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Dynamical symmetries and quantum transport through nanostructures

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We discuss the manifestation of dynamical symmetries in quantum transport through nanostructures. The dynamical symmetry $SO(4)$ manifested in the singlet-triplet excitations is shown to be responsible for several exotic effects in nano-devices: non-equilibrium Kondo effect in T-shape Double Quantum Dots, phonon-induced Kondo effect in transition-metal-organic complexes, Kondo shuttling in Nano-Electro-mechanical Single Electron Transistor. We consider the interplay between charge $U(1)$ and spin $SU(2)$ fluctuations in the vicinity of Stoner instability point and a non-monotonic behavior of a Tunneling Density of States in metallic quantum dots. The experiments showing important role of dynamical symmetries in nanostructures are briefly reviewed.

1 Introduction

The single electron tunneling through the quantum dot is studied in many details during the recent decade. Quantum dot [1, 2], being a little semiconductor box, can hold a small number of electrons. Quantum dots are often called artificial atoms since their electronic properties resemble those of real atoms. A voltage applied to one of the gate electrodes of semiconductor device controls the number of electrons in the dot. If such number is small (about 10), the properties of the dot are similar to the properties of the atom with few electrons. If the number of electrons is relatively large (about 100 and more), the statistics of levels in such artificial atom is determined by the Random Matrix Theory [2]. The system of coupled quantum dots is similar to an artificial molecule. Fabricating semiconductor devices with different geometries allows to consider different symmetries associated with the artificial molecule structure. There are many interesting effects already experimentally observed in quantum dots structures and many theoretical predictions which still wait for the experimental confirmation.

Among many interesting phenomena behind the unusual transport properties of mesoscopic systems is the Kondo effect in quantum dots, recently observed experimentally [3]. If the number of electrons trapped within a dot is odd, the total spin of a dot S is necessarily nonzero and has a minimum value at $S = 1/2$. Thus the system resembles a local spin interacting with electron seas associated with metallic reservoirs (leads). The Kondo effect is a result of exchange interaction of itinerant electrons in leads with the localized spin state in a dot. Being responsible for local spin polarization of the electron gas, the resonance Kondo scattering becomes significant at low temperatures. The antiferromagnetic exchange interaction between itinerant electrons and local spins gives rise to possibility of simultaneous change of spin projection both for electron and spin. As a result, the "Kondo cloud" is formed out when the temperature goes to zero. Formation of the "Kondo cloud" or "spin cloud" is due to the screening effects, associated with the

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processes when the free electron gas tends to screen the magnetic moments of the localized state. This phenomenon is responsible for the non-monotonic temperature behavior of the resistivity of metals with magnetic impurities at low temperatures (e.g. in many heavy fermion (HF) compounds). The inter-metallic alloys and HF compounds usually contain rare-earth atoms, which makes it difficult to produce the sample with adjustable parameters. In contrast, the modern nanoscience technologies allow one to produce the highly controllable systems based on quantum dot devices and possessing many of properties of strongly correlated electron systems.

Yet another interesting property of quantum dots is associated with the magnetic correlations between electrons confined in this nano-scale object. The question whether the itinerant ferromagnetism may occur in finite-size system in a presence of disorder attracts a lot of attention of theorists (see, e.g. [5]) and experimentalists [6]. The simplest model of ferromagnetism in metallic systems was proposed by Stoner long time ago [4]. The physical mechanism behind the instability is analogous to the familiar from atomic physics Hund's rule. The magnetic ordering in this model takes place when the increase of orbital energy due to promotion of electrons to higher energy states is smaller than the energy gain due to the exchange interaction. As soon as this happens, the system becomes unstable with respect to the transition to a state with the nonzero total magnetization. In contrast to bulk magnetic system where both orbital and exchange energies are self-averaging quantities, in a small (mesoscopic) systems they are sample specific. Therefore, one may expect strong mesoscopic fluctuations in a metallic quantum dots in the vicinity of the Stoner instability point. It is known, that in disordered metals the interaction between electrons in states which are close in energy is enhanced due to increase return probability and may satisfy the instability criterion for a weaker bare interaction. However, the question how the properties of isolated quantum dot as well as transport properties are affected by the disorder remains open.

The main goal of this paper is to demonstrate the relevance of dynamical symmetries [7] on the transport through the nanostructures and review last years progress in the analysis of the interplay between symmetries and strong correlations at the nanoscale. We elaborate on the role of dynamical symmetries in a special discipline of condensed matter physics, which enters under the name of correlated impurity problem. It is concerned with the physics which is exposed when the system is composed of strongly correlated localized electrons on the one hand and itinerant electrons on the other hand. We will review the role of dynamical symmetries and its manifestation in quantum dots, semiconducting nano-clusters and molecular electronic devices. As it turns out, the concept of dynamical symmetries is meaningful also in systems out of equilibrium, and the case of dynamical symmetries at finite frequencies will also be addressed.

The paper is organized as follows. In the Section 1 discuss how dynamical symmetries affect the transport through the semiconductor few-electron quantum dots and the molecular transistors. Section 2 is devoted to a description of magnetic instability in the metallic quantum dots. In conclusions the summary of discussions will be given and the perspectives of future research will be discussed.

