%
12	40%

3 particles, total energy = 3 (Arbitrary units)

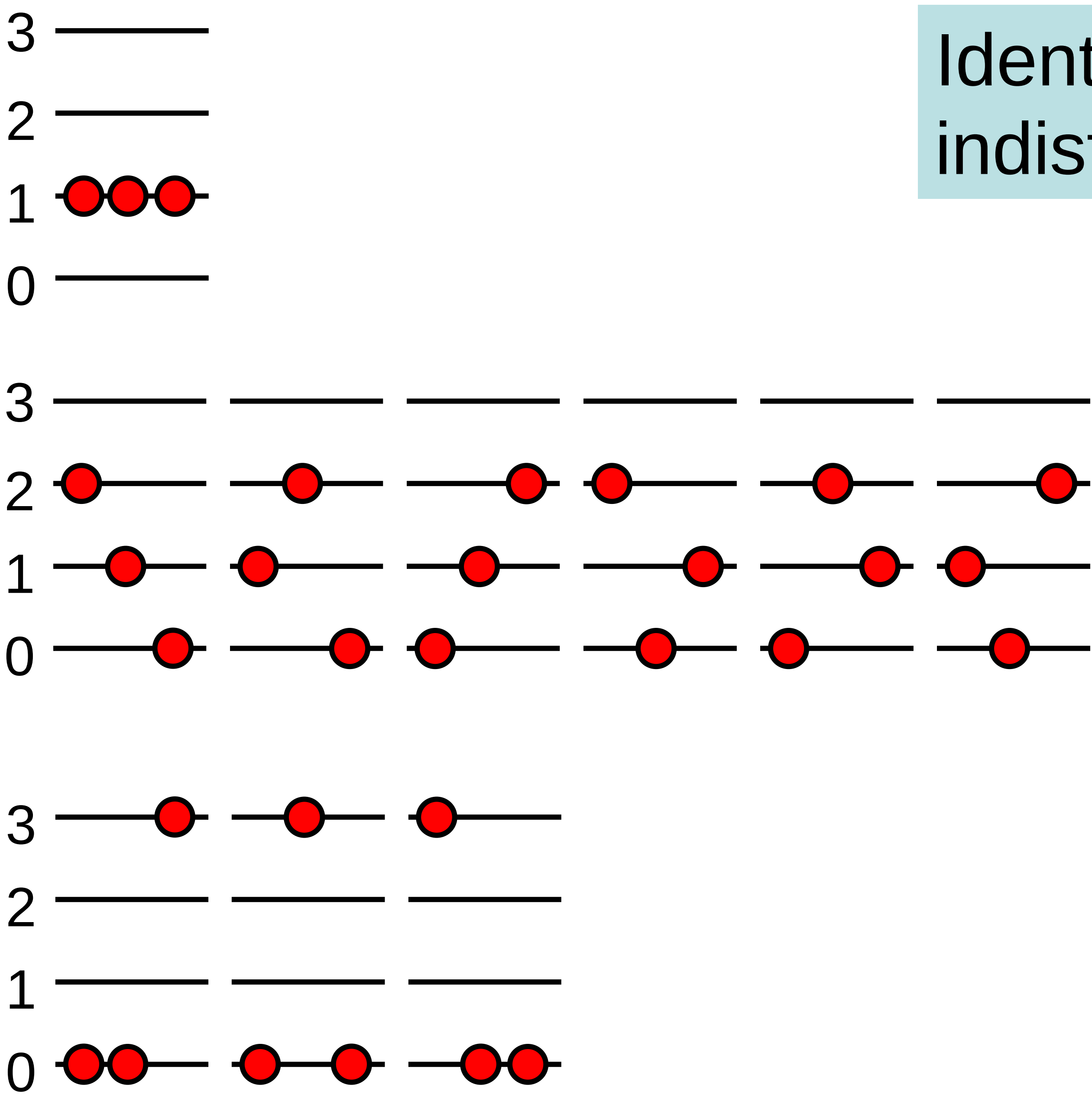


3 particles, total energy = 3 (Arbitrary units)



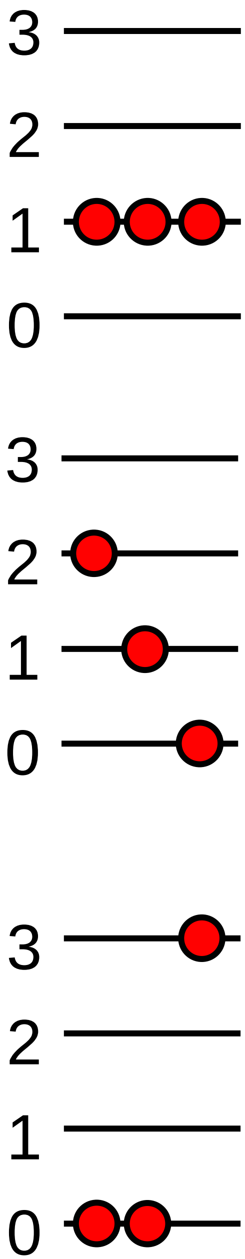
3 particles, total energy = 3 (Arbitrary units)

