Universal relations between thermoelectrics and noise in mesoscopic transport across a tunnel junction

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We develop a unified theory of weakly probed differential observables for currents and noise in transport experiments. Our findings uncover a set of universal transport relations between thermoelectric and noise properties of a system probed through a tunnel contact, with the Wiedemann-Franz law being just one example of such universality between charge and heat currents. We apply this theory to various quantum dot systems, including multichannel Kondo, quantum Hall and Sachdev-Ye-Kitaev quantum dots, and demonstrate that each of the microscopic theories is characterized by a set of universal relations connecting conductance and thermoelectrics with noise. Violations of these relations indicate additional energy scales emerging in a system.

Quantum transport probes are widely used for obtaining information about mesoscopic systems. Thermoelectric and shot noise measurements are employed for detecting signatures of Kondo physics [1–3] and characterization of quantum dots [4, 5], serve as direct quantum information probes [6]. In heavy fermion materials and strange metals, they provide experimental [7, 8] and theoretical [9–11] insights into the nature of the charge carriers and interactions dominating in different regimes. In holographic systems, the transport observables are related to thermodynamics of black holes [12–14]. In addition to widely used thermoelectric and shot noise measurements [15], delta-T noise, arising purely due to temperature bias, has been recently measured experimentally [16–18], which opens possibilities for utilizing it as an experimental probe. Nevertheless, it is often experimentally challenging to probe differential transport observables, as this procedure requires a simultaneous control over different biases applied to a system [1]. Furthermore, such probes may give ambiguous results about the microscopic properties of the system. For instance, violations of the Wiedemann-Franz (WF) law may have various underlying reasons [19–21].

In this *Letter*, we develop a linear response theory for transport through a tunnel contact that treats all currents and noise on the same footing. This theory provides a set of universal relations between different transport observables, leading to one-to-one connections between various Fano factors, establishes regimes of equivalence between thermoelectric and noise measurements. The obtained universality allows for experimental flexibility regarding a choice of observables for obtaining the same information about the probed system. At the same time, the breaking of the established universal relations provides a more nuanced information about the microscopic properties of the system comparing to existing analysis. After presenting the general theory, we illustrate its application to a wide variety of systems, including multichannel Kondo devices, quantum Hall systems, holographic systems (Sachdev-Ye-Kitaev (SYK) model and its generalizations [22, 23]). Generalizing the concept of the Lorenz number and the Lorenz ratio, we show that each of the microscopic theories results into a specific set of universal constants relating different transport observables to each other. Violations of these relations point to additional energy scales that effectively emerge within the system.

Model. – We consider a quantum system (S) weakly coupled through a tunnel junction to a metallic lead (L). This is a typical setup for tunneling spectroscopy and transport probes widely used in mesoscopic and nanoscopic experiments [1–5, 24–43]. The temperature of the system is T, while the temperature of the lead is $T + \Delta T$. There is a voltage bias ΔV between them, both ΔV and ΔT are assumed to be small enough, so the linear response theory can be justified. For now, we do not specify the nature of the probed system. It can be another metallic lead, a quantum dot of various types (e.g., a multiterminal Kondo device, an SYK quantum dot), or some extended non-Fermi liquid system. Throughout the paper, we put $\hbar = e = k_B = 1$.

In the following, we consider transport of charge and heat across the tunnel junction, induced due to the voltage and temperature bias. Within the considered setup, there are two types of current - charge current I_c and heat current I_h. For these two currents, there are three possible types of current-current correlations that constitute noise: charge noise S_c, heat noise S_h and mixed noise S_m (that accounts for correlations between charge and heat currents) [44–46]. This consideration can easily be extended to other types of transport (e.g., spin transport [47]), multiterminal setups [48, 49], and higher moments of the corresponding currents.

