

Noises in a two-channel charge Kondo model

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We investigate fluctuations of electric and heat currents, along with their cross-correlations, in a two-channel charge Kondo circuit driven by either a voltage bias or a temperature gradient applied across the weak link. The ratios of voltage-driven electric/heat noise to the applied voltage V exhibit oscillations with the gate voltage N , resembling the behavior of the thermoelectric coefficient G_T . In contrast, the ratios of temperature-driven electric/heat noise to the temperature difference ΔT vary with N in a manner analogous to the thermal coefficient G_H or the electric conductance G . The mixed noise, which is defined as the correlation function between electric and heat currents, displays behavior opposite that of the above noises. The logarithmic temperature dependence of these noises signals non-Fermi-liquid behavior, while their oscillations with gate voltage reflect the roles of particle-hole and time-reversal symmetries in thermoelectric transport. Our results demonstrate that the fundamental relations linking voltage- and temperature-induced noises to thermoelectric transport across a tunnel junction persist beyond the Fermi-liquid paradigm.

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I. INTRODUCTION

Thermoelectricity has become a significant topic in modern physics, driven by the growing demand for advanced thermoelectric materials [1–3]. One of the primary approaches in this search is the development of nanostructured devices [4]. Recent advances in the fabrication of these devices have opened up possibilities for exploring a wide range of quantum phenomena related to charge, spin, and thermoelectric effects [5,6]. The impact of quantum effects on the properties of quantum devices is generally studied initially in the single-electron transistor (SET) due to its ability to be precisely controlled by external fields, such as electric potential and magnetic fields [5,6]. SET devices provide valuable insights into the effects of strong electron interactions, interference, and resonant scattering on quantum transport.

A fundamental phenomenon that encapsulates both resonant scattering and strong interactions is the Kondo effect [7,8], where a local spin is coupled to conduction electrons. The Kondo physics in a SET is realized by transport measurements [9]. In the ground state of the single impurity $S = 1/2$ single-channel Kondo model (1CK), the spin of the dot is screened by a cloud of conduction electrons, forming a singlet state. The low-energy behavior of the remaining electrons is characterized by a local Fermi liquid (FL) theory [10]. Electrons are scattered by the singlet both elastically and inelastically. The ratio of elastic to inelastic scattering is fixed by universality. The Kondo temperature T_K is the only scale

that governs the low-energy properties of the model. However, the behavior of the M -orbital spin- S Kondo model at energies below the Kondo temperature T_K depends on how the mobile electrons screen the impurity spin. The system exhibits coherent behavior and FL properties when the system is fully or underscreened ($M \leq 2S$), while it is likely to exhibit non-FL (NFL) characteristics in the overscreened case ($M > 2S$) [11]. However, it is difficult to achieve the strong NFL regime at very low temperature, namely, $T \ll T^* < T_K$, where T^* is related to the perturbative expansion parameter $|r|$ as $T^* = |r|^2 T_K$. The system has a tendency to fall into the FL regime associated with the stable FL fixed point [12]. Nevertheless, in higher temperature regimes, $T^* < T < T_K$, the fingerprints of the weak NFL behavior can be observed [12].

Going beyond conventional knowledge that the Kondo effect is attributed to the spin degrees of freedom of the quantum impurity, many unconventional Kondo phenomena have been observed in a variety of systems [13–16]. For instance, the charge Kondo effect deals with an isospin implementation of the charge quantization [17–19]. This charge Kondo model has been implemented in pioneering breakthrough experiments that involve the edge currents of the integer quantum Hall effect [20,21]. These experiments mark a significant step forward in the study of multichannel charge Kondo effects [22,23]. Indeed, fairly recently, authors of another experimental study [24] have successfully implemented a tunable nanoelectronic circuit consisting of two coupled hybrid metallic-semiconductor islands.

Recently, thermoelectric transport through quantum dot (QD) systems has garnered significant attention from both theorists [25–33] and experimentalists [34–39]. Measurements of thermoelectric coefficients are challenging because they require heating the contacts [12,23,27,28,40]. Keeping the

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temperature drop small and controllable during measurements is particularly difficult for experimentalists. To address this, it is essential to relate the thermoelectric coefficients using established relations, such as the Cutler-Mott formula [41], which connects thermopower to electrical conductance [42], and the Wiedemann-Franz law [43], which links thermal conductance to electrical conductance [44].

Furthermore, since current noise measurement is known to provide valuable insights into the fundamental mechanisms of quantum transport and electron interactions [45–47], it is also a promising experimental method for investigating the properties of mesoscopic systems [48–52]. Moreover, noise measurements are capable of probing the out-of-equilibrium properties of the Kondo model [53,54], a field that has not been extensively studied due to the limited number of theoretical approaches [55–57] available for out-of-equilibrium situations.

Among the various types of noise observed in mesoscopic systems, shot noise—arising from the discrete nature of charge carriers—plays a crucial role, particularly in systems where transport is dominated by tunneling events (see the review in Ref. [58] and references therein). Recently, delta-T noise, associated with temperature differences, has garnered significant attention from physicists [59–69]. This type of noise is expected to serve as a probe of quantum effects that cannot be observed through shot noise measurements. Delta-T noise provides unique insights into the interplay between thermal effects and quantum correlations in transport phenomena. Notably, both shot noise and delta-T noise at low temperatures have the potential to yield valuable information about the statistical nature of charge transfer [48,49,69]. Furthermore, FL interactions can be extracted from the measurement of shot noise in a $SU(N)$ Kondo QD [70]. In this context, we investigate whether noise can probe the NFL characteristics in a multichannel charge Kondo model.

In this work, we calculate the voltage-driven electric/heat noise, temperature-driven electric/heat noise, and voltage/temperature-driven mixed noise (charge-heat cross correlations) of the electric and heat currents at the weak link connecting a reservoir to a two-channel charge Kondo (2CK) circuit [71]. The setup, illustrated in Fig. 1, consists of a large metallic QD that is weakly coupled to the left lead (LL) via a tunnel barrier and strongly coupled to the right lead (RL) through an almost transparent single-mode quantum point contact (QPC) [17–19]. The key idea behind mapping this system to a 2CK problem lies in treating the two degenerate charge states of the QD as a quantum impurity, while the position of the electrons inside or outside the dot defines an isospin degree of freedom. The spin projections of the electrons then naturally define the two screening channels in the Kondo model [27,28].

As discussed in Refs. [12,27,28,72,73], perturbative approaches, based on the assumption of weak backscattering at the QPCs ($|r| \ll 1$), are valid in the temperature regime $|r|^2 T_k \ll T \ll T_k$. Moreover, our model also allows for a nonperturbative treatment of $|r|$. Specifically, we compute the relevant correlation functions without assuming small reflection amplitudes, even though the charge Kondo model itself requires small $|r|$. Therefore, our results extend beyond the perturbative regime and remain valid even at temperatures $T \leq |r|^2 T_k$.

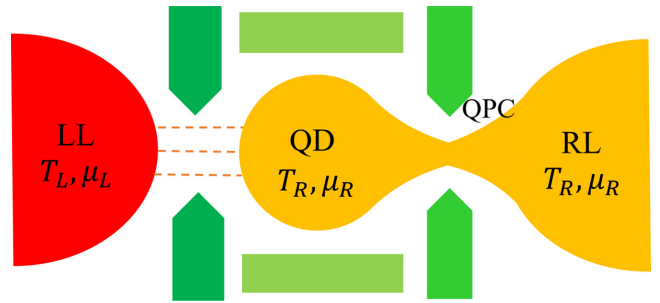


FIG. 1. Schematic of a SET device in which a large metallic QD is embedded into two-dimensional electron gas (2DEG) and connects weakly to the LL through a tunnel barrier and strongly coupled to the RL through an almost transparent single-mode QPC. The QD and the RL (yellow color) are at potential μ_R temperature $T_R = T$, while the LL (red color) is at higher voltage $\mu_L = \mu_R - eV$ and temperature $T_L = T + \Delta T$. The voltage or temperature drops at the weak link. The green patches demonstrate the gate voltage and the voltages controlling the tunnel barrier and the QPC.

Electric and heat currents, as well as their associated noises (electric, heat, and mixed noise), are computed up to the linear order in voltage or temperature bias—that is, we retain only the first-order terms of the expansions in eV or ΔT . We find that the voltage-driven electric and heat noises show oscillatory behavior with respect to the gate voltage N , mirroring the pattern observed in the thermoelectric coefficient G_T . On the other hand, the temperature-driven electric and heat noises display a dependence on N like that of the thermal coefficient G_H or the electric conductance G . In contrast, the mixed noise in both situations exhibits a trend opposite that of the aforementioned noises.

Notably, the logarithmic temperature dependence of voltage-driven electric and heat noises and temperature-driven mixed noise in the vicinities of Coulomb peaks provides clear evidence of the NFL behavior characteristic of the 2CK state. The distinctive behavior of the Fano factors further reinforces the connection between current noises and thermoelectric coefficients. Comparisons with the 1CK model across all noise types confirm the crossover flow from the 2CK to the 1CK regime.

The paper is organized as follows. We describe the theoretical model in Sec. II. Equations for the currents and noises are presented in Sec. III, while the correlation function and density of states (DOS) are introduced in Sec. IV. The main results are demonstrated and discussed in Sec. V. We conclude our work in Sec. VI.

II. MODEL

We analyze a SET device, as illustrated in Fig. 1 [12,27,28,72]. The central component consists of a large metallic QD operating in the weak Coulomb blockade (CB) regime. This QD is strongly coupled to the RL through a nearly transparent single-mode QPC. The QD-QPC structure is embedded within a two-dimensional electron gas (2DEG). We assume that the QD-QPC structure is in thermal equilibrium at potential μ_R and temperature $T_R = T$ (represented by the yellow region, the right electrode, in Fig. 1). The LL, also

part of the 2DEG, is connected to the QD via a tunnel barrier. It is assumed to be at a higher temperature $T_L = T + \Delta T$ (shown as the red region in Fig. 1, with a potential of $\mu - eV$). This setup is used to investigate the voltage and temperature bias-driven noises of electric and heat currents at the tunnel barrier. The temperature difference across the tunnel barrier ΔT is controlled using a current heating technique [36]. We assume that ΔT is small relative to the reference temperature T , which allows us to apply a perturbative expansion in terms of V and ΔT .