Identical, indistinguishable



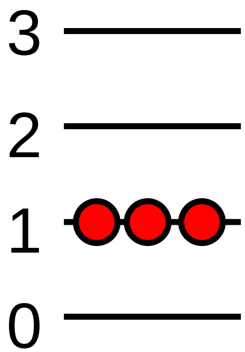
3 particles, total energy = 3 (Arbitrary units)

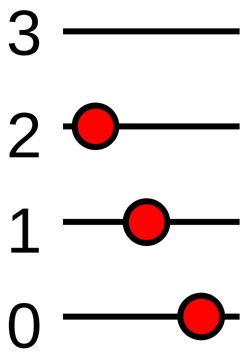
Bosons

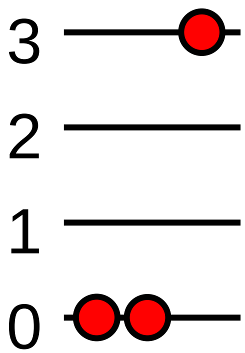


3 particles, total energy = 3 (Arbitrary units)

Bosons



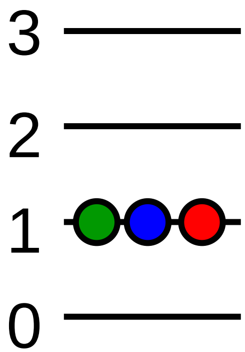




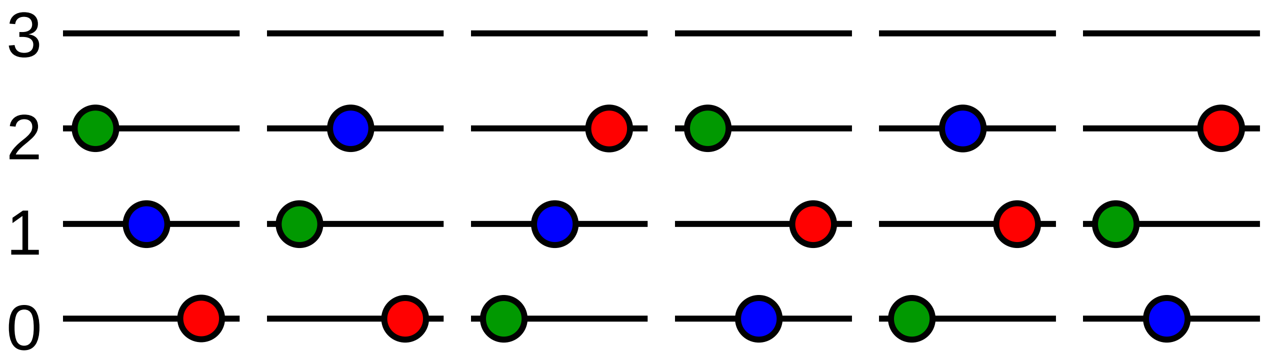
	bosons	classical
1	11%	10%
1	11%	20%
4	44%	30%
3	33%	40%

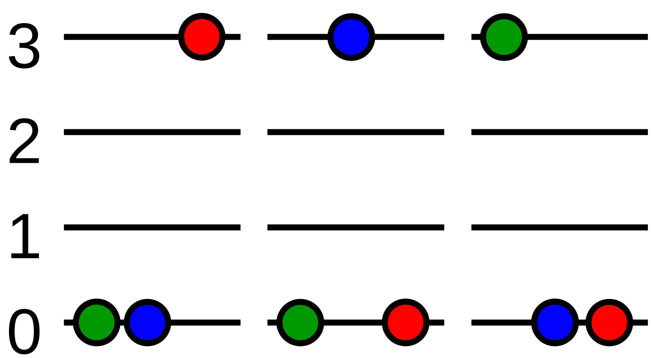
3 particles, total energy = 3 (Arbitrary units)

Classical



10 % probability
for triple occupancy

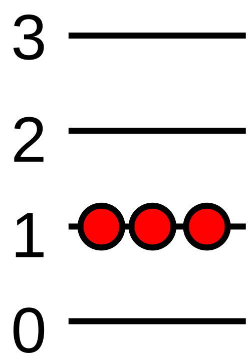




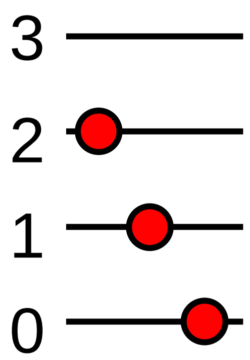
30 % probability
for double occupancy

3 particles, total energy = 3 (Arbitrary units)

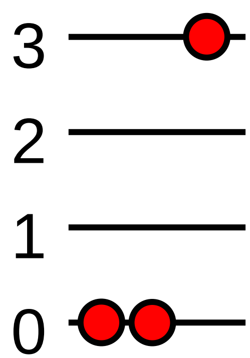
Bosons



33 % probability
for triple occupancy



Bosons like to get together!