Noise coefficients. – A steady state tunneling current and zero frequency noise power (which we further refer as noise for simplicity) across a tunnel junction connecting two systems depend on the local density of states (DoS) of the systems and their occupation numbers. To characterize the charge and heat transport through a tunneling barrier, it is convenient to introduce the energy-dependent transmission coefficient (more generally, imaginary part of the T-matrix)

$\mathcal{T}(\varepsilon)=2\pi|\lambda|^2\rho_L(\varepsilon)\rho_S(\varepsilon)$ [50–52], where λ is the tunneling amplitude through the barrier, $\rho_i(\varepsilon)\,(i=L,S)$ are the DoS of the lead and the system. Since the probe is a metallic lead, its DoS can be approximated (close to the Fermi level) by a constant value $\rho_L=(2\pi v_F)^{-1}$, where v_F is the Fermi velocity. In the weak tunneling limit one can consider the T-matrix in the lowest order of the tunneling amplitudes. Employing Keldysh Green's functions [53, 54], one can express the currents and the noise in this regime as

$$I_{c/h}=\int_{-\infty}^{\infty}d\varepsilon\,(\varepsilon-\Delta V)^n\,\mathcal{T}(\varepsilon)\left[n_L(\varepsilon-\Delta V,T+\Delta T)-n_S(\varepsilon,T)\right], \tag{1}$$

$$S_{c/m/h}=\int_{-\infty}^{\infty}d\varepsilon\,(\varepsilon-\Delta V)^l\,\mathcal{T}(\varepsilon)\left[n_L(\varepsilon-\Delta V,T+\Delta T)+n_S(\varepsilon,T)-2n_L(\varepsilon-\Delta V,T+\Delta T)n_S(\varepsilon,T)\right], \tag{2}$$

where $n=0,1$ for charge I_c and heat current I_h; $l=0,1,2$ for charge S_c, mixed S_m and heat noise S_h, correspondingly [49, 55]. ΔV and ΔT are voltage and temperature drops across the tunnel junction. $n(\varepsilon,T)=\left(e^{\frac{\varepsilon-\mu}{T}}+1\right)^{-1}$ is the Fermi-Dirac function (μ is the chemical potential).

The linear response theory of thermoelectric transport expresses currents via the transport coefficients that capture the system's response to small biases. These responses can be expressed in terms of the Onsager transport integrals [56]

$$\begin{pmatrix} I_c \\ I_h \end{pmatrix}=\begin{pmatrix} \mathcal{L}_0 & \frac{1}{T}\mathcal{L}_1 \\ \mathcal{L}_1 & \frac{1}{T}\mathcal{L}_2 \end{pmatrix}\begin{pmatrix} \Delta V \\ \Delta T \end{pmatrix}, \tag{3}$$

$$\mathcal{L}_n=\frac{1}{4T}\int_{-\infty}^{\infty}d\varepsilon\mathcal{T}(\varepsilon)\frac{\varepsilon^n}{\cosh^2\left(\frac{\varepsilon}{2T}\right)},\; n=0,1,2. \tag{4}$$

This defines charge conductance $G=\mathcal{L}_0$, thermoelectric coefficient $G_T=\frac{1}{T}\mathcal{L}_1$, heat conductance $G_H=\frac{1}{T}\mathcal{L}_2$, Peltier coefficient $\Pi=\frac{\mathcal{L}_1}{\mathcal{L}_0}$, Seebeck coefficient (a.k.a. thermopower) $\mathcal{S}=\frac{\mathcal{L}_1}{T\mathcal{L}_0}$, thermal conductance $K=G_H-TG_T\mathcal{S}$ [49]. With the same approach, we generalize this idea and introduce the *noise coefficients*

$$\mathcal{N}_n=\frac{1}{4T}\int_{-\infty}^{\infty}d\varepsilon\mathcal{T}(\varepsilon)\frac{\varepsilon^n\tanh\left(\frac{\varepsilon}{2T}\right)}{\cosh^2\left(\frac{\varepsilon}{2T}\right)},\; n=0,1,2,3. \tag{5}$$