2 Kondo effect and dynamical symmetries

2.1 Double qantum dot

The first example of the manifestation of dynamical symmetries is the non-equilibrium Kondo effect predicted [8, 10] and observed [11] in nano-structures characterized by the interplay between singlet/triplet excitations. An example of such system is a double quantum dot (Fig. 1, left panel). As was noticed in Ref. [12], quantum dots with even $\mathcal{N}$ possess the dynamical symmetry $SO(4)$ of *spin rotator* in the Kondo tunneling regime, provided the low-energy part of excitation spectrum is formed by a singlet-triplet (ST) pair, and all other excitations are separated from the ST manifold by a gap noticeably exceeding the tunneling rate Γ. A DQD with even $\mathcal{N}$ in a side-bound (T-shape) configuration where two wells are coupled by the tunneling v and only one of them (say, l) is coupled to metallic leads (L, R) is a simplest system satisfying this condition [12]. Such system was realized experimentally in Ref. [13]. Novel features introduced by the dynamical symmetry in Kondo tunneling are connected with the fact that unlike the case

of conventional $SU(2)$ symmetry of spin vector $\mathbf{S}$, the $SO(4)$ group possesses two generators $\mathbf{S}$ and $\mathbf{P}$. The latter vector describes transitions between singlet and triplet states of spin manifold (this vector is an analog of Runge-Lenz vector describing the hidden symmetry of hydrogen atom). As was shown in Ref. [14], this vector alone is responsible for Kondo tunneling through quantum dot with even $\mathcal{N}$ induced by external magnetic field. The Hamiltonian describing Kondo effect in DQD is given by

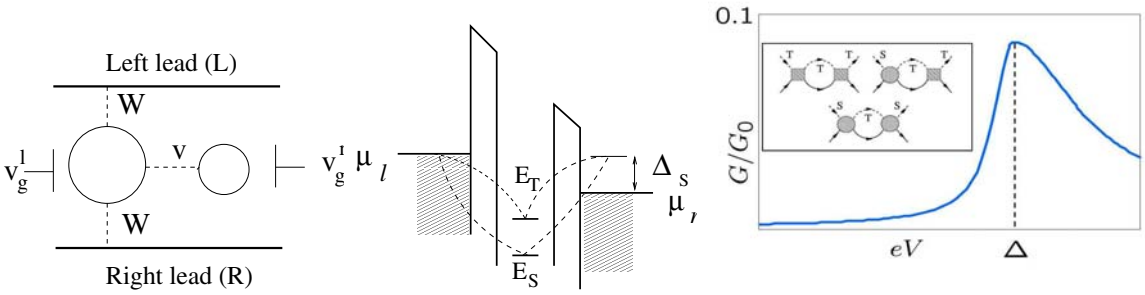

Fig. 1 (Color online) Left panel: Double quantum dot in a side-bound configuration. Central panel: cotunneling processes in biased DQD responsible for the resonance Kondo tunneling. Right panel: Typical shape of the differential conductance as the function of source-drain bias eV. The asymmetry of the conductance peak is attributed to finite repopulation of the triplet state for $eV > \Delta$ and also due to additional decoherence mechanisms associated with the triplet-triplet relaxation [8]. Insert shows the RG equations for the coupling constants defined in (1). The solution of the RG equation is given by (2).

$$H_{int} = \sum_{\alpha\alpha'} [(J^T_{\alpha\alpha'}\mathbf{S} + J^{ST}_{\alpha\alpha'}\mathbf{P}) \cdot \mathbf{s}_{\alpha\alpha'} + J^S_{\alpha\alpha'} N^{SS} n_{\alpha\alpha'}] \tag{1}$$

Here $\mathbf{s}_{\alpha\alpha'}=\sum_{kk'} c^\dagger_{k\alpha\sigma}\hat{\tau}c_{k'\alpha'\sigma'}$, $n_{\alpha\alpha'}=\sum_{kk'} c^\dagger_{k\alpha\sigma}\hat{1}c_{k'\alpha'\sigma}$, $\hat{\tau}$, $\hat{1}$ are the Pauli matrices and unity matrix respectively. The constants J^T, J^{ST} and J^S stand for the interaction between the total spin of the dot S, and the electrons in the leads $\alpha = L, R$.

We deal with the case, which was not met in the previous studies of non-equilibrium Kondo tunneling. The ground state of the system is singlet, and the Kondo tunneling in equilibrium is quenched at $T \sim \Delta$, where $\Delta = E_T - E_S$ stands for the energy difference between the ground (singlet) and excited (triplet) states. Thus, the elastic Kondo tunneling arises only provided $T_K \gg \Delta$ in accordance with the theory of two-impurity Kondo effect [12, 15]. However, the energy necessary for spin flip may be donated by external electric field eV applied to the left lead, and in the opposite limit $T_K \ll \Delta$ the elastic channel emerges at $eV \approx \Delta$. The processes responsible for resonance Kondo cotunneling at finite bias are shown in Fig. 1 (central panel).