The Hamiltonian describing the system in which the QD-QPC structure is weakly coupled to the LL is given by $H = H_L + H_R + H_T$. Here, $H_L = \sum_{\mathbf{k},\alpha} \epsilon_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}^\dagger c_{\mathbf{k},\alpha}$ describes the noninteracting LL (c denotes the electrons in the LL, $\alpha = \uparrow, \downarrow$ stands for the two spin projections of electrons), while H_R describes the right part of the junction, namely, the interacting QD strongly coupled to the RL. The tunneling from the LL to the dot is given by the tunneling Hamiltonian H_T which reads

$$H_T = \sum_{\mathbf{k},\alpha} t_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}^\dagger d_\alpha + \text{H.c.}, \quad (1)$$

where d corresponds to the electrons in the dot, and $t_{\mathbf{k},\alpha}/D \ll 1$ represents the tunneling amplitude (D being the bandwidth).

The QD-QPC structure is described by the following Hamiltonian:

$$\begin{aligned} H_R = & \sum_{\alpha} \epsilon_{\alpha} d_{\alpha}^{\dagger} d_{\alpha} + \sum_{\alpha} \frac{v_F}{2\pi} \int_{-\infty}^{\infty} \{[\Pi_{\alpha}(x)]^2 + [\partial_x \phi_{\alpha}(x)]^2\} dx \\ & + E_C \left[\hat{n} + \frac{1}{\pi} \sum_{\alpha} \phi_{\alpha}(0) - N(V_g) \right]^2 \\ & - \frac{D}{\pi} \sum_{\alpha} |r_{\alpha}| \cos [2\phi_{\alpha}(0)], \end{aligned} \quad (2)$$

where v_F is the Fermi velocity, ϕ_{α} is a bosonic displacement operator describing transport through the QPC with a scatterer at $x = 0$, and Π_{α} is the conjugated momentum $[\phi_{\alpha}(x), \Pi_{\alpha'}(x')] = i\pi \delta(x - x') \delta_{\alpha,\alpha'}$ [74]. The operator d_{α} is related to the fermionic field as $d_{\alpha} = \psi_{\alpha}(-\infty)$ with $\psi_{\alpha}(x) \sim \exp[i\phi_{\alpha}(x)]$ in the one-dimensional model that describes the QPC. The third term describes the Coulomb interaction in the dot where $E_C = e^2/2C$ is the charging energy (C is the QD capacitance), $N(V_g) = CV_g/e$ is a dimensionless parameter which is proportional to the gate voltage V_g , \hat{n} is the operator depicting the number of electrons entering the QD through the left tunnel barrier, while $\sum_{\alpha} \phi_{\alpha}(0)/\pi$ is the number of electrons entering through the QPC (from the RL) [75]. The last term demonstrates the backward scattering in the QPC with the small reflection amplitudes $|r_{\uparrow}| = |r_{\downarrow}| = |r|$ for the symmetric 2CK (no magnetic field is applied perpendicularly to the 2DEG plane).

III. GENERAL FORMULAS FOR CURRENTS AND NOISES

The device under consideration is out-of-equilibrium at the tunneling barrier (see Fig. 1). The resulting operator for the current flowing through this barrier is given by $I_C = edN_L/dt = ie[H_T, \sum_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}^\dagger c_{\mathbf{k},\alpha}] = -ie \sum_{\mathbf{k},\alpha} (t_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}^\dagger d_{\alpha} - t_{\mathbf{k},\alpha}^* d_{\alpha}^\dagger c_{\mathbf{k},\alpha})$, recovering the standard

expression for the current through a pointlike junction between two reservoirs. Similarly, the heat current flowing through the tunnel barrier can be defined as $I_Q = d(H_L - \mu_L N_L)/dt = -i \sum_{\mathbf{k},\alpha} (\epsilon_{\mathbf{k}} - \mu_L) (t_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}^\dagger d_{\alpha} - t_{\mathbf{k},\alpha}^* d_{\alpha}^\dagger c_{\mathbf{k},\alpha})$.

We consider the averages of both the charge current I_C and the heat current I_Q across the tunnel contact where a voltage bias V and temperature drop ΔT are applied [76]

$$\begin{pmatrix} \langle I_C \rangle \\ \langle I_Q \rangle \end{pmatrix} = \begin{pmatrix} G & G_T \\ TG_T & G_H \end{pmatrix} \begin{pmatrix} V \\ \Delta T \end{pmatrix}, \quad (3)$$

where $G = \partial \langle I_C \rangle / \partial V|_{\Delta T=0}$, $G_T = \partial \langle I_C \rangle / \partial \Delta T|_{V=0}$, and $G_H = \partial \langle I_Q \rangle / \partial \Delta T|_{V=0}$ are the electric conductance, thermoelectric coefficient, and thermal coefficient, respectively.

A. Electric current and noise

At the lowest order of the perturbative expansion in the tunneling amplitude $t_{\mathbf{k},\alpha}$, the average current reads [77,78]

$$\langle I_C \rangle = -2\pi e \sum_{\alpha} |t_{\alpha}|^2 \int_{-\infty}^{\infty} d\epsilon v_L(\epsilon) v_D(\epsilon) [f_L(\epsilon) - f_R(\epsilon)], \quad (4)$$

where v_L and v_D are the DOS of the LL and the QD at the tunnel barrier, respectively; and f_L and f_R are the corresponding Fermi distribution functions. Here, we assume the tunneling to be energy independent, allowing us to simplify $t_{\mathbf{k},\alpha} = t_{\alpha}$.

Electric noise, defined as $S_C(t - t') = \langle I_C(t) I_C(t') \rangle - \langle I_C(t) \rangle \langle I_C(t') \rangle$ or, alternatively, its Fourier transform $S_C(\omega)$ has been studied extensively. Although equilibrium zero-frequency noise $S_C = S_C(\omega = 0) = \int d\tau [S_C(\tau) + S_C(-\tau)] = 2 \int d\tau S_C(\tau)$ can be related to conductance by the fluctuation-dissipation theorem and does not carry additional information, out-of-equilibrium zero-frequency noise can yield information on charge fluctuations in the mesoscopic system [45–47]. The zero-frequency noise corresponds to the fluctuations of the tunneling charge current. It is proportional to the local DOS and related to the Fermi distributions as

$$\begin{aligned} S_C = & 4\pi e^2 \sum_{\alpha} |t_{\alpha}|^2 \int_{-\infty}^{\infty} d\epsilon v_L(\epsilon) v_D(\epsilon) \\ & \times [f_L(\epsilon) + f_R(\epsilon) - 2f_L(\epsilon)f_R(\epsilon)]. \end{aligned} \quad (5)$$

In the equilibrium case, we find the relation between zero-frequency noise S_C^{eq} and electric conductance G as expected $S_C^{\text{eq}} = 4TG$.

B. Heat current and noise

We similarly obtain the expression for the average heat current as [79,80]

$$\begin{aligned} \langle I_Q \rangle = & -2\pi \sum_{\alpha} |t_{\alpha}|^2 \int_{-\infty}^{\infty} d\epsilon (\epsilon - \mu_L) v_1(\epsilon) v_2(\epsilon) \\ & \times [f_L(\epsilon) - f_R(\epsilon)]. \end{aligned} \quad (6)$$

In the same spirit as for the charge noise introduced earlier, to probe fluctuations of the heat current, one should investigate heat noise, which is defined as $S_Q(t - t') = \langle I_Q(t) I_Q(t') \rangle - \langle I_Q(t) \rangle \langle I_Q(t') \rangle$, with I_Q being the tunneling

heat current operator. The zero-frequency noise corresponds to the fluctuations of the tunneling heat current and reads

$$S_Q = 4\pi \sum_{\alpha} |t_{\alpha}|^2 \int_{-\infty}^{\infty} d\epsilon (\epsilon - \mu_L)^2 v_L(\epsilon) v_D(\epsilon) \times [f_L(\epsilon) + f_R(\epsilon) - 2f_L(\epsilon)f_R(\epsilon)]. \quad (7)$$

We find that the zero-frequency heat noise at equilibrium S_Q^{eq} is related to the thermal coefficient G_H through $S_Q^{\text{eq}} = -4T^2 G_H$.

C. Mixed noise

At this stage, it is also possible to investigate the cross-correlations between charge and heat currents through the mixed noise defined as $S_M(t-t') = \langle I_C(t)I_Q(t') \rangle - \langle I_C(t) \rangle \langle I_Q(t') \rangle$, where I_C and I_Q are the tunneling charge and heat current operators, respectively. Along the same lines as the previous calculations for the charge and heat noises, the zero-frequency mixed noise can be written as

$$S_M = 4\pi e \sum_{\alpha} |t_{\alpha}|^2 \int_{-\infty}^{\infty} d\epsilon (\epsilon - \mu_L) v_L(\epsilon) v_D(\epsilon) \times [f_L(\epsilon) + f_R(\epsilon) - 2f_L(\epsilon)f_R(\epsilon)]. \quad (8)$$

The relation between zero-frequency mixed noise in the equilibrium situation S_M^{eq} and thermoelectric coefficient G_T is $S_M^{\text{eq}} = -4T^2 G_T$.

The computation of the average currents and zero-frequency noises in Eqs. (4)–(8) requires the explicit form of the DOS. Because the LL is a noninteracting Fermi sea, we can replace its DOS by its energy-independent value $v_{L,0}$ at the Fermi energy, while the DOS of the QD at the weak link is discussed in the next section.

IV. CORRELATION FUNCTION $K(\tau)$ AND DOS $v_D(\epsilon)$

For the convenience of the calculations, following Matveev-Andreev theory [28], we replace $d_{\alpha} \rightarrow d_{\alpha} \hat{F}$ in Eq. (1), where \hat{F} is the charge-lowering operator, which obeys the commutation relation $[\hat{F}, \hat{n}] = \hat{F}$. One should note that this substitution does not affect the form of the Hamiltonian H_L . The DOS $v_D(\epsilon)$ of the QD at the weak barrier is modified by the electron-electron interactions in the dot as

$$v_D(\epsilon) = v_{D,0} T_R \cosh\left(\frac{\epsilon}{2T_R}\right) \int_{-\infty}^{\infty} \frac{\exp(i\epsilon t) K\left(\frac{1}{2T_R} + it\right)}{\cosh(\pi T_R t)} dt, \quad (9)$$

where $v_{D,0}$ stands for the DOS of the QD which is no longer renormalized by the electron-electron interactions, while the correlation function $K(1/2T_R + it)$ characterizes these interactions [$K(\tau) = \langle T_{\tau} \hat{F}(\tau) \hat{F}^{\dagger}(0) \rangle$ (T_{τ} is the time-ordering operator; the imaginary time τ runs from 0 to $\beta = 1/T_R$)].