Within the linear response, each type of noise can be decomposed into three components: equilibrium (a.k.a. Johnson-Nyquist) noise $S^{JN}_{c,m,h}$, which is present at a finite temperature even without any bias; shot noise $\delta S^{SN}_{c,m,h}\equiv\left.\frac{\partial S_{c,m,h}}{\partial\Delta V}\right|_{\Delta T=0}$ induced due to the voltage bias; and delta-T noise $\delta S^{\Delta T}_{c,m,h}\equiv\left.\frac{\partial S_{c,m,h}}{\partial\Delta T}\right|_{\Delta V=0}$ induced due to the temperature bias [57]. It brings us to nine different types of noise, but only four of them are independent. Using Eqs. (4) and (5), all types of noise in the weak tunneling regime can be brought into a form similar to Eq. (3),

$$\begin{pmatrix} S_c \\ S_m \\ S_h \end{pmatrix}=\begin{pmatrix} 2T\mathcal{L}_0 & \mathcal{N}_0 & \frac{1}{T}\mathcal{N}_1 \\ 2T\mathcal{L}_1 & \mathcal{N}_1-2T\mathcal{L}_0 & \frac{1}{T}\mathcal{N}_2 \\ 2T\mathcal{L}_2 & \mathcal{N}_2-4T\mathcal{L}_1 & \frac{1}{T}\mathcal{N}_3 \end{pmatrix}\begin{pmatrix} 1 \\ \Delta V \\ \Delta T \end{pmatrix}. \tag{6}$$

Note that the Johnson-Nyquist noise is fully determined by the transport coefficients $\mathcal{L}_n$ [46, 58], and none of the mixed noise S_m components is independent, they are related to charge and heat noise as

$$\delta S^{SN}_m=T\,\delta S^{\Delta T}_c-S^{JN}_c,\; \delta S^{SN}_h=T\delta S^{\Delta T}_m-2S^{JN}_m \tag{7}$$

Detailed derivations of these results are given in [59]. The expressions of Eq. (7) extend the Onsager reciprocity relations [60] to noise and have the same origin.

For our purpose, it will be more convenient to represent the transmission coefficient as a function of real time. For that, one switches to the time domain and makes an analytic continuation that accounts for finite temperature, as detailed in [61],

$$\mathcal{T}(\varepsilon)=-\frac{1}{\pi}\cosh\frac{\varepsilon}{2T}\int_{-\infty}^{\infty}dt\mathcal{T}\left(\frac{1}{2T}+it\right)e^{i\varepsilon t}. \tag{8}$$

This expression can be substituted to Eqs. (4) and (5) by integrating out the energy dependence and expressing all transport coefficients through integrals in the time domain, which are calculated explicitly in [59]).

The transport integrals of Eq. (4) depend on the spectral symmetry of the system due to the structure of their kernels, which are either symmetric ($n=0,2$) of antisymmetric ($n=1$) functions of energy. For instance, the differential thermopower, which is proportional to $\mathcal{L}_1$, is zero for particle-hole symmetric systems [62]. The same symmetry considerations apply to the noise coefficients Eq. (5). Coefficients $\mathcal{L}_0$, $\mathcal{L}_2$, $\mathcal{N}_1$, $\mathcal{N}_3$ depend only on the symmetric part of $\mathcal{T}(\varepsilon)$, while coefficients $\mathcal{L}_1$, $\mathcal{N}_0$, $\mathcal{N}_2$ depend only on the antisymmetric part of $\mathcal{T}(\varepsilon)$.

Lorenz ratio. – There is a well-known relation between coefficients $\mathcal{L}_2$ and $\mathcal{L}_0$ that constitutes the WF law (conventionally, it is represented as a relation between K and G that holds for small thermoelectric coefficients and is violated by the strong thermoelectric effects [49]). As we show in the following, as long as the ratio between these two transport coefficients is universal, there are universal ratios between all coefficients within the symmetric and antisymmetric categories.

