SUPPLEMENTAL MATERIALS

This Supplemental Materials contains additional information about the charge current beyond the linear response theory and connections between the full fledged calculations performed using Kedlysh out-of-equilibrium approach and the results derived by means of the transport integrals method.

I. ELECTRIC CURRENT BEYOND THE LINEAR RESPONSE

A. Coupling asymmetry

If the quantum impurity is coupled to the leads with arbitrary coupling, the new variables (a and b) entering the FL Hamiltonian (6) [S1] are defined by Glazman-Raikh rotation [S2–S4] as follows:

$$\begin{pmatrix} b_{kr} \\ a_{kr} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} c_{Lkr} \\ c_{Rkr} \end{pmatrix}, \tag{S1}$$

where $\tan\theta{=}|t_R/t_L|$ is given by the ratio of tunnel lead-dot matrix elements [S4] of the Hamiltonian (3). The symmetric coupling corresponds to $\theta{=}\pi/4$. We introduce the parameter $\mathcal{C}{=}\cos 2\theta{=}(\Gamma_L-\Gamma_R)/(\Gamma_L+\Gamma_R)$ to characterize the asymmetry of the dot-lead coupling; $\Gamma_\alpha{=}\pi\nu N|t_\alpha|^2$ is intrinsic total local level width associated with the tunneling from/to the reservoirs (we assume that tunnel matrix elements are the same for all orbitals/flavours).

B. Phase shift

The phase shift expression in the presence of the finite voltage bias $e\Delta V$, finite temperature drop across the impurity ΔT is given by the equation [S5]:

$$\delta(\varepsilon) = \delta_0 + \alpha_1\varepsilon + \alpha_2\varepsilon^2 - \frac{N-1}{4}\phi_2 \times \mathcal{A}, \tag{S2}$$

where

$$\mathcal{A} = \frac{1}{6}\left[(\pi T_L)^2(1+\mathcal{C}) + (\pi T_R)^2(1-\mathcal{C}) + \frac{3}{2}(1-\mathcal{C}^2)(e\Delta V)^2\right]. \tag{S3}$$

The FL identities $\alpha_1 = (N-1)\phi_1$ and $\alpha_2 = 1/4(N-1)\phi_2$ follow from the Kondo floating paradigm [S5–S7]. The exact relation between α_1 and α_2 is given by the Bethe-Ansatz solution [S5, S7]:

$$\frac{\alpha_2}{\alpha_1^2} = \frac{N-2}{N-1}\frac{\Gamma(1/N)\tan(\pi/N)}{\sqrt{\pi}\Gamma\left(\frac{1}{2}+\frac{1}{N}\right)}\cot\delta_0. \tag{S4}$$

Here $\Gamma(x)$ is the Euler's gamma-function.

C. Elastic current

The elastic contribution to the charge current is performed by averaging the current with H_α of (6) which is equivalent to use of the Landauer-Büttiker formula [S8] containing the energy dependent transmission coefficient $\mathcal{T}(\varepsilon)$ computed with the scattering phase (S2, S3):

$$I_{el} = \frac{Ne}{h}\int d\varepsilon \mathcal{T}(\varepsilon)\Delta f(\varepsilon), \quad \mathcal{T}(\varepsilon) = \left(1-\mathcal{C}^2\right)\sin^2[\delta(\varepsilon)]. \tag{S5}$$