In conventional spin $S = 1/2$ quantum dots the Kondo regime out of equilibrium is affected by spin relaxation and decoherence processes, which emerge at $eV \gg T_K$ (see, e.g., [16–19]). These processes appear in the same order as Kondo co-tunneling itself, and one should use the non-equilibrium perturbation theory (e.g., Keldysh technique) to take them into account in a proper way. In our case these effects are expected to be weaker, because the nonzero spin state is involved in Kondo tunneling only as an intermediate virtual state arising due to S/T transitions induced by the second term in the Hamiltonian (1), which contains vector $\mathbf{P}$. The effects of repopulation of triplet state by external bias eV have been considered in details in [8]. It has been found that for $0 \leq eV \leq \Delta$ the repopulation of the triplet state is exponentially small. For large biases $eV > \Delta$ the effects of repopulation of the triplet state lead to inelastic cotunneling and should be taken into account. These effects [8] result in strong anisotropy of conductance line-shape (Fig. 1, right panel). The methods implemented for a weak-coupling Kondo regime in [8] are not applicable for the quantitative and even qualitative description of nonequilibrium

effects (full nonlinearconductance etc) at large biases $eV \gg \Delta$. The kinetic equation approach based on Schwinger-Keldysh diagrammatics [9, 10] might be appropriate tool to describe the regime of strong out of equilibrium in double quantum dots.

Having this in mind, we describe Kondo tunneling through DQD at finite $eV \sim \Delta$ within the quasi-equilibrium perturbation theory in a weak coupling regime (cf. the quasi-equilibrium approach to description of decoherence rate at large eV in Ref. [17]).

Using a Renormalization Group (RG) technique [8] (see also the insert in the right panel of Fig. 1) based on semi-fermionic representation of $SO(4)$ group generators [9,10,20] we find the following scaling dependencies of the exchange integrals:

$$J^{T}_{\alpha,\alpha'} = \frac{J^{T}_{0}}{1-\rho J^{T}_{0}\ln(D/T)},\ J^{ST}_{\alpha,\alpha'} = \frac{J^{ST}_{0}}{1-\rho J^{T}_{0}\ln(D/T)},\ J^{S}_{LR} = J^{S}_{0} - \frac{3}{4}\rho (J^{ST}_{0})^{2}\frac{\ln(D/T)}{1-\rho J^{T}_{0}\ln(D/T)}. \tag{2}$$

Here $\alpha = L$, $\alpha' = L, R$ and ρ is the density of states at the Fermi level of the contacts characterized by effective bandwidth D. One should note that the Kondo temperature is determined by triplet-triplet processes only in spite of the fact that the ground state is singlet. One finds from (2) that $T_K{=}D\exp[-1/(\rho J^{T}_{0})]$. This temperature is noticeably smaller than the "equilibrium" Kondo temperature T_{K0}, which emerges in tunneling through triplet channel in the ground state, namely $T_K \approx T^{2}_{K0}/D$. The reason for this difference is the reduction of usual parquet equations for T_K to a simple ladder series. In this respect our case differs also from conventional Kondo effect at strong bias [17], where the non-equilibrium Kondo temperature $T^{*} \approx T^{2}_{K0}/eV$ arises. In our model the finite bias does not enter T_K because of the compensation $eV \approx \Delta$ in spite of the fact that we take the argument $\omega = eV$ in our RG equations [8].

The differential conductance $G(eV,T)/G_0 \sim |J^{ST}_{LR}|^2$ (cf. Ref. [21]) is the universal function of two parameters T/T_K and eV/T_K, $G_0 = e^2/\pi\hbar$:

$$G/G_0 \sim \ln^{-2}\left(\max[(eV-\Delta),T]/T_K\right) \tag{3}$$

The resonance tunneling "flashes" at $eV \sim \Delta$ and dies away out of this resonance (Fig. 1, right panel). The decoherence effects for the finite bias processes have been studied in [8]. In [24] the gauge theory unifying the decoherence processes associated with the fluctuations of the constraint and exchange integrals has been constructed . It was shown that these processes do not suppress the Kondo effect in the weak coupling regime and the non-equilibrium peak in differential conductance remains intact.

An interesting question which arises here is whether the non-equilibrium Kondo effect falls into the class of strong-coupling regime. It has been extensively studied during the last few years (see discussion and references in [18, 19]). The same question when addressed to systems characterized by hidden dynamical symmetries allows a simple and straightforward answer: the strong coupling limit is not achievable in this situation. There is always an energy scale determined by an external bias, decoherence effects associated with AC or effects related to repopulation of the dot which prevent the system from both one-stage and two-stage Kondo scenario [22, 23] and suppress the Kondo effect in the ground state.

2.2 **Molecular transistor**

Another example of manifestation of dynamical symmetries is given by a Transition-Metal Organic Complexes (TMOC) with a transition metal ion secluded in a ligand cage [25, 26]. The cage is in tunnel contact with metallic reservoirs (surface, STM nanotip, or edges of metallic wire in electro-migration or break junction geometry). The left panel of Fig. 2 illustrates this setup. We consider a TMOC with even electron number N fixed by charge and energy quantization. Tunneling through single-molecule devices is a complex phenomenon involving vibrational motion and many-particle processes in metallic leads. Vibrational effects have been observed in the sequential [27–29] and strong tunneling regime [30–32]. Phonon satellites coexist with resonance Kondo co-tunneling [30] in transport through transition-metal (TM) organic complexes (TMOC). Kondo effect is a direct manifestation of strong correlation effects in tunneling [33].