The WF law is known to hold (up to subleading finite temperature corrections) whenever charge and heat are transferred by the same carriers, which (in the zero-T limit) is equivalent to the condition of existing a well-defined Fermi surface [20], and scattering processes in the probed system are elastic (so there are no competition between the elastic and inelastic processes) [7]. The generalized WF law, $\frac{\mathcal{L}_2}{T^2\mathcal{L}_0} = L_0 R_L$, goes beyond the Fermi liquid (FL) paradigm, and constitutes the same relation between the heat conductance and the charge conductance up to a modified proportionality constant [63]. Here, $L_0 = \frac{\pi^2}{3}$ is the Lorenz number, and R_L is the Lorenz ratio that accounts for the deviations from the FL relation ($R_L = 1$ for the FL).

The Lorenz ratio can be written as

$$R_L = \frac{6\int_{-\infty}^{\infty} dt \cosh^{-3}(\pi T t)\, \mathcal{T}\left(\frac{1}{2T} + it\right)}{\int_{-\infty}^{\infty} dt \cosh^{-1}(\pi T t)\, \mathcal{T}\left(\frac{1}{2T} + it\right)} - 3. \qquad (9)$$

The generalized WF law holds at finite temperatures (meaning that R_L is a constant universal number over a certain window of temperatures) as long as $\mathcal{T}$ in Eq. (9) scales as a function of a single parameter (the temperature). In general, an interplay of several energy scales for $\mathcal{T}$ and a competition between elastic and inelastic processes break this universality and violate the generalized WF law [21]. As an example, the WF law may be fulfilled at low and high temperatures, but broken at the scale of mesoscopic fluctuations [19]. A quantum dot below the Kondo temperature (which is proportional to the charging energy) with neglidgible level spacing δ (so $T \gg \delta$ [64]) is characterized by the transmission coefficient $\mathcal{T}\left(\frac{1}{2T} + it\right) \sim \frac{1}{\cosh^{\alpha}(\pi T t)}$. This structure of the transmission coefficient covers a broad variety of systems. E.g., $\alpha = 1$ for FL, $\alpha \in [1, 3]$ for a N-channel Kondo quantum dot [65] or a quantum Hall simulator [66] ($N \in [1, \infty)$, $\alpha = 1 + 2/N$). The same structure with $\alpha = \frac{1}{2}$ is applicable to a quantum dot with SYK interactions in the conformal regime [67]. In all these cases, the cosh-like structure of the T-matrix stems from the conformal symmetry [68]. The Lorenz ratio in this case reads

$$R_L = \frac{3\alpha}{2 + \alpha}. \qquad (10)$$

We illustrate these Lorenz ratios in Fig. 1. The Lorenz ratio for Kondo and quantum Hall devices is always enhanced comparing to the FL due to the Anderson's orthogonality catastrophe [65, 70–72]. In contrary, the reduced Lorenz ratio is realized for the SYK model, and an arbitrary small Lorenz ratio can be achieved within the double scaled SYK (DSSYK) model [73, 74], a q-body generalization of the SYK model (its saddle-point Green's function scales as $\cosh^{-\frac{2}{q}}(\pi T t)$, so $\alpha = \frac{2}{q}$), including $R_L = 0$ at $q \to \infty$. In this limit, DSSYK belongs to the same universality class as the Random Matrix Theory [75]. Furthermore, it can happen that the transmission coefficient is fully determined by inelastic processes, as is the case for an SYK dot deep within the Coulomb blockade [67]. In this case the universality of the generalized WF law can be restored whenever a single energy parameter governs the T-matrix (or at least its symmetric part) behavior over some range of temperatures (the detailed analysis is provided in [59]). We illustrate it for the SYK model in conformal and Schwarzian regimes in Fig. 1 (for comparison, we put the corresponding points at the same α values that are realized for their conformal counterparts). Thought the transmission coefficient of the system departs from the cosh-like behavior within the Schwarzian regime, the generalized WF law is still obeyed.

