Shuttle-promoted nano-mechanical current switch: Supplementary material

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TWO CRITICAL VOLTAGE v_2 AND v_3

In order to find stationary state without perpendicular magnetic fields, we linearize the problem, of stability of the NEM motion. The equation of motion Eq. (3) in main text can be expressed as three coupled first order differential equations by representing the time dependent $(x, \dot{x}, q_c)$ as three variables (x, y, z):

$$\dot{x} = y \tag{S1}$$

$$\dot{y} = -\frac{1}{Q_0}y - x + \frac{\alpha}{d^2}z^2 x \tag{S2}$$

$$\dot{z} = -\frac{1}{\tau_0}\{(\cosh(x) + \frac{1}{r_w})z - \frac{v}{r_w}\}. \tag{S3}$$

One can find the fixed point of system, $\{X^*, X^*_+, X^*_-\}$, by means of simple algebra from three conditions $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$. The result is: $X^* = (0, 0, \frac{v}{1+r_w})$, and $X^*_\pm = (\pm\cosh^{-1}(\frac{v\sqrt{\alpha/d^2}-1}{r_w}), 0, \sqrt{\frac{d^2}{\alpha}})$. The value of v_1 is also obtained from $X^*_\pm$. We evaluate the eigenvalues, λ, of Jacobian matrix for the system of first order differential equations. This matrix always possesses a real eigenvalue (λ_0), and two complex conjugate values $\lambda_\pm$ for $\{X^*, X^*_+, X^*_-\}$. In Fig. S1, the $X^*_\pm$ are shown when $v_1 < v$, and the eigenvalues are presented by red filled dots. The real part of eigenvalues of $X^*_\pm$ cross zero at some voltage (v_2) as shown in lower inset of Fig. S1 (a).

In order to find crossing point, we evaluate amplitude equation by means of ansatz $x(t) = x_\pm + A\sin(\omega t + \phi)$. Inserting this function into equation of motion and then averaging over a period with $\cos\omega t$, and $\sin\omega t$ yields phase

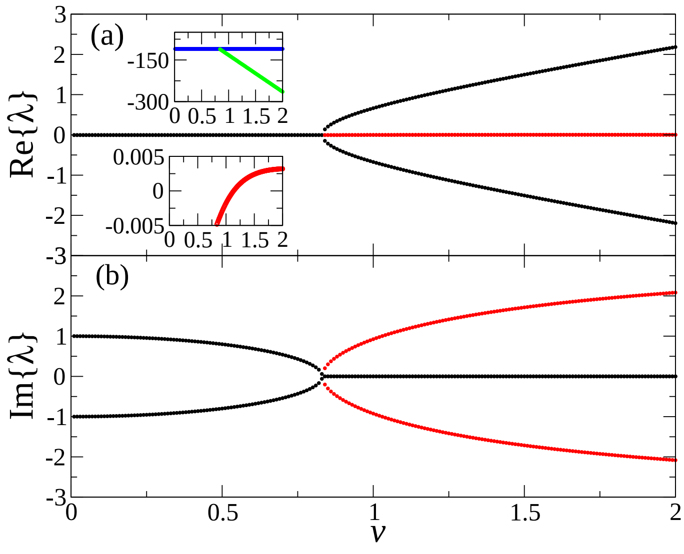

FIG. S1: Real part (a) and imaginary part (b) of eigenvalues of Jacobian matrix of the system, $\lambda_\pm$. The black (red) line represents $\lambda_\pm$ evaluated at the X^* $(X^*_\pm)$ as a function of v. Inset of (a): Upper : Large negative real value $lambda_0$ at X^* (blue) and $X^*_\pm$ (green). Lower: $X^*_\pm$ in the vicinity of v_2

equation,

$$\frac{\omega}{2}(\frac{1}{Q_0}\cos\phi - \omega\sin\phi) - \frac{\sin\phi}{8(1-\cosh x_\pm)^4}F(v,\frac{\alpha}{d^2},A,r_w) = 0 \quad \text{(S4)}$$

$$\frac{\omega}{2}(\omega\cos\phi + \frac{1}{Q_0}\sin\phi) - \frac{\cos\phi}{8(1-\cosh x_\pm)^4}F(v,\frac{\alpha}{d^2},A,r_w) = 0 \quad \text{(S5)}$$