33 % probability
for double occupancy

Thermodynamics of 3-dimensional Ideal Bose Gas.

$$N_{\varepsilon=0} = N\left(1-\left(\frac{T}{T_c}\right)^{3/2}\right), \qquad E = \sum_{\vec{k}} \varepsilon_k n_k = V\int_0^\infty \frac{k^2 dk}{2\pi^2}\frac{\hbar^2 k^2}{2m}\frac{1}{\exp\left(\frac{\hbar^2 k^2}{2mk_B T}\right)-1}$$

$$E = 0.770\, N k_B T\left(\frac{T}{T_c}\right)^{3/2} = 0.1289\frac{m^{3/2}(k_B T)^{5/2}}{\hbar^3}V,$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{5E}{2T}, \qquad F = E - TS = -\frac{2}{3}E$$

$$S = \int_0^T \frac{C_V}{T}dT = \frac{5E}{3T} \qquad P = -\left(\frac{\partial F}{\partial V}\right)_T = 0.0851\frac{m^{3/2}(k_B T)^{5/2}}{\hbar^3}$$

Pressure does not depend on the volume

$$P \xrightarrow{T\to 0} 0$$

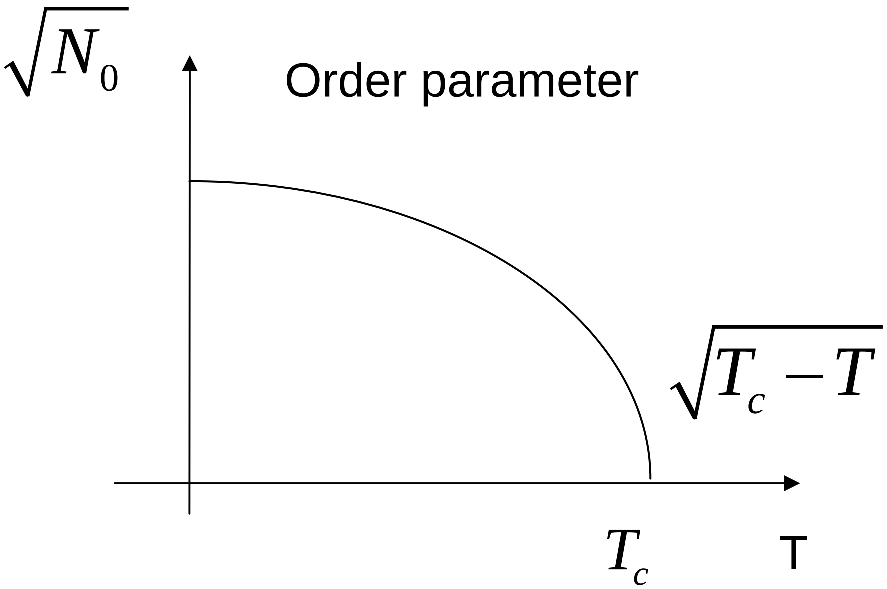
Summary:

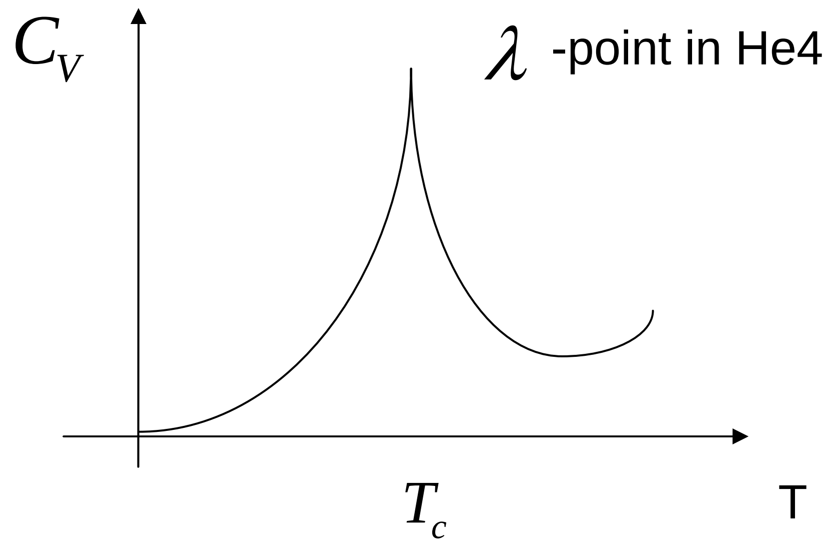
$$T < T_c = 3.31 \frac{\hbar^2}{mk_B}\left(\frac{N}{V}\right)^{2/3}$$ particles start to collect at lowest energy until at T=0 they are all there

Bose Gas undergoes a phase transition without any interaction!