The time-ordered correlation function is computed through the functional integration $K(\tau) = Z(\tau)/Z(0)$. We introduce the charge and spin fields $\phi_{c,s}(x,t) = [\phi_{\uparrow}(x,t) \pm \phi_{\downarrow}(x,t)]/\sqrt{2}$. Rewriting the Hamiltonian H_R in Eq. (2) in terms of these variables, we find that the correlation function $K(\tau)$ can be factorized into charge and spin components as $K(\tau) = K_c(\tau)K_s(\tau)$, where the main contribution to $K_c(\tau)$

can be obtained in the limit $|r| = 0$ (no backscattering at the QPCs). Authors of previous studies have shown that the thermoelectric properties of the system are controlled by charge and spin fluctuations at low temperatures $T \ll E_C$. However, the effect of small but finite $|r|$ on the charge modes is negligible due to the CB in the QD, while any small backscattering amplitude pins spin-mode fluctuations and dramatically changes their low-frequency dynamics. The correlation function $K_s(\tau)$ is computed nonperturbatively [28], enabling us to extend the results beyond perturbation theory for small $|r|$. The nonperturbative treatment involves the refermionization procedure [12,27,28,81]. The spin-mode model is mapped onto a time-dependent resonant scattering model.

In the end, the correlation function for the 2CK model in the nonperturbative treatment, concerning the reflection amplitudes of the QPCs, is expressed as [28]

$$K(\tau) = \frac{\pi T_R \Gamma}{\gamma E_C} \frac{1}{|\sin(\pi T_R \tau)|} \int_{-\infty}^{\infty} \frac{d\omega \exp(\omega \tau)}{(\omega^2 + \Gamma^2)[1 + \exp(\omega/T_R)]} - 4|r|^2 \frac{T_R}{E_C} \frac{\sin(2\pi N)}{|\sin(\pi T_R \tau)|} \ln\left(\frac{E_C}{T_R + \Gamma}\right) \times \int_{-\infty}^{\infty} d\omega \frac{\omega \exp(\omega \tau)}{(\omega^2 + \Gamma^2)[1 + \exp(\omega/T_R)]}, \quad (10)$$

where Γ is the Kondo-resonance width in the vicinity of Coulomb peaks

$$\Gamma(N) = \frac{8\gamma E_C}{\pi^2} |r|^2 \cos^2(\pi N), \quad (11)$$

and $\gamma = e^C \approx 1.78$ ($C \approx 0.577$ is Euler's constant). Following Refs. [12,82], the Kondo resonance width at a Coulomb peak is always finite for any asymmetry in the two Kondo channels. However, at this stage, we only study the symmetric case. One should also notice that the two terms in Eq. (10) determine the parity of the DOS of the QD. Indeed, substituting Eq. (10) back into Eq. (9), one has $v_D(\epsilon) = v_D^e(\epsilon) + v_D^o(\epsilon)$, with

$$v_D^e(\epsilon) = \frac{v_{D,0} T_R}{2\gamma E_C} \cosh\left(\frac{\epsilon}{2T_R}\right) \times \int_{-\infty}^{\infty} dx \frac{1}{\cosh\left(\frac{x}{2}\right)} \frac{p}{x^2 + p^2} \frac{x + \frac{\epsilon}{T_R}}{\sinh\left(\frac{x + \epsilon/T_R}{2}\right)},$$

$$v_D^o(\epsilon) = -\frac{2v_{D,0} T_R}{\pi E_C} |r|^2 \sin(2\pi N) \times \ln\left[\frac{E_C}{T_R(1+p)}\right] \cosh\left(\frac{\epsilon}{2T_R}\right) \times \int_{-\infty}^{\infty} dx \frac{1}{\cosh\left(\frac{x}{2}\right)} \frac{x}{x^2 + p^2} \frac{x + \frac{\epsilon}{T_R}}{\sinh\left(\frac{x + \epsilon/T_R}{2}\right)}, \quad (12)$$

where $p = \Gamma/T_R$. These DOS components, based on their parity properties with respect to energy, will determine the characteristics of thermoelectric coefficients and noises.

V. MAIN RESULTS: OUT-OF-EQUILIBRIUM SITUATIONS

In this section, we investigate both charge and heat transport. We compute voltage-driven electric and heat noises when a voltage bias is applied at the weak link, while

temperature-driven electric and heat noises are calculated when the temperature bias is enforced. Mixed noise, the electric current and heat current correlation function, is examined in both cases [71]. We calculate these quantities using the first-order approximation of the series expansion, based on the voltage or temperature difference across the weak link. Therefore, the condition for these results to be valid is that the voltage is sufficiently small, i.e., $V \ll E_C$, and the temperature difference is small enough, such that $\Delta T \ll T \ll E_C$.

A. Voltage bias and noises

We consider the generation of voltage-driven noise under a nonequilibrium condition, in which the two sides of the weak link have the same temperature $T_R = T_L = T$, but the chemical potentials satisfy $\mu_R - \mu_L = eV$. Expanding the combinations of Fermi functions entering the expressions for the current and the noise up to first order in eV , we are left with

$$f_L(\epsilon) - f_R(\epsilon) = -\frac{1}{2} \frac{1}{\cosh^2\left(\frac{\epsilon}{2T}\right)} \frac{eV}{2T} + O\left[\left(\frac{eV}{T}\right)^2\right], \quad (13)$$

and similarly

$$\begin{aligned} f_L(\epsilon) + f_R(\epsilon) - 2f_L(\epsilon)f_R(\epsilon) \\ = \frac{1}{2} \frac{1}{\cosh^2\left(\frac{\epsilon}{2T}\right)} \left[1 - \frac{eV}{2T} \tanh\left(\frac{\epsilon}{2T}\right)\right] + O\left[\left(\frac{eV}{T}\right)^2\right]. \end{aligned} \quad (14)$$

We point out that we have adopted the units $\hbar = k_B = 1$ and introduced the characteristic conductance $G_L = 2\pi e^2 \nu_{L,0} \nu_{D,0} \sum_{\alpha} |t_{\alpha}|^2$, which corresponds to the conductance of the barrier when the electrons in the dot are noninteracting. In the end, we obtain the following formula for the average electric current:

$$\langle I_C^V \rangle = V \frac{G_L}{2\gamma} \frac{T}{E_C} f_G\left(\frac{\Gamma}{T}\right), \quad (15)$$

where [83]

$$f_G(p) = p + \frac{\pi}{2} \left(1 - \frac{p^2}{\pi^2}\right) \psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right), \quad (16)$$

while the shot noise [71] contribution (we ignored the equilibrium thermal contribution) is given by

$$\begin{aligned} \Delta S_C^V = V \frac{eG_L}{\pi} \frac{T}{E_C} |r|^2 \sin(2\pi N) \ln\left[\frac{E_C}{T + \Gamma}\right] f_C^V\left(\frac{\Gamma}{T}\right), \quad (17) \\ f_C^V(p) = \left[12 - \frac{8p}{\pi} \psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right) \right. \\ \left. + \left(1 - \frac{p^2}{\pi^2}\right) \psi^{(2)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)\right], \quad (18) \end{aligned}$$

where $\psi^{(1)}(x) = \sum_{n=0}^{\infty} (x+n)^{-2}$ is the trigamma function, and $\psi^{(2)}(x) = \partial_x \psi^{(1)}(x)$ is the tetragamma function.

From Eq. (15), we can obtain the formula of electric conductance G [12]. It is interesting that the shot noise in Eq. (17) is proportional to the thermoelectric coefficient G_T (see formula (69) in Ref. [28]). This can be understood that both properties are affected by the scattering processes and

energy transport in the system. The thermoelectric coefficient (and thermopower) involves energy transport, and fluctuations in this energy transport contribute to shot noise. The electric conductance [can be extracted from the formula in Eq. (4)], on the other hand, is primarily related to charge transport and does not directly account for these energy-dependent fluctuations that affect thermoelectric properties. Therefore, when both charge and energy transport are considered, we find that shot noise is related to the conductance G at zeroth order and to the thermoelectric coefficient G_T at first order in the expansion with respect to the voltage bias V (see also Refs. [84,85]).

In studying the heat-related transport properties, we also focused only on the excess noises by subtracting the corresponding equilibrium contribution. Since there are contributions of order $O(eV)$ arising from both the term in $\epsilon - \mu_L$ and the one involving Fermi functions, ΔS_Q and ΔS_M contain two terms. The average heat current is written as

$$\langle I_Q^V \rangle = V \frac{G_L}{3\pi e} \frac{T^2}{E_C} |r|^2 \sin(2\pi N) \ln\left[\frac{E_C}{T + \Gamma}\right] f_{G_T}\left(\frac{\Gamma}{T}\right), \quad (19)$$

where

$$f_{G_T}(p) = \frac{8\pi^2}{3} - 2p^2 + (p^2 - \pi^2) \frac{p}{\pi} \psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right), \quad (20)$$

while the corresponding voltage-driven heat noise is

$$\Delta S_Q^V = V \frac{G_L}{3\pi e} \frac{T^3}{E_C} |r|^2 \sin(2\pi N) \ln\left[\frac{E_C}{T + \Gamma}\right] f_Q^V\left(\frac{\Gamma}{T}\right), \quad (21)$$

with

$$\begin{aligned} f_Q^V(p) = \frac{82\pi^2}{3} - 14p^2 - 8\pi p \left(2 - \frac{p^2}{\pi^2}\right) \psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right) \\ + \frac{1}{2} \left(3\pi^2 - 4p^2 + \frac{p^4}{\pi^2}\right) \psi^{(2)}\left(\frac{1}{2} + \frac{p}{2\pi}\right), \quad (22) \end{aligned}$$

and the voltage-driven mixed noise is expressed as

$$\Delta S_M^V = V \frac{G_L}{3\gamma} \frac{T^2}{E_C} f_M^V\left(\frac{\Gamma}{T}\right), \quad (23)$$

where

$$\begin{aligned} f_M^V(p) = 5p + \pi \left(1 - \frac{3p^2}{\pi^2}\right) \psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right) \\ + \frac{p}{4} \left(1 - \frac{p^2}{\pi^2}\right) \psi^{(2)}\left(\frac{1}{2} + \frac{p}{2\pi}\right). \quad (24) \end{aligned}$$

From Eqs. (19) and (21), we observe that the voltage-driven heat noise of the heat current is proportional to the thermoelectric coefficient G_T because $G_T = \langle I_Q^V \rangle / V$ in the linear regime. This can be explained as follows: The thermoelectric coefficient dictates the coupling between charge and energy, affecting both the steady-state heat current and the fluctuations in energy transport. The average heat current, driven by the charge current and the Seebeck coefficient, establishes the scale for energy fluctuations (shot noise) in the system. Both the thermoelectric coefficient and the average heat current play essential roles in determining the magnitude of shot noise, as they influence the transport of charge and