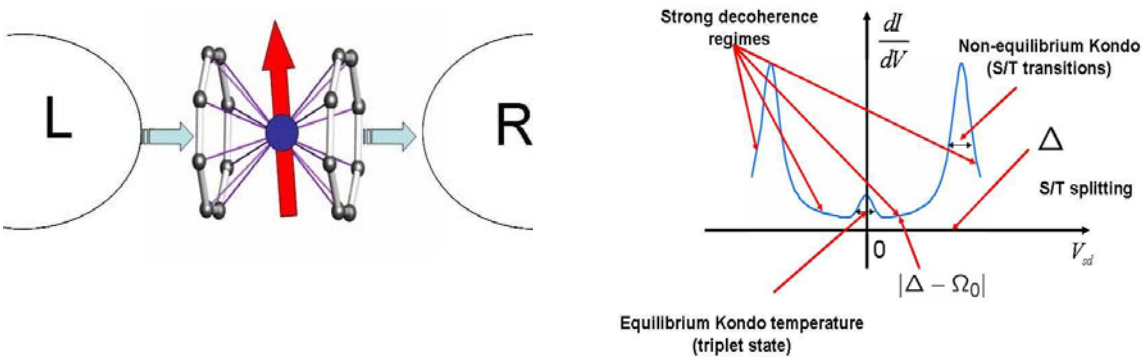

Fig. 2 (Color online) Left panel: Electrode tunnel-coupled to a Transition-Metal Organic Complex. Charging of the complex by a tunnel process deforms the outer part of the ligand cage without strongly affecting the direct coordination-sphere of the metal ion and thereby the ligand-field splitting. We assume that the extra electron is localized mainly on the cage. Electrons tunnel onto the ion through the tails of the molecular state centered on the ion, which includes admixtures of the outer shell electronic states. Therefore the main effect of the charging is the *modulation of the tunnel barrier* between the ion-centered states and electrode. Right panel: Typical shape of the differential conductance as the function of source-drain bias eV. The central peak corresponds to Zero-Bias Anomaly while the two finite bias peaks are attributed to the non-equilibrium Kondo effect.

The ground state of TMOC is supposed to be a spin singlet, and the energy of the lowest triplet excitation Δ exceeds Kondo temperature T_K. The linear conductance is thus suppressed. To investigate how intramolecular vibrations may *induce* transport through a Kondo effect, in the first place one should incorporate a vibronic mode in a generic tunneling Hamiltonian

$$H = H_{mol} + H_{res} + H_{tun} \tag{4}$$

Here H_{mol} includes the $3d$ electron levels in a ligand field of the cage electrons, the molecular orbitals of these ligands, as well as interactions within the $3d$ shell and within the cage. One should take into account the three most relevant charge states including their dependence on the vibrational coordinate of the cage Q:

$$H_{mol} = H_Q^{(N)} + H_Q^{(N+1)} + H_Q^{(N-1)} + T_n \tag{5}$$

The last term T_n is the kinetic energy of the cage distortion. The eigenstates of $H_Q^{(N\pm1)}$ are admixed to those of $H^{(N)}$ by the tunneling H_{tun} of electrons from the reservoir H_{res}.

The effective Hamiltonian accounting for dynamical $SO(4)$ symmetry has the form

$$H_{eff} = H_{res} + \frac{1}{2}\Delta \mathbf{S}^2 + J^T \mathbf{S}\cdot\mathbf{s} + J^{ST}\mathbf{P}\cdot\mathbf{s} + T_n \tag{6}$$

The electron spin operator is given by the conventional expansion $\mathbf{s} = \frac{1}{2}\sum_{kk'}\sum_{\sigma\sigma'} c^\dagger_{k\sigma}\boldsymbol{\tau}_{\sigma\sigma'}c_{k'\sigma'}$ where $\boldsymbol{\tau}$ is the Pauli vector. The exchange coupling constants $J^{T,ST}(Q)$ describe the Q-dependent resonance scattering of the electrons in the leads on the S-T $SO(4)$ multiplet. The main source of phonon emission/absorption in our case is the tunneling rate. Expanding $J^{ST}(Q) = J^{ST} + j_P Q$ in the quantized displacement operator $Q = (b^\dagger + b)/\sqrt{2}$ and assuming that the optical phonon mode is approximated by single Einstein phonon with frequency Ω we come to phonon assisted exchange Kondo Hamiltonian.

We calculate the phonon assisted Kondo-renormalization of the differential conductance $dI/dV \sim |\gamma|^2$ [25,26] using the RG technique developed in [8]:

$$\gamma \sim (j_P)^2 \rho \left[\frac{\log\left(\frac{D}{\max[T, |\Delta - \hbar\Omega|]}\right)}{1 - J^T A\rho \log\left(\frac{D}{\max[T, |\Delta - \hbar\Omega|]}\right)} \right] \tag{7}$$