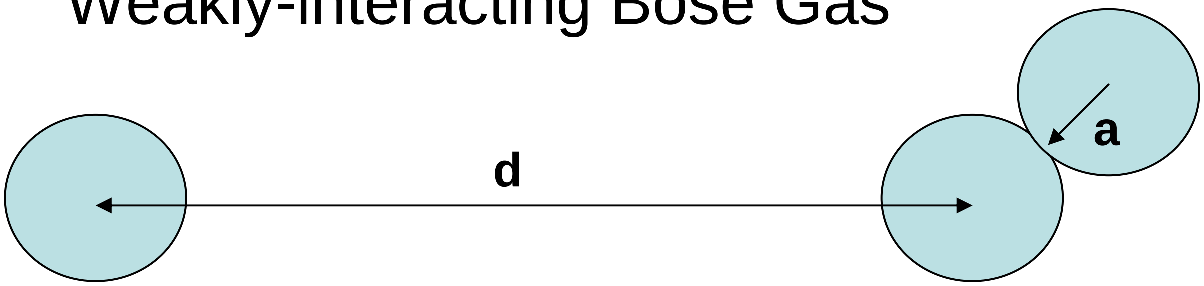
All the thermodynamic quantities are continuous at the transition point.

3rd - order phase transition





Weakly-interacting Bose Gas



$$U(q) \approx \frac{4\pi\hbar^2}{m} a$$

Weak interactions: $a \ll d$

a – scattering length,
d – average distance

Ideal Bose Gas

$$\varepsilon(p) = \frac{p^2}{2m}$$

Interacting Bose Gas

$$\varepsilon(p) = \sqrt{\frac{4\pi n_0 a}{m^2}\hbar^2 p^2 + \left(\frac{p^2}{2m}\right)^2} \approx \begin{cases} \frac{\hbar p}{m}\sqrt{4\pi n_0 a}, & \frac{p^2}{2m} \ll \frac{\hbar^2 n_0 a^3}{m a^2} \\ \frac{p^2}{2m}, & \frac{p^2}{2m} \gg \frac{\hbar^2 n_0 a^3}{m a^2} \end{cases}$$

$$n_0 = \frac{N}{V}$$

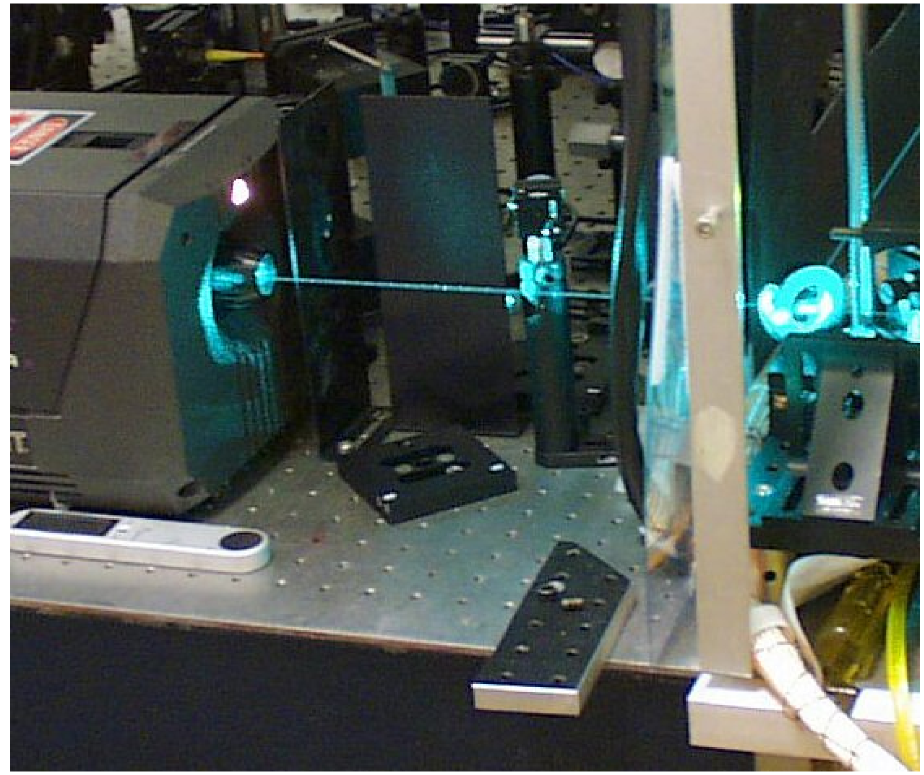
$$\frac{N}{N_0} \approx 1 - \frac{8}{3\sqrt{\pi}} a^{3/2} \left(\frac{N}{V}\right)^{1/2}$$

Particles of a non-ideal Bose gas **do not** all have zero momentum, **even in the ground state.**