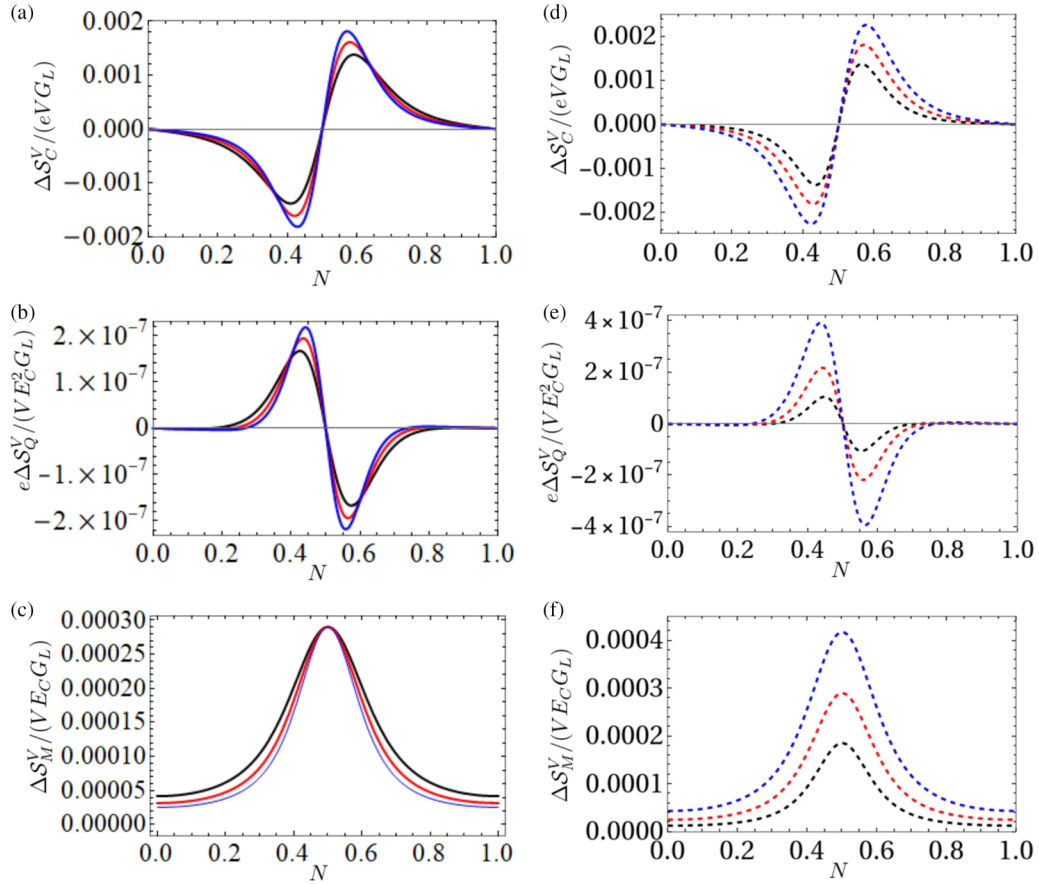


FIG. 2. (a) and (d) Voltage-driven electric noise $\Delta S_C^V/(eVG_L)$, (b) and (e) voltage-driven heat noise $e\Delta S_Q^V/(E_C^2G_L)$, and (c) and (f) voltage-driven mixed shot noise $\Delta S_M^V/(E_CG_L)$ over the voltage difference V between two sides of the weak link as a function of the gate voltage N . For the plots on the left [(a)–(c)], $T/E_C = 0.01$, black, red, and blue lines correspond to $|r|^2 = 0.06$, $|r|^2 = 0.08$, and $|r|^2 = 0.1$. For the plots on the right [(d)–(f)], $|r|^2 = 0.1$, black, red, and blue lines correspond to $T/E_C = 0.008$, $T/E_C = 0.01$, and $T/E_C = 0.012$.

energy as well as the fluctuations that arise in these processes within the heat current.

The gate voltage dependence of the voltage-driven noises ΔS_C^V , ΔS_Q^V , and ΔS_M^V for both electric and heat currents, under the condition of a small voltage bias applied across the tunnel barrier, is shown in Fig. 2. While the voltage-driven noises ΔS_C^V and ΔS_Q^V of individual electric and heat currents are odd functions, the voltage-driven mixed noise ΔS_M^V of these two currents is an even function of the gate voltage. Increasing the backscattering amplitude of the QPC leads to a slight elevation at the peaks in ΔS_C^V and ΔS_Q^V , but no such effect is observed for ΔS_M^V . This is demonstrated in Figs. 4(a)–4(c).

B. Temperature bias and noises

We now turn to the situation of a temperature bias across the tunnel barrier. We set the temperatures $T_L = T + \Delta T$ and $T_R = T$, while enforcing the same chemical potentials $\mu_R = \mu_L$. Expanding the combinations of Fermi functions entering the expressions for the current and the noise up to first order in ΔT (linear response regime), we are left

with

$$f_L(\epsilon) - f_R(\epsilon) = \frac{1}{2} \frac{1}{\cosh^2\left(\frac{\epsilon}{2T}\right)} \frac{\epsilon \Delta T}{2T^2} + O\left[\left(\frac{\Delta T}{T}\right)^2\right], \quad (25)$$

and

$$\begin{aligned} f_L(\epsilon) + f_R(\epsilon) - 2f_L(\epsilon)f_R(\epsilon) \\ = \frac{1}{2} \frac{1}{\cosh^2\left(\frac{\epsilon}{2T}\right)} \left[1 + \frac{\epsilon \Delta T}{2T^2} \tanh\left(\frac{\epsilon}{2T}\right)\right] + O\left[\left(\frac{\Delta T}{T}\right)^2\right]. \end{aligned} \quad (26)$$

Substituting the expression in Eq. (25) into Eq. (4) yields the formula for the average current

$$\begin{aligned} \langle I_C^{\Delta T}(t) \rangle = -\Delta T \frac{G_L}{3e\pi} |r|^2 \sin(2\pi N) \frac{T}{E_C} \ln\left[\frac{E_C}{T + \Gamma}\right] \\ \times f_{G_T}\left(\frac{\Gamma}{T}\right). \end{aligned} \quad (27)$$

We notice that Eqs. (19) and (27), together with the expressions for G and G_T in Ref. [28], precisely satisfy the Onsager relation as presented in Eq. (3). Meanwhile, plugging the expression in Eq. (26) into Eq. (5) gives the formula for the

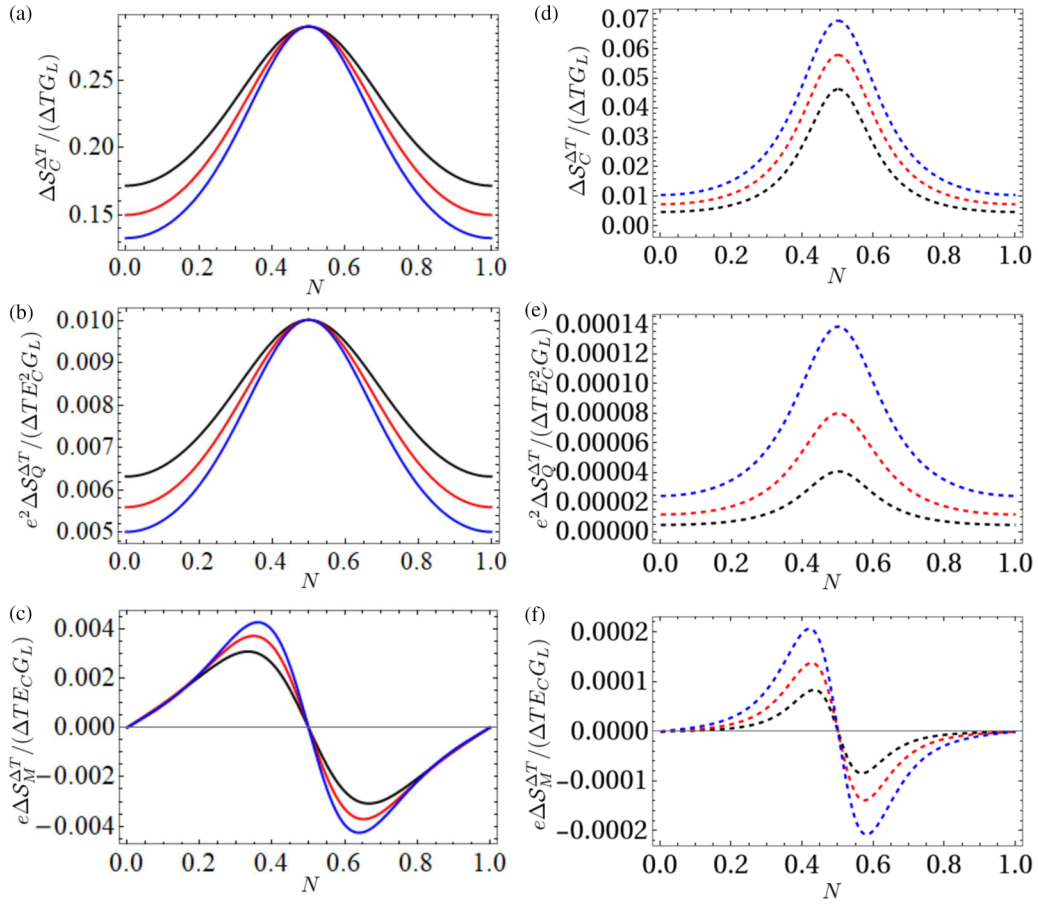


FIG. 3. (a) and (d) Temperature-driven charge noise (ΔT noise) $\Delta S_C^{\Delta T}/G_L$, (b) and (e) temperature-driven heat noise $e^2 \Delta S_Q^{\Delta T}/(E_C^2 G_L)$, and (c) and (f) temperature-driven mixed noise $e \Delta S_M^{\Delta T}/(E_C G_L)$ over the temperature difference ΔT between two sides of the weak link $\Delta T/E_C$ as a function of the gate voltage N . For the plots on the left [(a)–(c)], $T/E_C = 0.01$, black, red, and blue lines correspond to $|r|^2 = 0.06$, $|r|^2 = 0.08$, and $|r|^2 = 0.1$. For the plots on the right [(d)–(f)], $|r|^2 = 0.1$, black, red, and blue lines correspond to $T/E_C = 0.008$, $T/E_C = 0.01$, and $T/E_C = 0.012$.