where $F(v,\frac{\alpha}{d^2},A,r_w)$ is

$$F(v,\frac{\alpha}{d^2},A,r_w) = v^2\frac{\alpha}{d^2}(1-v\sqrt{\frac{\alpha}{d^2}}+r_w)\{3A^2(v\sqrt{\frac{\alpha}{d^2}}-1-r_w) - 8v\sqrt{\frac{\alpha}{d^2}}\sqrt{\frac{v\sqrt{\alpha/d^2}-1-r_w}{v\sqrt{\alpha/d^2}-1+r_w}}\cosh^{-1}(\frac{v\sqrt{\frac{\alpha}{d^2}}-1}{r_w})\}$$

Here, we use first order term in expansion of $\cosh(x)$ at $x_\pm$ for $q_c(t) = v/(1+r_w\cosh(x(t)))$. Typically, $r_w(=R_w/R_0)$ can be controlled in the range of $r_w \ll 1$, since the tunnel resistance R_0 is order of $G\Omega$. This condition allow us to assume $1 \pm r_w \approx 1$ which implies, $\sinh(\cosh^{-1}(x)) = \sqrt{\frac{x-1}{x+1}}(x+1) \approx (x+1)$. Then, $F(v,\frac{\alpha}{d^2},A,r_w)$ simplifies:

$$F'(v,\frac{\alpha}{d^2},A,r_w) = v^2\frac{\alpha}{d^2}(1-v\sqrt{\frac{\alpha}{d^2}})\{3A^2(v\sqrt{\frac{\alpha}{d^2}}-1) - 8v\sqrt{\frac{\alpha}{d^2}}\cosh^{-1}(\frac{v\sqrt{\frac{\alpha}{d^2}}-1}{r_w})\}$$

Using this approximate function $F'(v,\frac{\alpha}{d^2},A,r_w)$, the amplitude square is obtained from the sum of squared Eqs (S4) and (S5):

$$A^2 = \frac{4}{3\left(v\sqrt{\frac{\alpha}{d^2}}-1\right)^4}\left(\sqrt{\omega^2\left(\frac{1}{Q_0^2}+\omega^2\right)\left(v\sqrt{\frac{\alpha}{d^2}}-v^2\frac{\alpha}{d^2}\right)^4} + 2v\sqrt{\frac{\alpha}{d^2}}\left(v\sqrt{\frac{\alpha}{d^2}}-1\right)^3\cosh^{-1}\left(\frac{\sqrt{\frac{\alpha}{d^2}}-1}{r_w}\right)\right)$$

The values of v_2 and v_3 can be defined from conditions $A^2 = 0$ for v_2, and $A^2 = x_\pm^2$ for v_3. Applying $\frac{1}{Q_0^2} = 0$, one can rewrite these conditions in terms of v_2 and v_3;

$$v_2\sqrt{\frac{\alpha}{d^2}} = \frac{6\left(\cosh^{-1}\left(\frac{v_2\sqrt{\alpha/d^2}-1}{r_w}\right)\right)^2}{\left(2\cosh^{-1}\left(\frac{v_2\sqrt{\alpha/d^2}-1}{r_w}\right)-\omega^2\right)^2}, \quad \text{(S6)}$$

$$v_3\sqrt{\frac{\alpha}{d^2}} = \frac{18\left(\cosh^{-1}\left(\frac{v_3\sqrt{\alpha/d^2}-1}{r_w}\right)\right)^2}{D_1(v,\frac{\alpha}{d^2},r_w)+D_2(v,\frac{\alpha}{d^2},r_w)} \quad \text{(S7)}$$

where,

$$D_1(v,\frac{\alpha}{d^2},r_w) = 3\cosh^{-1}\left(\frac{v_3\sqrt{\alpha/d^2}-1}{r_w}\right)\left(3\cosh^{-1}\left(\frac{v_3\sqrt{\alpha/d^2}-1}{r_w}\right)-8\right) - 12\omega^2 + 32,$$

$$D_2(v,\frac{\alpha}{d^2},r_w) = 4\sqrt{(4-3\omega^2)\left(3\cosh^{-1}\left(\frac{v_3\sqrt{\alpha/d^2}-1}{r_w}\right)-4\right)}.$$

We also apply $\omega = \omega_0$ when constructing the phase diagrams shown in Fig 4(a) of the main text.