Ordinary light

divergent
incoherent
many small waves
many modes

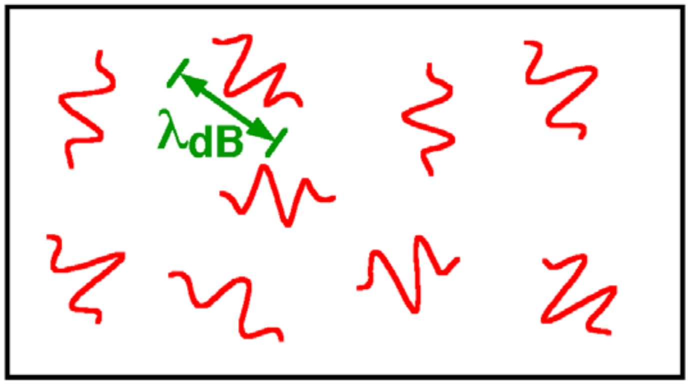
Laser light

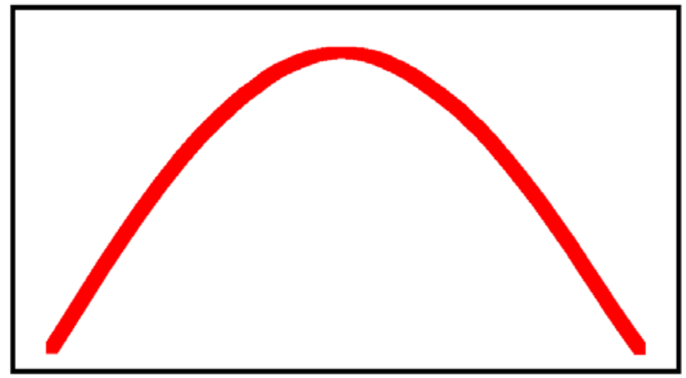


diffraction limited (directional)
coherent
one big wave
single mode (monochromatic)

Ordinary gas

Bose-Einstein condensate





atoms move around randomly

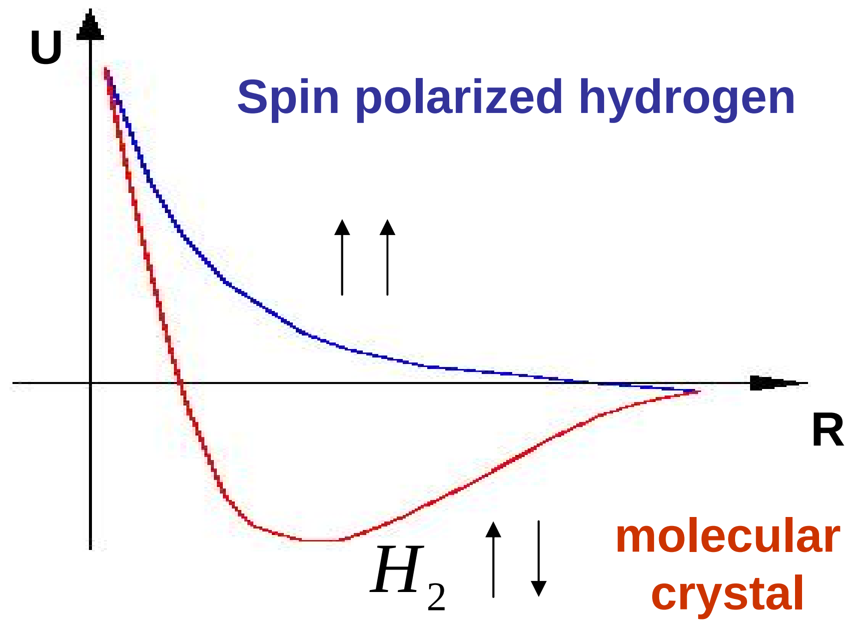
atoms in a coherent state

divergent
incoherent
many small waves
many modes

diffraction limited (directional)
coherent
one big wave
single mode (monochromatic)

Possible candidates for the experimental observation of the Bose-Einstein Condensation