Computing integrals with Fermi distribution function we obtain the elastic current:

$$I_{el} = \frac{Ne}{h}\left(1-\mathcal{C}^2\right)\left[(\mathcal{T}_0 - \sin 2\delta_0\alpha_2\mathcal{A})\mathcal{J}_0 - \sin 2\delta_0\alpha_1\mathcal{J}_1 + (\cos 2\delta_0\alpha_1^2 + \sin 2\delta_0\alpha_2)\mathcal{J}_2\right], \tag{S6}$$

where we use short-hand notations:

$$\begin{aligned} \mathcal{J}_0 &= e\Delta V, \quad \mathcal{J}_1 = \frac{1}{6}\left[(\pi T_L)^2 - (\pi T_R)^2 - 3(e\Delta V)^2\mathcal{C}\right], \\ \mathcal{J}_2 &= \frac{e\Delta V}{6}\left[(\pi T_L)^2(1-\mathcal{C}) + (\pi T_R)^2(1+\mathcal{C}) + \frac{1}{2}(e\Delta V)^2(1+3\mathcal{C}^2)\right]. \end{aligned} \tag{S7}$$

D. Inelastic current

For computing the inelastic contribution to the current we use the general equation for the self-energies

$$\Sigma^{\eta_1,\eta_2}(t) = \left(\frac{\phi_1}{\pi\nu^2}\right)^2 \sum_{k_1,k_2,k_3} G_{bb}^{\eta_1,\eta_2}(k_1,t) G_{bb}^{\eta_2,\eta_1}(k_2,-t) G_{bb}^{\eta_1,\eta_2}(k_3,t) \tag{S8}$$

expressed in terms of the fermionic Green's functions (which replace (12) in [S1]):

$$G_{bb}^{+-}(t) = -\frac{\pi\nu}{2}\left[\frac{T_L(1+\mathcal{C})e^{-i\mu_L t}}{\sinh(\pi T_L t)} + \frac{T_R(1-\mathcal{C})e^{-i\mu_R t}}{\sinh(\pi T_R t)}\right], \tag{S9}$$

$$G_{ab/ba}(t) = -\frac{\pi\nu}{2}\sin 2\theta \left[\frac{T_L e^{-i\mu_L t}}{\sinh(\pi T_L t)} - \frac{T_R e^{-i\mu_R t}}{\sinh(\pi T_R t)}\right], \tag{S10}$$

where $\mu_L = \frac{e\Delta V}{2}(1-\mathcal{C})$ and $\mu_R = -\frac{e\Delta V}{2}(1+\mathcal{C})$. We also take into account the coefficient $\sin 2\theta$ in front of the definition of the total current in Eq. 6 and $\sin^2 2\theta$ in front of the inelastic current in Eq. 11 in [S1].

The inelastic current for symmetrical dot-lead coupling is given by the relation:

$$\delta I_{in} = \frac{N(N-1)e\pi}{2h}\left(\frac{\phi_1}{\pi\nu^2}\right)^2 2\cos 2\delta_0 \left(\frac{\pi\nu}{2}\right)^4 \times \left\{[\mathcal{L}(T_L,T_R,z) - \mathcal{L}(T_L,T_R,0)] - [\mathcal{L}(T_R,T_L,-z) - \mathcal{L}(T_R,T_L,0)]\right\}, \tag{S11}$$

where

$$\mathcal{L}(x,y,z) = \int_{-\infty-i\gamma}^{+\infty-i\gamma} \left[x^4 \frac{e^{-izt}+e^{2izt}}{\sinh^4(\pi x t)} + x^3 y \frac{3+2e^{izt}-e^{-2izt}}{\sinh^3(\pi x t)\sinh(\pi y t)} + \frac{3}{2}x^2y^2 \frac{e^{izt}-e^{-izt}}{\sinh^2(\pi x t)\sinh^2(\pi y t)}\right] dt, \tag{S12}$$

Here we denoted z=$e\Delta V$ and introduced the point splitting parameter γ [S5, S9] to regularize the integrals (S12) divergent at t=0. The parameter γ is chosen to satisfy the conditions $\gamma e\Delta V \ll 1$, $\gamma T \ll 1$ and $T_K/D \ll \gamma T_K \ll 1$ [S5] (D is a bandwidth of conduction band, $T \ll T_K$, $T_K/D \propto \sqrt{(\Gamma_L+\Gamma_R)/D}\exp[-c\cdot U/(\Gamma_L+\Gamma_R)]$, $c \sim 1$).