zero-frequency excess ΔT noise as follows:

$$\Delta S_C^{\Delta T} = \Delta T \frac{G_L}{3\gamma} \frac{T}{E_C} f_C^{\Delta T} \left(\frac{\Gamma}{T} \right), \quad (28)$$

where

$$f_C^{\Delta T}(p) = p + 2\pi \psi^{(1)} \left(\frac{1}{2} + \frac{p}{2\pi} \right) - \frac{p}{4} \left(1 - \frac{p^2}{\pi^2} \right) \psi^{(2)} \left(\frac{1}{2} + \frac{p}{2\pi} \right). \quad (29)$$

Concerning the fluctuations, together with the ΔT noise in Eq. (28), it is also important to investigate both the temperature-driven heat and mixed noises. By applying the zero-voltage and finite-temperature bias conditions into Eqs. (6)–(8), we derive the formula for the average heat current as

$$\langle I_Q^{\Delta T} \rangle = -\Delta T \frac{G_L}{48e^2\gamma} \frac{T^2}{E_C} f_{G_H} \left(\frac{\Gamma}{T} \right), \quad (30)$$

where

$$f_{G_H}(p) = 4p \frac{13\pi^2}{3} - 4p^3 + (3\pi^4 - 4\pi^2 p^2 + p^4) \times \frac{2}{\pi} \psi^{(1)} \left(\frac{1}{2} + \frac{p}{2\pi} \right). \quad (31)$$

The temperature-driven heat noise is obtained as

$$\Delta S_Q^{\Delta T} = \Delta T \frac{G_L}{120e^2\gamma} \frac{T^3}{E_C} f_Q^{\Delta T} \left(\frac{\Gamma}{T} \right), \quad (32)$$

with

$$f_Q^{\Delta T}(p) = 92\pi^2 p - 12p^3 + 112\pi^3 \psi^{(1)} \left(\frac{1}{2} + \frac{p}{2\pi} \right) - p \left(17\pi^2 - 20p^2 + \frac{3p^4}{\pi^2} \right) \psi^{(2)} \left(\frac{1}{2} + \frac{p}{2\pi} \right). \quad (33)$$

The temperature-driven mixed noise in this situation is written as

$$\Delta S_M^{\Delta T} = -\Delta T \frac{G_L T^2}{6\pi e E_C} |r|^2 \sin(2\pi N) \ln \left[\frac{E_C}{T + \Gamma} \right] \times f_M^{\Delta T} \left(\frac{\Gamma}{T} \right), \quad (34)$$

where

$$f_M^{\Delta T}(p) = 12\pi^2 - 16\pi p \psi^{(1)} \left(\frac{1}{2} + \frac{p}{2\pi} \right) + 4p^2 + \left(3\pi^2 - 4p^2 + \frac{p^4}{\pi^2} \right) \psi^{(2)} \left(\frac{1}{2} + \frac{p}{2\pi} \right). \quad (35)$$

The temperature-driven electric and heat noises in Eqs. (28) and (32) are completely different from the voltage-driven electric and heat noises in Eqs. (17) and (21) for both electric and heat transport. The temperature-driven electric and heat noises are associated with the thermal coefficient G_H (because $G_H = \langle I_Q^{\Delta T} \rangle / \Delta T$ in the linear regime), following the Wiedemann-Franz law [43,44]; they are also associated with the electric conductance G (where $G = \langle I_C^V \rangle / V$ in the linear regime), while voltage-driven electric and heat noises

are related to the electric coefficient G_T . (The expressions of G , G_H , and G_T can be seen easily in formulas of average currents.) Figure 3 shows the gate voltage dependence of temperature-driven electric, heat, and mixed noises $\Delta S_C^{\Delta T}$, $\Delta S_Q^{\Delta T}$, and $\Delta S_M^{\Delta T}$, in the presence of a temperature drop ΔT at the tunnel barrier. Temperature- and voltage-driven noise have the same partition origin but are activated by different stimuli; they thus behave completely different from each other.

Although the 2CK fixed point is known to be unstable against channel anisotropy [12] and perfect symmetry is difficult to achieve experimentally [20], our results remain robust for realistic devices. As discussed in detail in Ref. [12], asymmetry between the two channels gives rise to a nontrivial contribution to the Kondo resonance width Γ in the vicinity of Coulomb peaks (see Eq. (11) and eq. (11) of Ref. [12]). Consequently, an additional term appears in the correlation function $K(\tau)$ in Eq. (10), which has the same functional form as the second term but with a prefactor proportional to $|\delta r|^2$, where $|\delta r|$ characterizes the channel asymmetry. Since $|\delta r|$ is always smaller than $|r|$ —the average scattering amplitude at the two QPCs (or, equivalently, the average Kondo exchange coupling)—the new energy scale $\tilde{T} \sim |\delta r|^2 E_C$ is

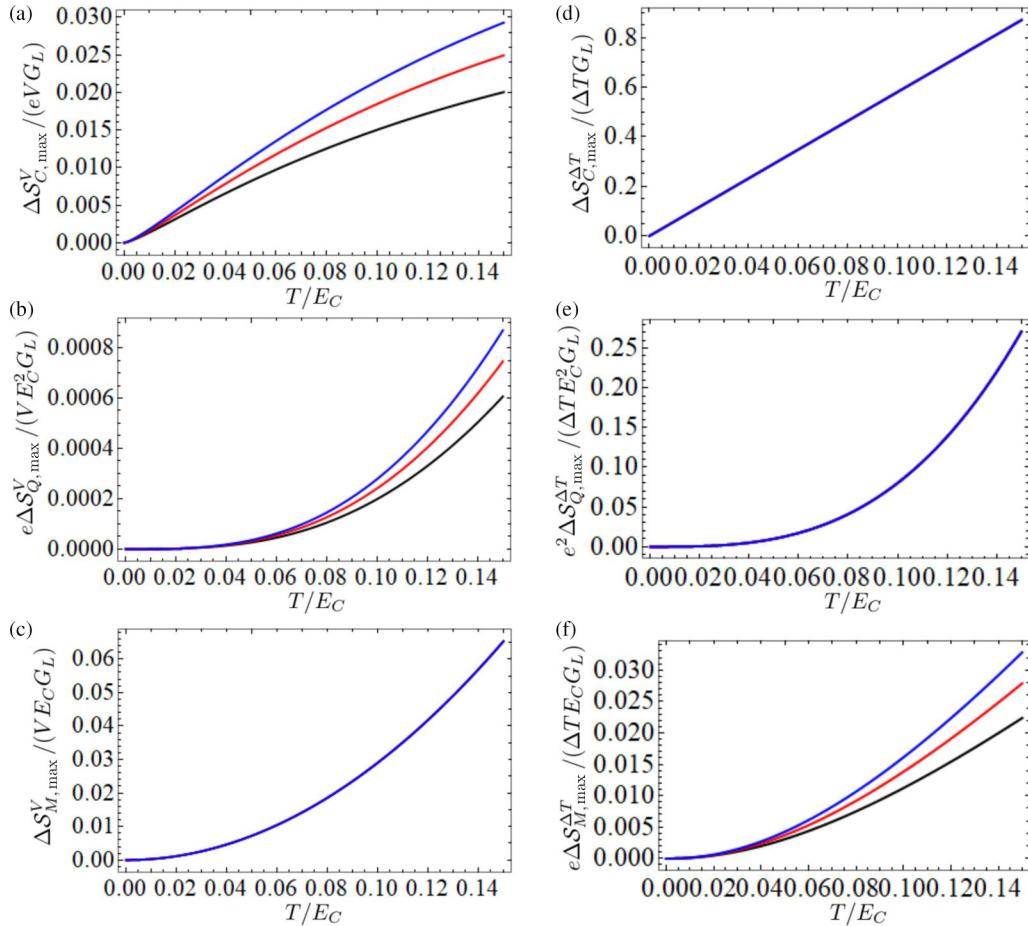


FIG. 4. (a) Maximum of voltage-driven electric noise (shot noise) $\Delta S_{C,\max}^V / (eG_L)$, (b) maximum of voltage-driven heat noise $e\Delta S_{Q,\max}^V / (E_C^2 G_L)$, (c) maximum of voltage-driven mixed noise $\Delta S_{M,\max}^V / (E_C G_L)$ over the voltage difference V between two sides of the weak link, (d) maximum of temperature-driven electric noise $\Delta S_{C,\max}^{\Delta T} / (G_L)$, (e) maximum of temperature-driven heat noise $e^2 \Delta S_{Q,\max}^{\Delta T} / (E_C^2 G_L)$, and (f) maximum of temperature-driven mixed noise $e\Delta S_{M,\max}^{\Delta T} / (E_C G_L)$ over the temperature difference ΔT between two sides of the weak link as functions of temperature T/E_C are plotted. For all plots, black, red, and blue lines are corresponding to $|r|^2 = 0.06$, $|r|^2 = 0.08$, and $|r|^2 = 0.1$.

always smaller than $T^* \sim |r|^2 E_C$. The anisotropy-induced corrections to the noises and thermoelectric coefficients therefore become important at very low temperatures $T \ll \tilde{T}$, in the vicinity of $N \approx 1/2 \pm \pi|\delta r|/4|r|$, where FL behavior dominates. By contrast, in the intermediate temperature window $\tilde{T} \ll T \ll E_C$, the renormalization-group flow has not yet reached the FL fixed point. In this regime, the system exhibits weak NFL behavior characteristic of the vicinity of the symmetric 2CK critical point, allowing the logarithmic noise signatures to persist.

Because the electric current is carried by discrete charge carriers, its fluctuations can be directly accessed using standard electrical measurement techniques. In contrast, heat current corresponds to the transport of energy rather than particles, making heat noise and mixed noise intrinsically difficult to detect. The corresponding signals are typically extremely small, and their measurement is further hindered by detector backaction and uncontrolled coupling to environmental degrees of freedom. Recent theoretical advances have shown that heat noise and mixed noise can nevertheless be accessed indirectly via fluctuation spectroscopy, which probes not only average charge and heat currents but also their frequency-dependent fluctuations [86]. In this framework, a weakly coupled floating probe—such as a QD or a metallic island—serves as a sensitive detector whose temperature and electrochemical potential fluctuate in response to the intrinsic current noise of the interacting region. Analysis of the frequency dependence of these probe-induced fluctuations allows one to reconstruct the auto- and cross-correlations of charge and heat currents in driven mesoscopic conductors [86,87].