$$T_c = 3.31\frac{\hbar^2}{2mk_B}\left(\frac{N}{V}\right)^{2/3}$$



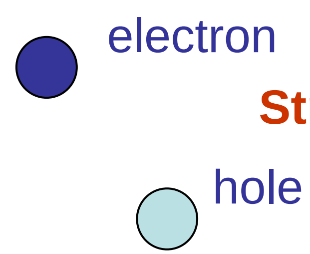
Atomic Hydrogen

Helium

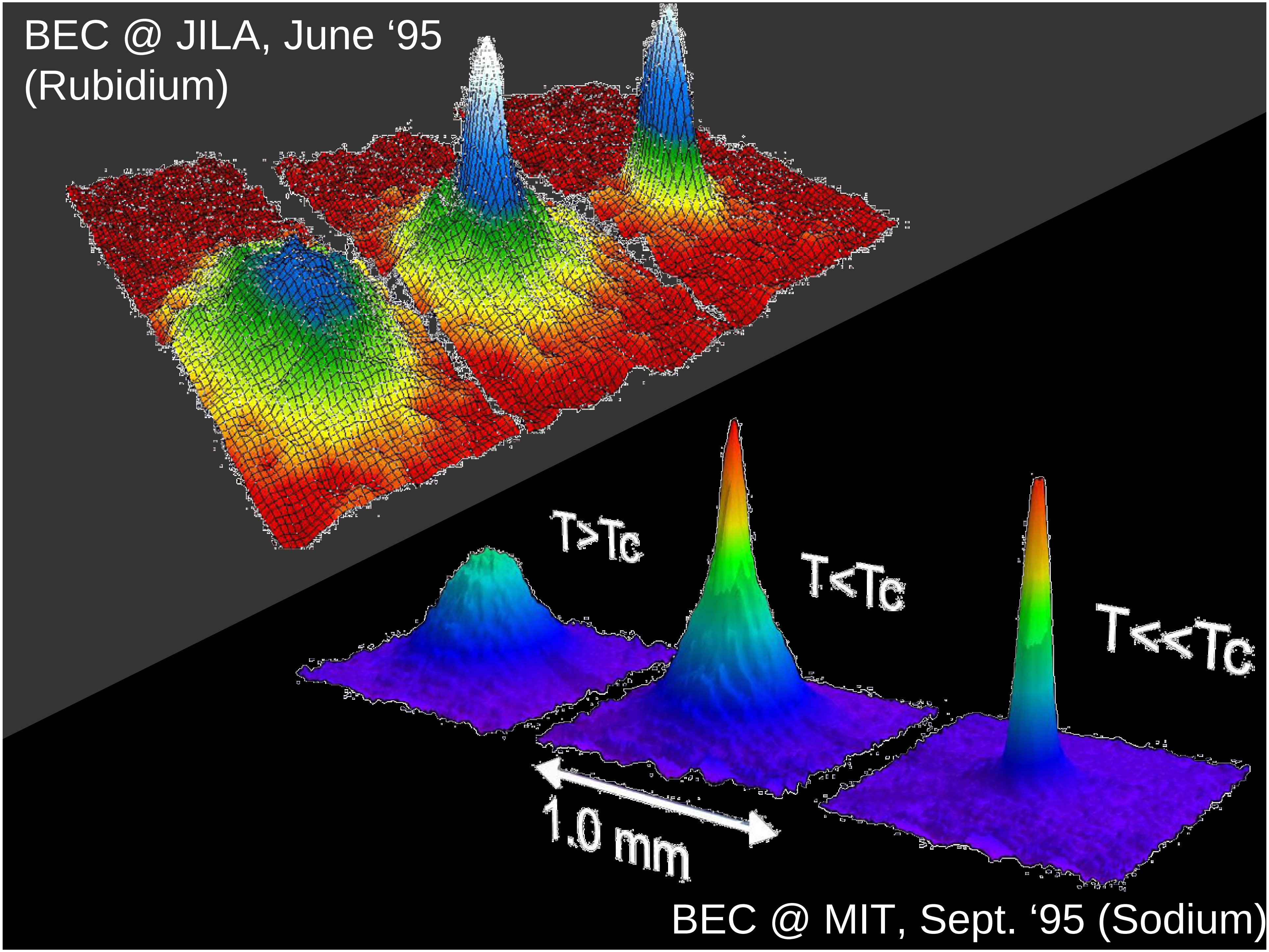
Only 8% of particles in the condensate

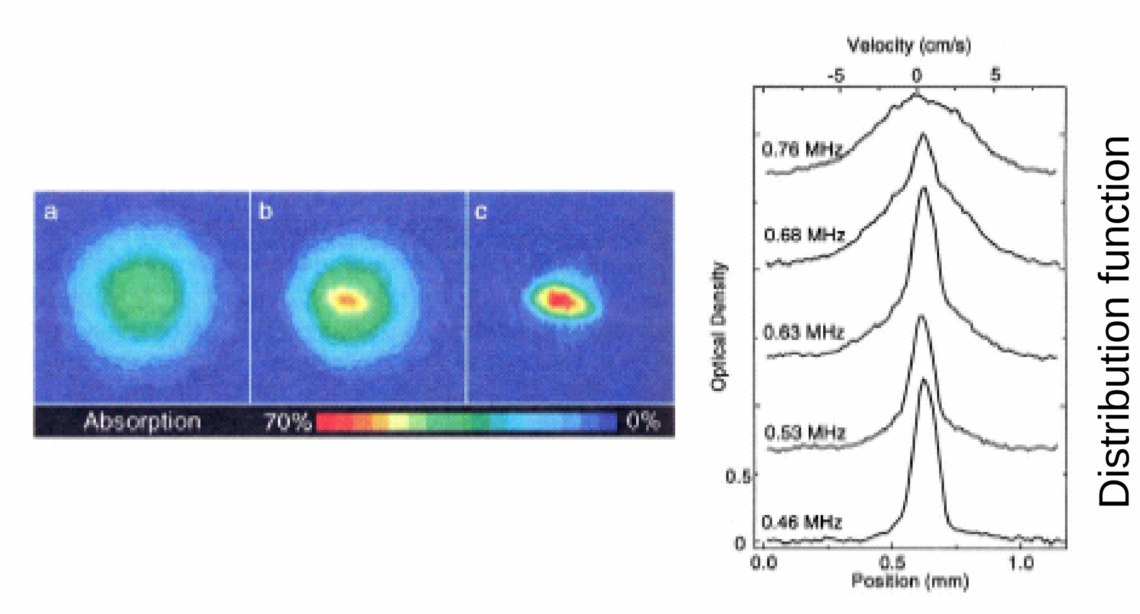
Strongly interacting Bose system!