We show on Fig. S1 the thermo-voltage ΔV as a function of two temperatures T_L and T_R of the left-right leads for three important cases discussed in the paper: i) $m=1$ SU(2); ii) $m=1$ SU(4) and iii) $m=2$ SU(4). One can see similarity of the plots i) and iii) describing the broken by potential scattering particle-hole symmetric regimes. The density plot visualises the non-linearity of the thermo-voltage at low compared to T_K temperatures.

II. TRANSPORT INTEGRALS

We illustrate the application of the textbook [S10, S11] method of transport integrals to the thermoelectric transport through the SU(N) quantum impurity assuming the symmetric dot-leads coupling for simplicity. The

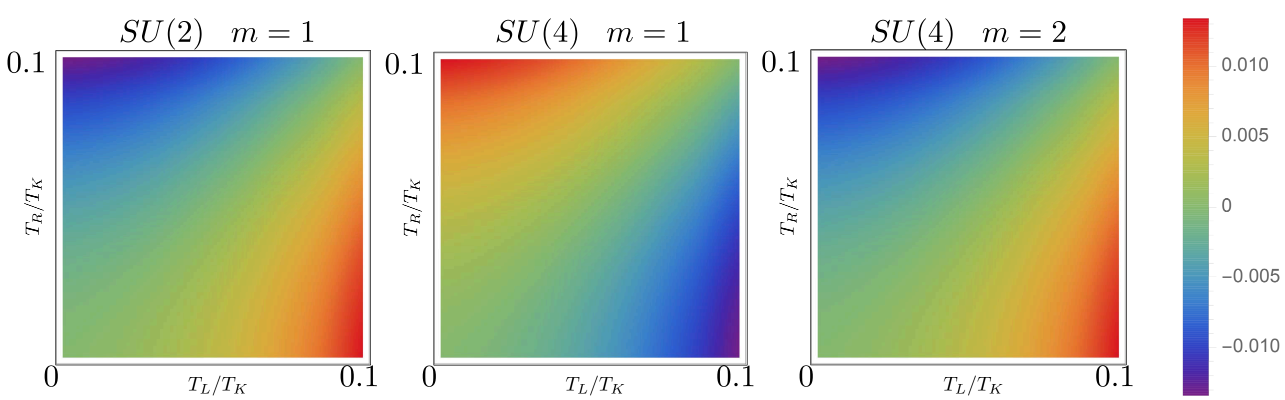

FIG. S1. (Color online) Density plot showing the thermo-voltage $e\Delta V/T_K$ obtained at the zero-current conditions as a function of the temperatures of L/R contacts. Left panel: SU(2) PH-symmetric Kondo regime $m=1$; central panel: SU(4) PH-non-symmetric Kondo regime $m=1$; right panel: SU(4) PH-symmetric Kondo regime $m=2$; for all plots: δ_P=0.3, γT_K =0.001 and $\mathcal{C}=0$.

charge and the heat currents in the linear response theory are connected by equations:

$$\begin{pmatrix} I_{charge} \\ I_{heat} \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta V \\ \Delta T \end{pmatrix}. \tag{S13}$$

Differential conductance $G(T)$ and differential thermopower $S(T)$ are defined as follows:

$$G(T) = \lim_{\Delta V \to 0} \left. \frac{I_{charge}}{\Delta V} \right|_{\Delta T=0} = L_{11}, \tag{S14}$$

$$S(T) = -\lim_{\Delta T \to 0} \left. \frac{\Delta V}{\Delta T} \right|_{I_{charge}=0} = L_{12}/L_{11}. \tag{S15}$$