Here $A{\sim}1$ is a constant determined by spin algebra, D is the effective width of the electron conduction band and ρ is the density of states on the Fermi level. The Kondo temperature extracted from this equation reads $T_K{\sim}D\exp\bigl(-1/(A\rho J^T)\bigr)$. One concludes from these calculations that the single-phonon processes are sufficient to compensate the energy of the S/T splitting and induce resonance tunneling through the TMOC provided the local vibration mode with appropriate frequency satisfying the condition

$$|\hbar\Omega - \Delta| \sim T_K \tag{8}$$

exists in the cage. One can expect in this case a significant enhancement of the tunnel conductance already at $T > T_K$ according to the law $G/G_0 \sim \ln^{-2}(T/T_K)$ [33], where G_0 is the conductance at unitarity limit $T \to 0$. We emphasize that in spite of the fact that the Kondo effect exists in our case *only under phonon assistance* (c.f. [34]), the Kondo temperature T_K is *the same* as in the usual Kondo effect. Since T_K is high enough ($\sim$ 10 K) in electro-migrated junction experiments with a TMOC deposited between contacts [30, 31], the effect predicted in this work seems to be easily observable. The crucial point is the existence of phonon satisfying condition (15) in a TMOC with the S/T multiplet as a lowest spin excitation. One should note, however, that even if this condition is not exactly satisfied, one may tune the system by applying the magnetic field. Then the triplet is split, and only the level $E_{T,-1} = E_T - E_Z$ is involved in the phonon induced Kondo tunneling (E_Z is the Zeeman energy). In this case Δ in (15) is substituted for $\Delta_Z = \Delta - E_Z$, and E_Z may be tuned to satisfy the inequality (15). Thus the vibration gives rise to a magnetic field induced Kondo effect at Zeeman energies which can be much smaller than Δ. The only difference is that in this case the effective spin of the TMOC is one half instead of one [35].

The differential conductance as a function of the bias eV is shown in Fig. 2 (right panel). The central peak is suppressed at $eV{\sim}T_K$ due to the decoherence effects associated with the electrical current across the TMOC. The conductance grows again at $eV{\to}\Delta$, due to *non-equilibrium effects* occurring when the resonance tunneling is restored at $eV{=}\Delta$ [8]. One of essential ingredients of our theory is that we use the dynamical symmetry of the TMOC, which characterizes both the spin algebra of localized spin itself and transitions between various levels of different spin multiplets [12].

2.3 **Kondo shuttle**

The Nano-Electromechanical (NEM) devices represent yet another class of nanostructures where the manifestations of dynamical symmetries can be seen in the transport experiments [36]. Building on the analogy with shuttling experiments of [37, 38], we consider the device where an isolated nanomachined island oscillates between two electrodes (Fig.3, left panel). We, however, are interested in a regime where the applied voltage is low enough so that the field emission of many electrons, which was the main mechanism of tunneling in those experiments, should be neglected. Note further that the characteristic de Broglie wave length associated with the dot should be much shorter than typical displacements allowing thus for a classical treatment of the mechanical motion of the nano-particle. The condition $\hbar\Omega{\ll}T_K$, necessary to eliminate decoherence effects, requires for e.g. planar quantum dots with the Kondo temperature $T_K{\sim}100$ mK, the condition $\Omega{\sim}1$ GHz for oscillation frequencies to hold; this frequency range is experimentally feasible [37, 38]. The shuttling island then is to be considered as a "mobile quantum impurity", and transport experiments will detect the influence of mechanical motion on a differential conductance. If the dot is small enough, then the Coulomb blockade guarantees the single electron tunneling or cotunneling regime, which is necessary for realization of Kondo effect [33, 39]. Cotunneling process is accompanied by the

change of spin projection in the process of charging/discharging of the shuttle and therefore is closely related to the spin/charge pumping problem [40].

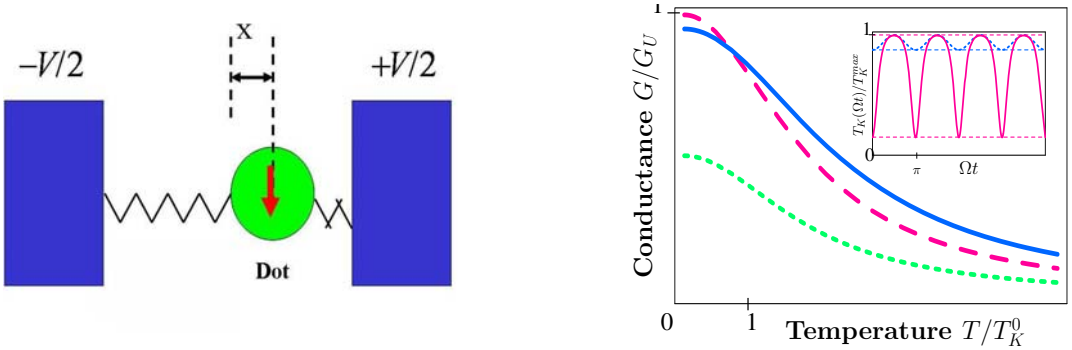

Fig. 3 (Color online) Left panel: Nanomechanical resonator with the odd number of electrons as a "mobile quantum impurity". Right panel: Differential conductance of a Kondo shuttle Γ_0/U=0.4. Solid line denotes G for the shuttle Γ_L=Γ_R, A=λ_0, dashed line: the static nano-island $\Gamma_L = \Gamma_R$, A=0, dotted line: Γ_L/Γ_R=0.5, A=0. Insert shows the time oscillations of T_K for small A=0.05λ_0 (dotted line) and large A=2.5λ_0 (solid line) shuttling amplitudes.