Excitons in semiconductors



Strong many-body effects

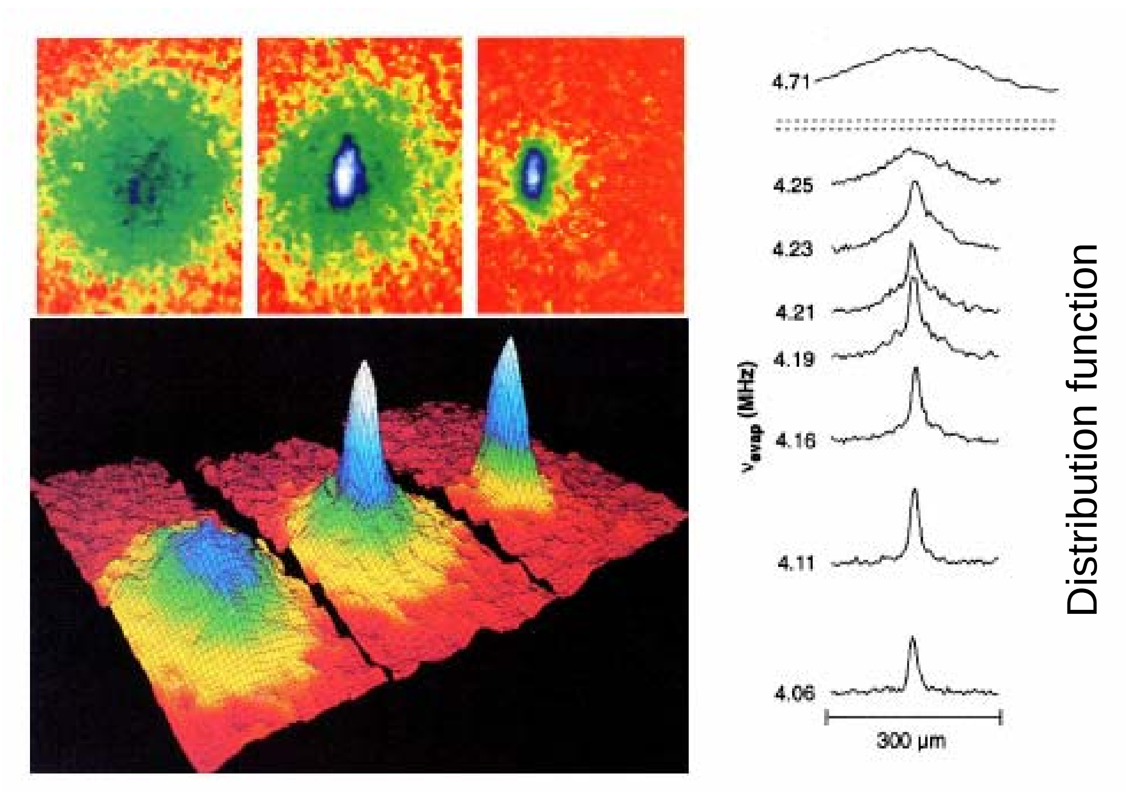

BEC @ JILA, June ‘95
(Rubidium)
T>Tc
T<Tc
T<<Tc
1.0 mm
BEC @ MIT, Sept. ‘95 (Sodium)