C. Symmetry properties

The first two terms in the Hamiltonian describing the QD-QPC structure [see Eq. (2)] exhibit electron-hole (PH) symmetry. The third term, representing the charging energy, is invariant under the electron-hole transformation accompanied by the change of sign of the gate voltage $N \rightarrow -N$. When N is changed to $-N$, the form of the Hamiltonian (without scattering at the QPCs) remains unchanged, provided the electrons are simultaneously converted into holes. However, the PH symmetry in this 2CK model is broken by scattering at the QPCs [28], leading to an imbalance in the thermal distributions of particles and holes. This imbalance explains why, under voltage bias, the heat current across the tunnel barrier is an odd function of the gate voltage N , while the electric current, which reflects charge transport rather than energy transport, does not exhibit the same symmetry. In contrast, when a temperature difference is applied, the roles reverse: It is the electric current that becomes an odd function of N , reflecting the asymmetry in energy transport.

The voltage- and temperature-driven noises, which arise from random fluctuations in both heat and charge transport due to the voltage bias and temperature difference, reflect fluctuations in transport rather than the net flow of carriers. Therefore, the voltage-driven noises in Eqs. (17) and (21), as well as the temperature-driven noises in Eqs. (28) and (32), are unaffected by whether the system is PH symmetric or asymmetric. Namely, ΔS_C^V , ΔS_Q^V , and $\Delta S_M^{\Delta T}$ vanish

when the system is PH symmetric. The other noises remain finite.

Regarding the gate voltage dependence, the ratios $\Delta S_C^V/V$, $\Delta S_Q^V/V$, and $\Delta S_M^{\Delta T}/\Delta T$ are odd functions of N . In contrast, the ratios $\Delta S_M^V/V$, $\Delta S_C^{\Delta T}/\Delta T$, and $\Delta S_Q^{\Delta T}/\Delta T$ are even functions of N , as demonstrated in Figs. 2 and 3. This can be understood by considering time-reversal symmetry (TRS). Under time reversal, the voltage bias V changes sign, whereas the temperature difference ΔT remains unchanged. Although both the electric current I_C and the heat current I_Q are odd under TRS, they originate from different physical mechanisms. As a result, their cross-correlation $\langle I_C(t)I_Q(t') \rangle$ is odd under TRS, while the autocorrelations $\langle I_C(t)I_C(t') \rangle$ and $\langle I_Q(t)I_Q(t') \rangle$ are even under TRS.

Figure 4 demonstrates the monotonic temperature dependence of the maxima of voltage- and temperature-driven noise for electric current, heat current, and their combination, for different values of reflection amplitudes at the QPC. When the noise expressions as a function of N are even, the noises exhibit the same maximum value at $N = 0.5$ for all r . However, when the noise expressions are odd, the maximum values increase with r . In terms of thermoelectric properties, we observe that, as the reflection at the QPC increases, both the maximum of the thermopower [$S_{\max} \sim |r|^2 \ln(|r|^2)$] and the figure of merit [$ZT_{\max} \sim |r|^4 \ln^2(|r|^2)$] are enhanced [28,73,82]. Simultaneously, voltage-driven noise of both the electric current and heat current increases, while the temperature-driven noise remains unchanged. These findings are promising when investigating 2CK as a thermoelectric material.

D. Fano factors

In this subsection, we examine the Fano factor, a dimensionless quantity that describes the statistical properties of fluctuations in particle counting. It is defined as $F_C = \Delta S_C/2e\langle I_C \rangle$ for charge transport and $F_Q = \Delta S_Q/2T\langle I_Q \rangle$ for heat transport.

The Fano factors (a) F_C^V , (b) F_Q^V , (c) $F_C^{\Delta T}$, and (d) $F_Q^{\Delta T}$ as a function of the gate voltage are plotted for different temperatures in Fig. 5. Interestingly, the Fano factors F_C^V and $F_C^{\Delta T}$ either vanish or diverge in the Coulomb peaks, but their product is always finite, yielding

$$\begin{aligned} F_C^V F_C^{\Delta T} &= -\frac{12 - \frac{8p}{\pi}\psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right) + \left(1 - \frac{p^2}{\pi^2}\right)\psi^{(2)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)}{p + \frac{\pi}{2}\left(1 - \frac{p^2}{\pi^2}\right)\psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)} \\ &\quad \times \frac{4p + 8\pi\psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right) - p\left(1 - \frac{p^2}{\pi^2}\right)\psi^{(2)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)}{16\left[\frac{4\pi^2}{3} - 16p^2 - 8\pi p\left(1 - \frac{p^2}{\pi^2}\right)\psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)\right]}, \end{aligned} \quad (36)$$

which reduces, in the two limiting regimes, to

$$F_C^V F_C^{\Delta T} \xrightarrow{\Gamma \ll T} -3 \frac{12 + \psi^{(2)}\left(\frac{1}{2}\right)}{4\pi^2} \approx 0.37, \quad (37)$$

$$F_C^V F_C^{\Delta T} \xrightarrow{\Gamma \gg T} \frac{15}{4\pi^2} \approx 0.38. \quad (38)$$

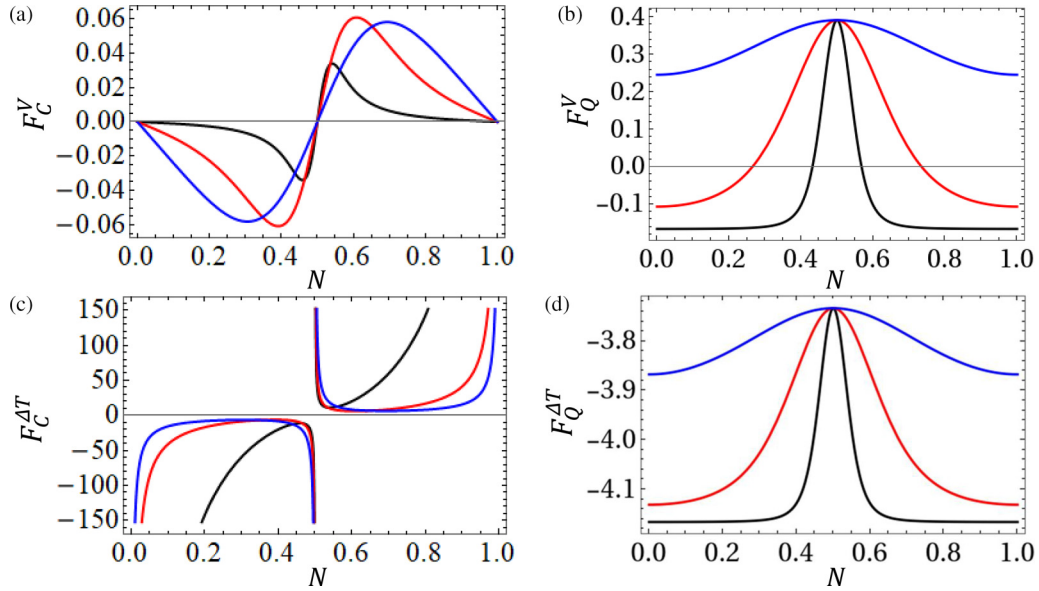


FIG. 5. Fano factors as functions of the gate voltage N : (a) F_C^V , (b) F_Q^V , (c) $F_C^{\Delta T}$, and (d) $F_Q^{\Delta T}$ are plotted with different temperatures $T/E_C = 0.001, 0.01, \text{ and } 0.1$ corresponding to black, red, and blue lines. For all plots, $|r|^2 = 0.1$.

In fact, by relating the Fano factors with the thermopower in the linear response regime $S = G_T/G$ of the system, we can make a conjecture as $F_C^V \times S = \text{constant} \times (T/E_C)^\alpha$, while $F_C^{\Delta T}/S = \text{constant} \times (T/E_C)^{-\alpha}$ with $\alpha > 0$ is a constant [where for the 2CK system, $\alpha = 1/2$ in the nonperturba-

tive (low temperature) regime and $\alpha = 0$ in the perturbative (higher temperature) regime]. Therefore, the product ($F_C^V F_C^{\Delta T}$) is a universal constant.

The heat Fano factors F_Q^V Fig. 5(b) and $F_Q^{\Delta T}$ Fig. 5(d) reach their maximum/minimum absolute values in the vicinity of the Coulomb peaks. Their product is

$$F_Q^V F_Q^{\Delta T} = -\frac{41\pi^2 - 21p^2 - \frac{12p}{\pi}(2\pi^2 - p^2)\psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right) + \frac{3}{4}(3\pi^2 - 4p^2 + \frac{p^4}{\pi^2})\psi^{(2)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)}{8\pi^2 - 6p^2 + \frac{3p}{\pi}(p^2 - \pi^2)\psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)} \times \frac{92\pi^2 p - 12p^3 + 112\pi^3 \psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right) - p(17\pi^2 - 20p^2 + \frac{3p^4}{\pi^2})\psi^{(2)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)}{5\left[4p\frac{13\pi^2}{3} - 4p^3 + \frac{2}{\pi}(3\pi^4 - 4\pi^2 p^2 + p^4)\psi^{(1)}\left(\frac{1}{2} + \frac{p}{2\pi}\right)\right]}, \quad (39)$$

which also reduces, in the two limiting regimes, to

$$F_Q^V F_Q^{\Delta T} \xrightarrow{\Gamma \ll T} -\frac{287}{15} + \frac{63}{60}\psi^{(2)}\left(\frac{1}{2}\right) \approx -1.4631, \quad (40)$$

$$F_Q^V F_Q^{\Delta T} \xrightarrow{\Gamma \gg T} \frac{25}{36} \approx 0.6944. \quad (41)$$

This universal behavior is in full agreement with the general theory establishing the universal relations between thermoelectrics and noise in mesoscopic transport across a tunnel junction [84].

From Fig. 5, we observe that, near the Coulomb peaks, a voltage bias suppresses both charge and heat current fluctuations, as indicated by Fano factors less than one. In contrast, a temperature bias significantly enhances these fluctuations, resulting in Fano factors greater than one. Under a voltage bias, the charge Fano factor vanishes and changes sign at the Coulomb peak, while the heat Fano factor reaches its maximum at the same point. At low temperatures, the heat Fano factor can even approach zero and become negative in the vicinity of the Coulomb peak. When a temperature difference is applied, the charge Fano factor exhibits a discontinuous

behavior at the Coulomb peak. Meanwhile, the heat Fano factor again attains its maximum at the peak but remains negative. In both cases, whether driven by voltage or temperature bias, the charge Fano factor displays antisymmetric behavior around the Coulomb peak, while the heat Fano factor remains symmetric.

E. Comparison with 1CK model

Authors of previous studies [12,28] have shown that thermoelectric transport properties differ significantly between 1CK and 2CK systems. In the 1CK regime, the thermoelectric coefficients are consistent with FL behavior, whereas in the 2CK regime, they exhibit NFL characteristics. Therefore, in this subsection, we compare the results of the 2CK circuit with those of the 1CK case to gain a deeper understanding of the distinct physical signatures and emergent phenomena associated with multichannel Kondo systems.