The Hamiltonian of the shuttle H=H_0+H_{tun} is given by

$$H_0 = \sum_{k,\alpha} \varepsilon_{k\sigma,\alpha} c^\dagger_{k\sigma,\alpha} c_{k\sigma,\alpha} + \sum_{i\sigma} [\epsilon_i - e\mathcal{E}x] d^\dagger_{i\sigma} d_{i\sigma} + Un^2$$
$$H_{tun} = \sum_{ik\sigma,\alpha} T^{(i)}_\alpha(x) [c^\dagger_{k\sigma,\alpha} d_{i\sigma} + H.c], \tag{9}$$

where $c^\dagger_{k\sigma}$, $d^\dagger_{i\sigma}$ create an electron in the lead α=L,R, or the dot level $\varepsilon_{i=1,2}$, respectively, n=$\sum_{i\sigma} d^\dagger_{i\sigma} d_{i\sigma}$, $\mathcal{E}$ is the electric field between the leads. The tunnelling matrix element $T^{(i)}_{L,R}(x)$= $T^{(i,0)}_{L,R} \exp[\mp x(t)/\lambda_0]$, depends exponentially on the ratio of the time-dependent displacement $x(t)$ (which is considered to be a given harmonic function of the time) and the electronic tunneling length λ_0.

We begin with the discussion of an odd N, $S = 1/2$, case. Then only the state with i=1 retains in (9), and hereafter we omit this index. In order to find an analytic solution, we assume that if $x(t)$ varies adiabatically slow (on the scale of the tunneling recharging time), there is no charge shuttling due to multiple recharging processes [41], but the Kondo resonance cotunneling occurs. The time-dependent tunneling width is $\Gamma_\alpha(t) = 2\pi\rho_0 |T_\alpha(x(t))|^2$ [21], where ρ_0 is the density of states at the leads Fermi level. The adiabaticity condition reads: $\hbar\Omega \ll T_K \ll \Gamma$, with Γ=min$[\sqrt{\Gamma^2_L(t) + \Gamma^2_R(t)}]$. We apply the time-dependent Schrieffer-Wolff transformation and obtain the time-dependent Kondo Hamiltonian [21] as

$$H = H_0 + \sum_{k\alpha\sigma,k'\alpha'\sigma'} \mathcal{J}_{\alpha\alpha'}(t) [\vec{\sigma}_{\sigma\sigma'} \vec{S} + \frac{1}{4}\delta_{\sigma\sigma'}] c^\dagger_{k\sigma,\alpha} c_{k'\sigma',\alpha'} \tag{10}$$

where $\mathcal{J}_{\alpha,\alpha'}(t)$=$\sqrt{\Gamma_\alpha(t)\Gamma_{\alpha'}(t)}/(\pi\rho_0 E_d(t))$ and $\vec{S}$=$\frac{1}{2} d^\dagger_\sigma \vec{\sigma}_{\sigma\sigma'} d_{\sigma'}$. In the adiabatic regime the time can be treated as an external parameter, and the renormalization group equations for the Hamiltonian (10) can be solved the same manner as those for the equilibrium [21]. As a result, the Kondo temperature becomes time oscillating:

$$T_K(t) = D(t) \exp\left[-\frac{\pi U}{8\Gamma_0 \cosh(2x(t)/\lambda_0)} \right]. \tag{11}$$

Neglecting the weak time-dependence of the effective bandwidth $D(t)\approx D_0$, we arrive at the following expression for the time-averaged Kondo temperature:

$$\langle T_K\rangle = T_K^0\left\langle \exp\left[\frac{\pi U}{4\Gamma_0}\frac{\sinh^2(x(t)/\lambda_0)}{1+2\sinh^2(x(t)/\lambda_0)}\right]\right\rangle. \tag{12}$$

Here $\langle ...\rangle$ denotes averaging over the period of the mechanical oscillation. The time-dependence of the Kondo temperatures results in a remarkable effect: the competition between Breit-Wigner (BW) resonance responsible for the transparency of the nano-device and the Abrikosov-Suhl (AS) resonance related to the quasi-particle Kondo peak occurring at the Fermi level of the leads. While BW resonance has a maximum at the most symmetrical regime $\Gamma_L=\Gamma_R$ corresponding to the central position of the island in its classical trajectory, the sharpest AS resonance occurs at the turning points of the trajectory when the Kondo temperature reaches its minimal value. In the weak coupling regime $T_K^{max}\ll T\ll D_0$ the zero bias anomaly (ZBA) in the tunneling conductance is given by

$$G(T)=\frac{3\pi^2}{16}G_U\left\langle \frac{4\Gamma_L(t)\Gamma_R(t)}{(\Gamma_L(t)+\Gamma_R(t))^2}\frac{1}{[\ln(T/T_K(t))]^2}\right\rangle. \tag{13}$$

The two competing effects of BW and AS resonances lead to the effective enhancement of G at high temperatures (see Fig.3, right panel):

$$\frac{\delta G_K}{G_K^0}=\frac{G(T)-G_K^0}{G_K^0}=2\frac{\delta T_K}{T_K^0}\frac{1}{\ln(T/T_K^0)}. \tag{14}$$