The coefficients L_{ij} are expressed in terms of the transport integrals (see e.g.[S11] for details of the derivation):

$$\mathcal{I}_n(T) = \frac{1}{h} \sum_\sigma \int d\varepsilon \cdot \varepsilon^n \left(-\frac{\partial f}{\partial \varepsilon} \right) \cdot \mathrm{Im}\left[-\pi\nu T_\sigma(\varepsilon) \right]. \tag{S16}$$

Here $L_{11} = e^2 \mathcal{I}_0$, $L_{12} = -e\mathcal{I}_1/(2T)$ and T_σ is a diagonal part of a single-particle T-matrix defined by the Dyson equation [S4]:

$$\mathcal{G}_{\sigma k}(\varepsilon) = \mathcal{G}^0_{\sigma k}(\varepsilon) + \mathcal{G}^0_{\sigma k}(\varepsilon) T_\sigma(\varepsilon) \mathcal{G}^0_{\sigma k}(\varepsilon), \tag{S17}$$

where $\mathcal{G}^0_{\sigma k}(\varepsilon)$ and $\mathcal{G}_{\sigma k}(\varepsilon)$ are bare and full electron Green's functions.

The full T-matrix consists of the elastic part

$$T^{el}_\sigma(\varepsilon) = -\frac{i}{2\pi\nu} \left(1 - e^{2i\delta_\sigma(\varepsilon)} \right), \tag{S18}$$

and the inelastic part

$$T^{in}_\sigma(\varepsilon) = -\frac{i}{2\pi\nu} (N-1) e^{2i\delta_0} \phi_1^2 \left[\varepsilon^2 + (\pi T)^2 \right]. \tag{S19}$$

Here we used the FL Hamiltonian (6) of [S1] describing the SU(N) Kondo model at the strong coupling fixed point. We compute the transport integrals by Taylor-expanding the T-matrix

$$-\pi\nu \mathrm{Im} T_\sigma(\varepsilon) = \mathcal{A}_1 + \mathcal{A}_2 \varepsilon + \mathcal{A}_3 \varepsilon^2 + ... \tag{S20}$$

where the coefficients $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$ are defined as follows:

$$\mathcal{A}_1 = \left[\mathcal{T}_0 + \frac{(\pi T)^2 (N-1)}{2} \left(\phi_1^2 \cos 2\delta_0 - \sin 2\delta_0 \frac{\phi_2}{6} \right) \right], \tag{S21}$$

$$\mathcal{A}_2 = \alpha_1 \sin 2\delta_0, \tag{S22}$$

$$\mathcal{A}_3 = \left[\cos 2\delta_0 \left(\alpha_1^2 + \frac{(N-1)\phi_1^2}{2} \right) + \sin 2\delta_0 \alpha_2 \right]. \tag{S23}$$

The transport integrals for $n = 0, 1, 2$ are given by equations:

$$\mathcal{I}_0 = \frac{N}{h} \left[\mathcal{A}_1 + \frac{\mathcal{A}_3}{3} (\pi T)^2 \right], \qquad \mathcal{I}_1 = \frac{N \mathcal{A}_2}{3h} (\pi T)^2, \qquad \mathcal{I}_2 = \frac{N}{h} \left[\frac{\mathcal{A}_1}{3} (\pi T)^2 + \frac{7\mathcal{A}_3}{15} (\pi T)^4 \right]. \tag{S24}$$

The transport coefficients: the electrical conductance $G(T)$, thermopower $S(T)$ and thermal conductance $K_e(T)$ read as:

$$G = e^2 \mathcal{I}_0, \quad S = -\frac{1}{eT} \frac{\mathcal{I}_1}{\mathcal{I}_0}, \quad K_e = \frac{1}{T} \left[\mathcal{I}_2 - \frac{\mathcal{I}_1^2}{\mathcal{I}_0} \right]. \tag{S25}$$

The electronic contribution to the thermoelectric figure of merit ZT and the normalized power factor PF are expressed in terms of thermoelectric properties defined in Eq.(S25) via $ZT = S^2 GT/K_e$ and $PF = S^2 G/G_0$.

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