The charge Kondo model with 1CK exhibits FL behavior and can be treated perturbatively [28]. The DOS of the QD at

the weak link is given by

$$v_D^{\text{1CK}}(\epsilon) = \frac{\nu_{D0}\pi^2}{2\gamma^2 E_C^2} \left\{ [1 - 2\gamma\xi|r|\cos(2\pi N)](\epsilon^2 + \pi^2 T_R^2) + \frac{4\pi\gamma\xi}{3E_C} |r|\sin(2\pi N)\epsilon(\epsilon^2 + \pi^2 T_R^2) \right\}, \quad (42)$$

where ξ is a numerical constant $\xi \approx 1.59$. The perturbative solution is valid in the temperature regime $|r|^2 E_C \ll T \ll E_C$, with the requirement $|r| \ll 1$.

1. Voltage bias and noises

The average charge current is

$$\langle I_C^{V,1CK} \rangle = V[1 - 2\gamma\xi|r|\cos(2\pi N)] \frac{2\pi^4 G_L T^2}{3\gamma^2 E_C^2}. \quad (43)$$

The equilibrium charge noise is $S_{C,\text{eq}}^{\text{1CK}} = [1 - 2\gamma\xi|r|\cos(2\pi N)]8G_L\pi^4 T^3/3\gamma^2 E_C^2 = 4GT$. The shot noise (linear in eV) is

$$\Delta S_C^{V,1CK} = -V \frac{8e\pi^5 \xi G_L}{3\gamma} |r| \left(\frac{T}{E_C} \right)^3 \sin(2\pi N), \quad (44)$$

with the corresponding Fano factor

$$F_C^{V,1CK} = -2\pi\xi\gamma \frac{T}{E_C} |r|\sin(2\pi N). \quad (45)$$

To compare with the 2CK case in the same temperature regime $|r|^2 E_C \ll T \ll E_C$ (i.e., $\Gamma \ll T \ll E_C$), we write the explicit formula for the shot noise as shown in Eq. (17) as

$$\Delta S_C^{V,2CK} \sim -V|r|^2 \frac{T}{E_C} \ln\left(\frac{E_C}{T}\right) \sin(2\pi N). \quad (46)$$

The charge Fano factor for 2CK in this case is

$$F_C^{V,2CK} \approx \frac{4\gamma}{\pi^4} \left[12 + \Psi^{(2)}\left(\frac{1}{2}\right) \right] \times |r|^2 \ln\left(\frac{E_C}{T}\right) \sin(2\pi N). \quad (47)$$

Similarly, the heat current of the 1CK setup is

$$\langle I_Q^{V,1CK} \rangle = V \frac{8\pi^7 \xi G_L T^4}{15e\gamma E_C^3} |r|\sin(2\pi N). \quad (48)$$

The equilibrium noise $S_{Q,\text{eq}}^{\text{1CK}} = [1 - 2\gamma\xi|r|\cos(2\pi N)]8G_L\pi^6 T^5/5\gamma^2 E_C^2 = 4T^2 G_H$, and the voltage-induced part reads

$$\Delta S_Q^{V,1CK} = -V \frac{8\pi^7 \xi G_L T^5}{45e\gamma E_C^3} |r|\sin(2\pi N). \quad (49)$$

The Fano factor is a constant $F_Q^{V,1CK} = -1/6 \approx -0.1667$.

For 2CK in this condition, from Eq. (21), we find the voltage-induced part as

$$\Delta S_Q^{V,2CK} \sim V|r|^2 \frac{T^3}{E_C} \ln\left(\frac{E_C}{T}\right) \sin(2\pi N), \quad (50)$$

and the corresponding Fano factor $F_Q^{V,2CK} = 41/8 + 9\psi^{(2)}(1/2)/32 \approx 0.39$.

The mixed noise is a bit different from the above two noises. It includes the equilibrium term $S_{M,\text{eq}}^{\text{1CK}} = (32G_L\pi^7 \xi T^5/15\gamma E_C^3)|r|\sin(2\pi N) = -4G_T T^2$, and the voltage-induced component

$$\Delta S_M^{V,1CK} = V[1 - 2\gamma\xi|r|\cos(2\pi N)] \frac{2\pi^4 G_L T^3}{3\gamma^2 E_C^2}, \quad (51)$$

while the mixed noise of 2CK in this temperature regime is obtained from Eq. (23) as

$$\Delta S_M^{V,2CK} \sim V \frac{T^2}{E_C}. \quad (52)$$

2. Temperature bias and noises

The average of the electric current in this situation is

$$\langle I_C^{\Delta T,1CK} \rangle = -\Delta T \frac{8\pi^7 \xi G_L}{15e\gamma} \left(\frac{T}{E_C} \right)^3 |r|\sin(2\pi N). \quad (53)$$

The equilibrium delta-T noise $S_{C,\text{eq}}^{\Delta T,1CK} = [1 - 2\gamma\xi|r|\cos(2\pi N)]8G_L\pi^4 T^3/3\gamma^2 E_C^2 = 4GT$, and the ΔT -induced part is

$$\Delta S_C^{\Delta T,1CK} = \Delta T \frac{2\pi^4 G_L}{\gamma^2} \left(\frac{T}{E_C} \right)^2 [1 - 2\gamma\xi|r|\cos(2\pi N)]. \quad (54)$$

The Fano factor $F_C^{\Delta T,1CK}$ is

$$F_C^{\Delta T,1CK} \approx -\frac{15}{8\pi^3 \gamma \xi} \frac{E_C}{T} \frac{1}{|r|\sin(2\pi N)}. \quad (55)$$

From Eqs. (45) and (55), we find that

$$F_C^{V,1CK} F_C^{\Delta T,1CK} \approx \frac{15}{4\pi^2} \approx 0.38. \quad (56)$$

This is again in full agreement with the general theory [84].

For 2CK, from Eq. (28), we get (in the relevant regime $\Gamma \ll T$)

$$\Delta S_C^{\Delta T,2CK} \sim \Delta T \left\{ \frac{T}{E_C} + |r|^2 [1 + \cos(2\pi N)] \right\}, \quad (57)$$

and the Fano factor is

$$F_C^{\Delta T,2CK} \approx -\frac{3\pi^2}{16|r|^2 \sin(2\pi N) \ln\left(\frac{E_C}{T}\right)}. \quad (58)$$

We also reobtain the product of Fano factors, which has been shown in Eq. (37). Interestingly, the product of the Fano factors for charge transport under voltage and temperature bias in both 1CK and 2CK models are numbers.

For heat transport, the average current is

$$\langle I_Q^{\Delta T,1CK} \rangle = -\Delta T \frac{2G_L\pi^6 T^3}{5e^2\gamma^2 E_C^2} [1 - 2\gamma\xi|r|\cos(2\pi N)]. \quad (59)$$

The temperature-driven heat noise includes $S_{Q,\text{eq}}^{\Delta T,1CK} = [1 - 2\gamma\xi|r|\cos(2\pi N)]8G_L\pi^6 T^5/5\gamma^2 E_C^2 = -4T^2 G_H$ and

$$\Delta S_Q^{\Delta T,1CK} = \Delta T \frac{10G_L\pi^6 T^4}{e^2 3\gamma^2 E_C^2} [1 - 2\gamma\xi|r|\cos(2\pi N)]. \quad (60)$$

TABLE I. A comparison table highlighting the qualitative differences in the temperature scaling of nonequilibrium noise contributions between the 1CK and 2CK cases.

Quantity	NFL-2CK: $ r ^2 E_C \ll T \ll E_C$	Giant FL-2CK: $T \ll r ^2 E_C$	FL-1CK: $ r ^2 E_C \ll T \ll E_C$	Experimental signature
$\Delta S_C^V/V$	$ r ^2 \frac{T}{E_C} \ln \left[\frac{E_C}{T} \right]$	$\frac{1}{ r ^2} \left[\frac{T}{E_C} \right]^3 \ln \left[\frac{E_C}{T} \right]$	$ r \left[\frac{T}{E_C} \right]^3$	NFL: log-enhanced; FL: cubic
$\Delta S_Q^V/V$	$ r ^2 \frac{T^3}{E_C} \ln \left[\frac{E_C}{T} \right]$	$\frac{1}{ r ^2} \frac{T^5}{E_C^3} \ln \left[\frac{E_C}{T} \right]$	$ r \frac{T^5}{E_C^3}$	Indicating charge and energy transport
$\Delta S_M^V/V$	$\frac{T^2}{E_C}$	$\frac{1}{ r ^2} \frac{T^3}{E_C^2}$	$\frac{T^3}{E_C^2}$	NFL: quadratic; FL: cubic
$\Delta S_C^{\Delta T}/\Delta T$	$\frac{T}{E_C}$	$\frac{1}{ r ^2} \left[\frac{T}{E_C} \right]^2$	$\left[\frac{T}{E_C} \right]^2$	NFL: linear- T ; FL: quadratic
$\Delta S_Q^{\Delta T}/\Delta T$	$\frac{T^3}{E_C}$	$\frac{1}{ r ^2} \frac{T^4}{E_C^2}$	$\frac{T^4}{E_C^2}$	Indicating charge and energy transport
$\Delta S_M^{\Delta T}/\Delta T$	$ r ^2 \frac{T^2}{E_C} \ln \left[\frac{E_C}{T} \right]$	$\frac{T^4}{E_C^3} \ln \left[\frac{E_C}{T} \right]$	$ r \frac{T^4}{E_C^3}$	NFL: log-enhanced; FL: T^4

Analogous to the voltage-driven scenario, the heat Fano factor in response to a temperature bias is also found to be constant, $F_Q^{\Delta T, 1CK} = -25/6 \approx -4.1667$.

For 2CK, from Eq. (32), we find

$$\Delta S_Q^{\Delta T, 2CK} \sim \Delta T \left\{ \frac{T^3}{E_C} + T^2 |r|^2 [1 + \cos(2\pi N)] \right\}, \quad (61)$$

and the Fano factor $F_Q^{\Delta T, 2CK} = -56/15 \approx -3.73$. The product of the heat Fano factors in the high-temperature regime of 1CK is c.f. [84]

$$F_Q^{V, 1CK} F_Q^{\Delta T, 1CK} = \frac{25}{36} \approx 0.6944. \quad (62)$$

The dependence of the Fano factors on system parameters shows similar characteristics in both 1CK and 2CK systems.