Next we turn to the case of even N in the island. In this case one may refer to the *excited-state Kondo features* [39], where the KR tunneling is possible only during the time intervals where

$$\Delta_{ST}(t)=\delta(t)-J_{ex}(t)<T_K(t). \tag{15}$$

The level spacing $\delta(t)=\epsilon_2(t)-\epsilon_1(t)$ may reduce due to the tunneling-induced Friedel shift

$$\epsilon_i(t)=\epsilon_i^0-\sum_{\alpha=L,R}|T_\alpha^{(i)}(t)|^2 Re\int\frac{\rho_0 d\varepsilon}{\epsilon_i-\epsilon_\alpha}, \tag{16}$$

provided $T_\alpha^{(2)}>T_\alpha^{(1)}$, which is usually the case [39]. This effect is maximal near the turning points of shuttle motion. Thus, if the condition (15) is valid for the certain time intervals during the oscillation cycle, the Kondo tunneling is possible for a part of this cycle, where the shuttle is close to one of the leads. It should be emphasized that in this regime only the weak-coupling Kondo effect may be observed at $T\gg T_K$, whereas at $T\to 0$ the triplet state is quenched and the dot behaves as a zero spin nano-particle [23]. The full scale Kondo effect may arise only if the variation of $|T_\alpha^{(i)}(t)|^2$ induces the crossover from a singlet to a triplet ground state of a shuttle. The singlet/triplet crossover induced by the variation of gate voltages was observed on a static planar dot [42]. Unlike conventional level crossing, this crossover does not violate adiabaticity because it conserves the $SO(4)$ symmetry of singlet/triplet manifold [12, 39].

3 Stoner instability and dynamical symmetries

As one decreases the effective dimensionality of a conductor, the role of electron-electron interactions — notably in the charge and spin channels — is enhanced. In one-dimension (d=1) these two channels, responsible for a widely ranged spectrum of effects, often decouple. It is of obvious interest to study the counterpart of this physics in d=0 quantum dots (QDs). An easily accessible scheme is the "Universal Hamiltonian" [2, 43] where, in addition to the (impurity and geometry dependent) single-particle Hamiltonian, only zero-mode interactions (charge and spin (exchange) in our case) are included. The former leads

to the phenomenon of the Coulomb blockade, while the latter leads to the Stoner instability [4] which is modified in mesoscopic systems [43]. Attention has been given to the intriguing interplay between the charge $U(1)$ [46] and the spin $SU(2)$ [47, 48] channels. This is manifest, *e.g.*, in the suppression of certain Coulomb peaks due to "spin-blockade" [44]. In a recent theoretical study [45] the effect of the the spin channel on Coulomb peaks has been analyzed employing a master equation in the classical limit. Notwithstanding the success of this approach, quantum effects are expected to play an important role. A full fledged quantum mechanical analysis of the charge-spin interplay in zero dimensions is thus called for [47, 48].

Our QD of linear size L is in the "metallic regime" (either diffusive ($\ell \ll L$) or ballistic-chaotic ($\ell \approx L$)). The Thouless energy and the mean level-spacing satisfy $g \equiv E_{Th}/\Delta \gg 1$. We consider the following terms of the Universal Hamiltonian:

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} + H_C + H_S. \tag{17}$$

The spin (σ) degenerate levels of the single-particle Hamiltonian obey the Wigner-Dyson statistics. For simplicity we confine ourselves to the Gaussian Unitary Ensemble (GUE) case. The charging interaction $H_C = E_c\,(\hat{n} - N_0)^2$ accounts for the Coulomb blockade. Here $\hat{n}$ is the number operator; N_0 represents the positive background charge and is tuned to the Coulomb valley regime. The term

$$H_S = -J\left[\left(\sum_\alpha S^z_\alpha\right)^2 + \gamma\left\{\left(\sum_\alpha S^x_\alpha\right)^2 + \left(\sum_\alpha S^y_\alpha\right)^2\right\}\right]$$

represents spin, $\vec{S}_{\sigma\sigma'}=\frac{1}{2}\sum_\alpha a^\dagger_{\alpha,\sigma}\vec{\sigma}_{\sigma\sigma'}a_{\alpha,\sigma'}$, interactions within the dot. Below we allow for an easy axis anisotropy, $\gamma=J_\perp/J<1$, reducing the original $SU(2)$ symmetry to $SO(2)$. There are several possible sources for such an anisotropy: geometrical, molecular anisotropy etc. The degree of anisotropy can be controlled by introducing magnetic impurities into the system, or by applying anisotropic mechanical pressure [49].

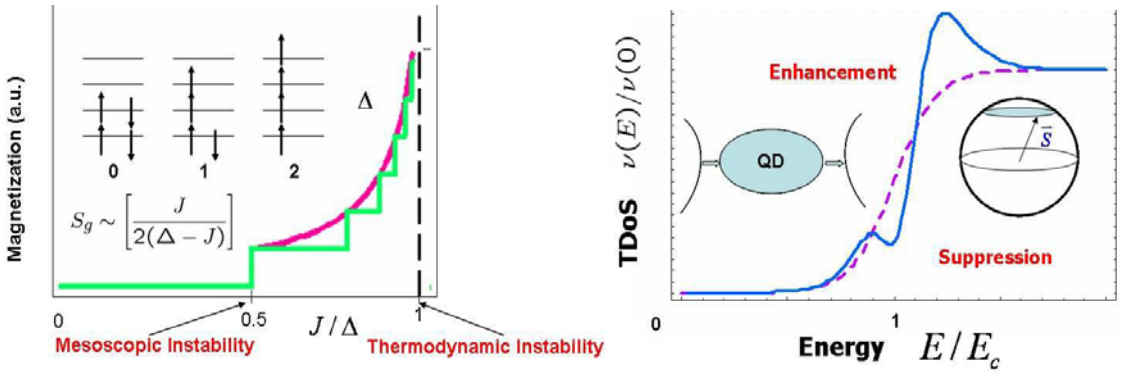

Fig. 4 (Color online) Left panel: Spin of the ground state S_g as a function of the spin exchange coupling. The inset shows spin configurations for the $S = 0, 1, 2$ states. Right panel: The spin normalized tunneling density of states shown as function of the energy $E_c/T = 10$. Dashed curve: $J = 0$. Solid curve: $J/\Delta = 0.92$, $J/T = 0.1$, $\gamma = 0.93$. Left inset shows the QD setup. Right inset shows the spin rotation on the Bloch sphere.