Sodium, MIT Group, September 1995

$\sim 10^6$ particles in BEC

$$T_c \sim 1 \mu K$$



field of view

$$200 \mu m \times 270 \mu m$$

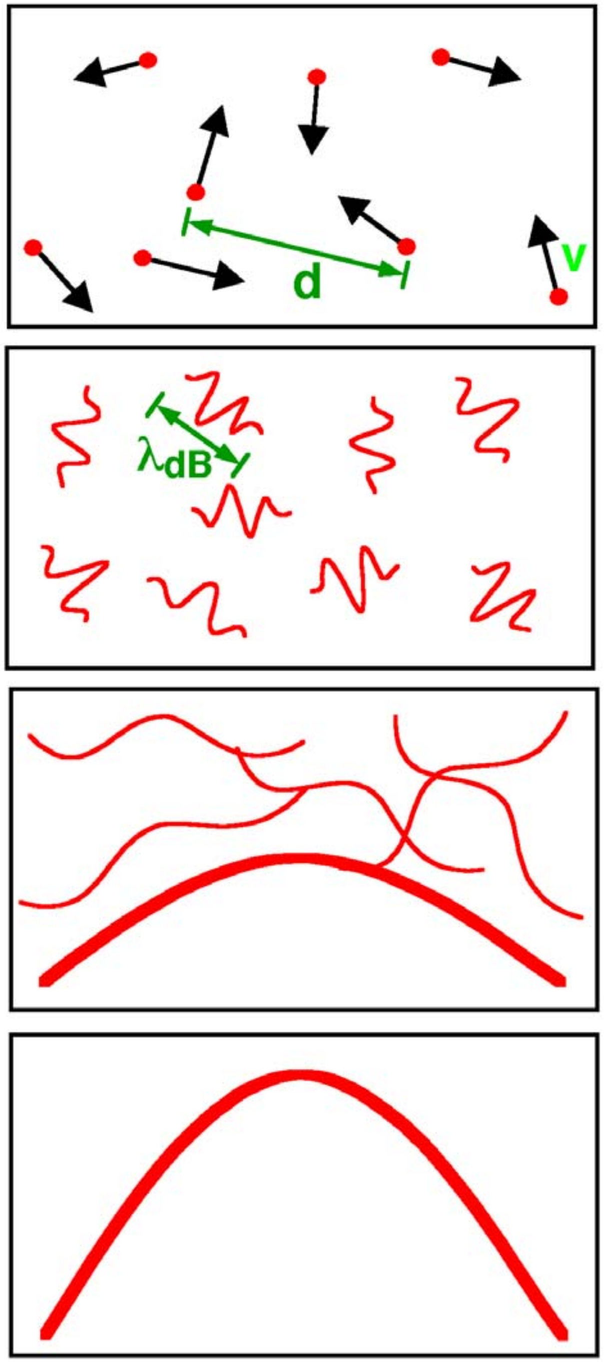
time – 1/20 s

Rubidium, JILA Group, June 1995

$\sim 10^3$ particles in BEC

JILA –Joint Institute for Laboratory Astrophysics

What is Bose-Einstein condensation (BEC)?



High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength

$\lambda_{dB} = h/p_T \propto T^{-1/2}$

"Wave packets"

$T=T_{crit}$: Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"

T=0: Pure Bose condensate

"Giant matter wave"

BEC=Tool for knowledge

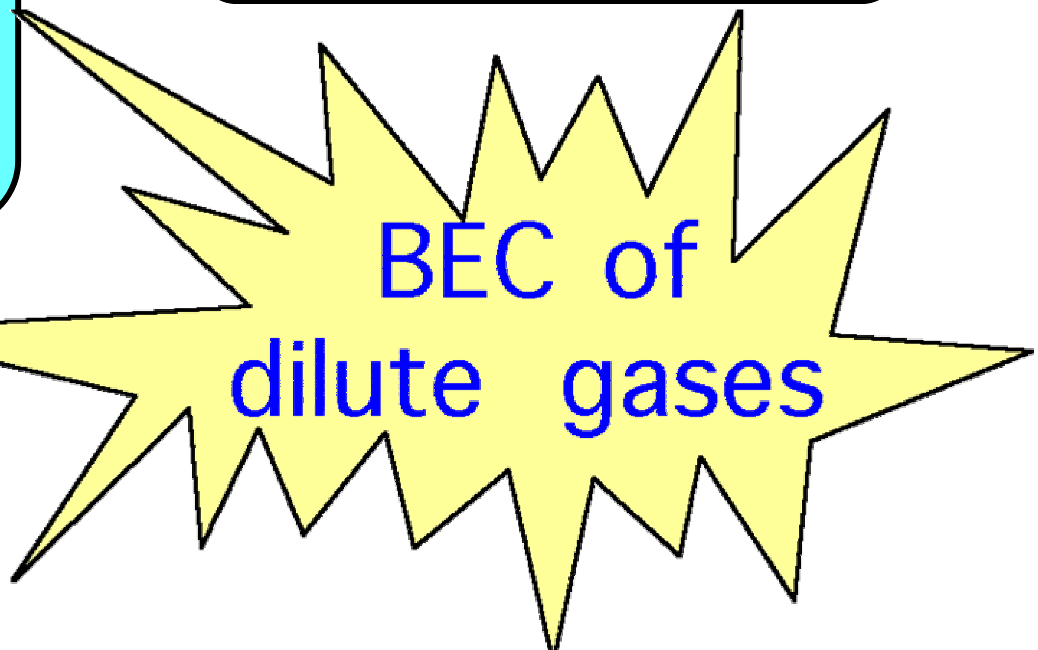
BEC=Tool for applications

Condensed matter physics
Many-body physics
Statistical physics

- Superfluidity
- Quantum gases
- Mesoscopic physics

Metrology

- Atomic clocks
- Matter wave sensors



Collisional physics

- Ultracold collisions
- Cold chemistry

Quantum optics

- Coherence of atoms
- Atom laser
- Entanglement

Visionary long-term goals

- Atom deposition – nanotechnology
- Concepts for quantum computer