In addition to the equilibrium term $S_{M,eq}^{\Delta T, 1CK} = (32G_L\pi^7 \xi T^5/15\gamma E_C^3)|r| \sin(2\pi N) = -4G_T T^2$, the temperature-bias-induced contribution to the mixed noise in the 1CK model is given by

$$\Delta S_M^{\Delta T, 1CK} = \Delta T \frac{G_L}{e} \frac{40\pi^7 \xi}{9\gamma} \frac{T^4}{E_C^3} |r| \sin(2\pi N). \quad (63)$$

The corresponding contribution for the 2CK model can be evaluated from Eq. (34) as

$$\Delta S_M^{\Delta T, 2CK} \sim -\Delta T |r|^2 \frac{T^2}{E_C} \ln \left(\frac{E_C}{T} \right) \sin(2\pi N). \quad (64)$$

We find that the symmetry of both the noise and the Fano factor with respect to the gate voltage N is identical in the 1CK and 2CK models. Moreover, shot noise and delta- T noise of the charge current exhibit a behavior distinct from that of the corresponding differential electric conductance, much like the thermoelectric coefficient. In contrast, mixed noise closely follows the behavior of the differential conductance.

For the heat current, the voltage- and temperature-driven noise behave like the corresponding differential heat conductance. However, this correspondence does not hold for the mixed noise, which deviates from the behavior of the differential heat conductance.

Regarding PH symmetry, the 1CK and 2CK models exhibit consistent behavior. Both the voltage-driven charge and heat noises vanish when PH symmetry is broken, whereas the voltage-driven mixed shot noise remains finite even under PH-symmetric conditions. For temperature-driven noise,

the charge and heat contributions persist under PH symmetry in both models, while the mixed noise vanishes when PH symmetry is broken. These results indicate that mixed noise provides complementary information compared with charge or heat noise, particularly in relation to PH symmetry breaking.

The Fano factors of the heat currents in both cases (voltage and temperature bias) are constant in both charge Kondo models because both the heat current and its noise scale similarly in the low-temperature limit ($T \ll E_C$), governed by universal scattering processes around the Kondo fixed point. This leads to a universal constant ratio, a hallmark of the underlying quantum critical behavior in 2CK and FL behavior in 1CK.

Table I summarizes the distinct temperature scalings of nonequilibrium noise in the 1CK and 2CK regimes and highlights clear signatures of NFL physics. In the NFL-2CK regime ($|r|^2 E_C \ll T \ll E_C$), the noise contributions $\Delta S_C^V/V$, $\Delta S_Q^V/V$, and $\Delta S_M^{\Delta T}/\Delta T$ display anomalous power-law behavior accompanied by logarithmic enhancements. These features originate from impurity overscreening and the absence of well-defined quasiparticles. At lower temperatures ($T \ll |r|^2 E_C$), the system crosses over into a giant FL regime, characterized by conventional power-law scaling but with strongly enhanced amplitudes [12,27,28]. These large prefactors are inherited from the nearby NFL fixed point and reflect the renormalization-group flow connecting the NFL-2CK and FL-1CK regimes. By contrast, the FL-1CK case exhibits similar scaling exponents to those of the FL-2CK regime, but without enhanced prefactors and, unlike the NFL-2CK regime, without logarithmic corrections, consistent with standard quasiparticle transport.

Notably, for both the 1CK and 2CK regimes, the temperature dependence of heat noise differs from that of charge noise, obeying

$$\frac{\Delta S_Q^V}{\Delta S_C^V} \propto \frac{\Delta S_Q^{\Delta T}}{\Delta S_C^{\Delta T}} \propto \frac{T^2}{e^2}, \quad (65)$$

indicating a nontrivial separation between charge and energy transport. The universal relation in Eq. (65) provides a direct means to calibrate voltage- or temperature-induced heat noise from the corresponding charge noise. Two-channel charge Kondo devices in the IQH regime have already been fabricated [20,21], and temperature-induced charge noise has been

measured experimentally [61]. Based on these advances, we expect that voltage- and temperature-induced heat noise can be accessed with current experimental techniques.

Overall, the qualitative comparison of scaling behaviors summarized in Table I reveals robust and experimentally relevant distinctions between the 1CK and 2CK regimes, providing clear guidance for identifying NFL physics in charge Kondo nanodevices.

F. General reciprocity relations for noises

In this subsection, we briefly recapitulate the universal reciprocity relations established in Ref. [84] for the general case of mesoscopic transport across a tunnel junction and illustrated by the theory for the noise signatures of a charged Sachdev-Ye-Kitaev dot [85].

The general expressions for the charge, heat, and mixed noise in the linear response regime can be derived for both the 1CK and 2CK charge Kondo models as follows:

$$\begin{aligned} S_C &= 4TG - V \frac{eG_L}{2T} \mathcal{N}_0 + \Delta T \frac{G_L}{2T^2} \mathcal{N}_1, \\ S_Q &= -4T^2 G_H - V \left[8T^2 G_T + \frac{G_L}{2eT} \mathcal{N}_2 \right] + \Delta T \frac{G_L}{2e^2 T^2} \mathcal{N}_3, \\ S_M &= -4T^2 G_T - V \left[-4TG + \frac{G_L}{2T} \mathcal{N}_1 \right] + \Delta T \frac{G_L}{2eT^2} \mathcal{N}_2, \end{aligned} \quad (66)$$

where for $l = 0, 1, 2, 3$,

$$\mathcal{N}_l = \int_{-\infty}^{\infty} \epsilon^l \frac{v_D(\epsilon)}{v_{D,0}} \frac{\sinh\left(\frac{\epsilon}{2T}\right)}{\cosh^3\left(\frac{\epsilon}{2T}\right)} d\epsilon. \quad (67)$$

The ratios of the thermal (equilibrium) noises to the Onsager coefficients satisfy the so-called Onsager relations [fluctuation-dissipation theorem (FDT)].

$$\frac{S_C^{\text{eq}}}{G} = -\frac{1}{T} \frac{S_Q^{\text{eq}}}{G_H} = -\frac{1}{T} \frac{S_M^{\text{eq}}}{G_T} = 4T. \quad (68)$$

These ratios exhibit universal behavior: They follow from conservation laws and microscopic reversibility. They are entirely independent of system parameters (for the 1CK and 2CK cases, these parameters are the number of channels, reflection amplitudes, etc.) and are determined solely by the temperature. Their origin lies in equilibrium statistical mechanics, where the same microscopic scattering processes responsible for irreversible dissipation under an external drive also generate equilibrium current fluctuations. Consequently, it can be straightforwardly shown that the Wiedemann-Franz law can be re-expressed in terms of the thermal noises, and the corresponding Lorenz ratio satisfies

$$R = -\frac{3}{\pi^2 T^2} \left[\frac{S_Q^{\text{eq}}}{S_C^{\text{eq}}} + \left(\frac{S_M^{\text{eq}}}{S_C^{\text{eq}}} \right)^2 \right]. \quad (69)$$

In short, Johnson-Nyquist noise is not an imperfection. It is the equilibrium fingerprint of electrical dissipation, mandated by the FDT.

Out of equilibrium, detailed balance is broken and the FDT no longer holds, so current noises are not fixed by dissipation alone and include additional contributions. In the

out-of-equilibrium linear response regime, the mixed, voltage- and temperature-driven noises are not independent [84,85]. The general reciprocity relations for the noises are expressed as follows (see more details of the derivation in Refs. [84,85]):

$$\frac{\partial S_M}{\partial V} + T \frac{\partial S_C}{\partial \Delta T} = 4TG, \quad (70)$$

$$\frac{\partial S_Q}{\partial V} + T \frac{\partial S_M}{\partial \Delta T} = -8T^2 G_T. \quad (71)$$

Equations (70) and (71) are exactly satisfied for Kondo models with an arbitrary number of channels. The validity of these reciprocal relations is not restricted by the specific model or by whether the system exhibits FL or NFL behavior. Moreover, these equations establish not only universal reciprocity relations among the three types of noise but also a universal connection between nonequilibrium noise and thermoelectric transport.

More generally, the fundamental connection between current correlations and transport coefficients follows from charge and energy conservation together with Onsager reciprocity, rendering these relations robust across a broad class of interacting quantum systems. Even out of equilibrium, where the strict form of the FDT is violated, noise correlations continue to encode thermoelectric transport properties, underscoring that current fluctuations constitute a universal and powerful probe of transport and interactions rather than a mere byproduct of dissipation.

From an experimental perspective, the reciprocity relations in Eqs. (70) and (71) enable access to heat and mixed noise in charge Kondo circuits using electrical measurements alone. High-precision charge-noise and noise-thermometry techniques have already been implemented in charge Kondo devices based on floating metallic islands coupled to quantum Hall edge channels, where both symmetric and asymmetric two-channel regimes and voltage- and temperature-induced charge noise have been realized [20,21,61]. These devices share the same floating-probe architecture that allows electronic temperature and noise measurements [61,88,89]. Combined with the universal reciprocity relations, these techniques make it feasible to extract heat noise and mixed noise in charge Kondo circuits with current experimental capabilities.

VI. CONCLUSIONS

In summary, we have presented a detailed analysis of both electric and heat currents, as well as electric, heat, and mixed noises under small voltage and temperature biases. These noise characteristics exhibit distinct NFL behavior in the 2CK model, in contrast with the conventional FL behavior seen in the 1CK case. Importantly, noise signals are closely related to the thermoelectric coefficients in the linear response regime, establishing a direct link between fluctuations and transport properties.

We highlight unique features of exotic charge Kondo systems that are now accessible in experiments. By combining measurements of the thermoelectric coefficients and noises, we have provided valuable insights into the interplay between charge and heat transport in strongly correlated mesoscopic systems. The universality observed in the Fano factor

products for electric and heat currents under both voltage and temperature biases reflects fundamental aspects of reversible thermoelectric effects and fixed-point physics.

The logarithmic temperature-dependence characteristic of the 2CK model is further manifested in the thermopower, suggesting that nonlinear Seebeck coefficient and noise measurements offer sensitive probes of electron correlations in nanostructures. Although shot noise (voltage-driven electric noise) has been well studied in many mesoscopic and nanoscopic systems, both theoretically and experimentally, our focus on voltage-driven electric/heat noise, voltage-driven mixed noise, and temperature-driven noises in the NFL regime of the two-channel charge Kondo quantum simulators uncovers complementary information about temperature-induced fluctuations and nonequilibrium dynamics. Our analysis also illustrates the nonequilibrium reciprocity relations and a thermal-noise analog of the Wiedemann-Franz law recently reported in Refs. [84,85], demonstrating a profound universality governed solely by temperature, independent of system-specific details.

Overall, in this work, we advance the fundamental understanding of quantum transport and noises in strongly

correlated charge Kondo circuits, offering experimentally testable predictions that can guide future studies and the development of nanoscale thermoelectric devices operating beyond linear response.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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