We recall that beyond the thermodynamic Stoner instability point, $J_{th} = \Delta$ (see Fig. 4, left panel), the spontaneous magnetization is an extensive quantity. At smaller values of the exchange coupling,

$J_{mesoscopic} < J < J_{th}$, finite magnetization shows up (see the inset on the left panel of Fig.4), which, for finite systems, does not scale linearly with the size of the latter [43]. Its non-self-averaging nature gives rise [5] to strong sample-specific mesoscopic fluctuations.

The full fledged quantum mechanical analysis of the interplay between charge $U(1)$ and transverse $SO(2)$ fluctuations is given in [47, 48]. Skipping the details of the field-theoretical treatment of the Universal Hamiltonian based on $0+1$ functional bosonization approach, I present below the key prediction for the Tunneling density of states.

The conductance g_T is related to the TDoS ν through

$$g_T = \frac{e}{\hbar}\int d\epsilon \nu(\epsilon)\Gamma(\epsilon)\left(-\frac{\partial f_F}{\partial \epsilon}\right)$$

where f_F is the Fermi distribution function at the contact and Γ is the golden rule dot-lead broadening. The TDoS is written in terms of the QD electrons Green's Function averaged by both charge and spin fluctuations and given by [50]

$$\nu(\epsilon) = -\frac{1}{\pi}\cosh\left(\frac{\epsilon}{2T}\right)\int_{-\infty}^{\infty}\sum_{\sigma}\langle G_\sigma\left(\frac{1}{2T}+it\right)\rangle_{k,m}e^{i\epsilon t}dt. \tag{18}$$

where $\langle ...\rangle_{k,m}$ denotes a summation over all winding numbers for Coulomb and longitudinal zero-modes [51]. Examples for the temperature and energy dependence of the TDoS (for various γ) are depicted in Fig. 4 (right panel). The energy dependent TDoS shows an intriguing non-monotonic behavior at energies comparable to the charging energy E_c. This behavior, absent for $J = 0$, is due to the contribution of the transverse spin susceptibility (see [47, 48] for detail). The oscillating (in real time) factor in the dynamic transverse susceptibility describes Bloch precessions in an anisotropic easy axis spin model (see Fig.4, right panel inset). The oscillations are amplified in the vicinity of the Stoner Instability point, and signals the effect of collective spin excitations (incipient ordered phase). One of possible experimental realizations of predicted effect is transport measurements in magnetic QD [6].

We have found [47,48] that (i) As the spin modes renormalize the Coulomb blockade (CB), they modify the tunneling density of states (TDoS) – hence the differential conductance – of the dot (Fig. 4, right panel). For an Ising-like spin anisotropy the longitudinal mode partially suppresses the CB. Quantum fluctuations, manifest through the transverse $SO(2)$ modes, act qualitatively in the same way, but as one approaches the Stoner instability (from the disordered phase) their effect reverses its sign, giving rise to suppression of the conductance (i.e., *enhancement* of the CB). This results in a *non-monotonic behavior of the TDoS* ; (ii) *The longitudinal spin susceptibility diverges* at the thermodynamic Stoner instability point, while *the transverse susceptibility is enhanced* by gauge fluctuations (but remains finite).

4 Conclusions

Today, the concept of dynamical symmetry is ubiquitous in many branches of modern physics, such as quantum field theory, nuclear physics, quantum optics and condensed matter physics in low dimensions. Quantum dots are especially suitable objects for the group theoretical approach because the fully discrete spectrum of low-lying excitations in these systems often may be characterized by the definite dynamical symmetry, and the interaction with the metallic reservoir of metallic electrons in the leads provides a powerful tool of symmetry breaking.

In this review we concentrated on the spin excitations in quantum dots. We discussed the quantum transport through the Single Electron Transistor, molecular electronic devices and nanoelectromechanical shuttles. Many interesting properties, like thermoelectric transport through few-electron quantum dots [52] or magnetic control on the Kondo oscillations [53] has not been discussed in this review due to the space limitations for the publication.

Another promising class of nanoobject for applications of the ideas of dynamical symmetries which has not been considered in this paper is spin ladders and spin chains. Such kind of quasi-1D structures can be fabricated as an array of artificial atoms or nano-crystals. In this case the role of object with definite dynamical symmetry is played by a single rung or pair of neighboring rungs bound by diagonal bonds, whereas the longitudinal modes violate this symmetry. The application of of dynamical symmetry approaches in this field are seldom enough as yet [54]- [60], but the field seems to be really wide.

At the end, I hope to convince the reader of the beauty and relevance of dynamical symmetries in the transport through the nanostructures and to stress its relation with experiments.

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