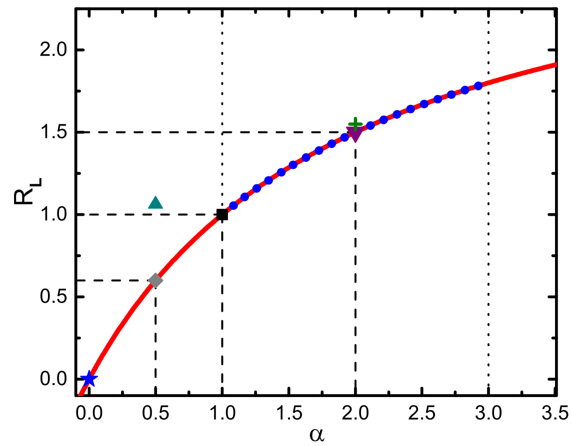

FIG. 1. Lorenz ratio given by Eq. (9). Red solid line - Lorenz ratio given by Eq. (10). Blue dotted line - a range of Lorenz ratios for an N-channel Kondo/quantum Hall simulator; vertical dotted lines at $\alpha = 1$ and $\alpha = 3$ depict the range of α for the Kondo/quantum Hall simulator (they values reproduce the results obtained in [65, 66, 69] for these systems). Black square corresponds to the Fermi liquid regime. Gray diamond - SYK in the conformal regime. Downward violet triangle - inelastic tunneling regime for the conformal SYK. Blue star - large-q conformal regime of the double-scaled SYK. Upward cyan triangle - SYK dot in the Schwarzian regime. Green cross - Schwarzian SYK regime with only inelastic tunneling. Dashed lines are used as eyeguides.

Within the elastic tunneling regime, the Lorenz ratio effectively probes the DoS in the vicinity of the zero energy. For FL, the DoS saturates in this area to some finite value and can be approximated by a constant (which results in $\alpha = 1$). If DoS is suppressed due to strong electron-electron correlations, approaching zero at zero energy, the Lorenz ratio is boosted, as is the case for the multichannel Kondo simulators and quantum Hall devices (resulting in $\alpha > 1$). The conformal regime of the (double scaled) SYK model provides an opposite example - the DoS diverges in this case, leading to $\alpha < 1$ and $R_L > 1$. The SYK model provides a particular well-controlled example here, since its DoS switches between $\varepsilon^{-\frac{1}{2}}$ divergence within the conformal regime to $\varepsilon^{\frac{1}{2}}$ convergence within the Schwarzian regime, and the corresponding Lorenz ratio immediately changes from $R_L = \frac{3}{5} < 1$ to $R_L \simeq 1.06 > 1$ (gray diamond and cyan upward triangle in Fig. 1). The vanishing Lorenz ratio $R_L = 0$ for the DSSYK at $q \to \infty$ arises from ε^{-1} divergence of the DoS [76]. Generally speaking, other scaling laws for divergent DoS may be realized. For instance, some conformal field theories exhibit the logarithmic DoS divergency [77]. The van Hove singularities [78] result into the logarithmic or power-law divergencies [79–82] of the DoS (see also connections to multicritical Lifshitz points [83]), that strongly affect their transport properties [84–87]. The emergent noise relations for these theories may provide tools for distinguishing different divergencies in experiments, but such a consideration is beyond the scope of this work.

Universal noise relations. – As long as the universality of Eq. (9) holds (i. e., the generalized WF law is obeyed), one can construct similar relations involving other transport coefficients. For instance, one can define the ratio between the delta-T charge noise and the electric conductance. This is identical to the ratio of the following coefficients $\delta S_c^{\Delta T}/G = \frac{\mathcal{N}_1}{T\mathcal{L}_0} = L_1 R_C^{\Delta T}$. Here we denoted $L_1 = 1$ as the first extended Lorenz number for FL, $R_C^{\Delta T}$ is the deviation of this ratio from the FL value,

$$R_C^{\Delta T} = 2 - \frac{2\int_{-\infty}^{\infty} dt\, \pi T t \frac{\sinh(\pi T t)}{\cosh^2(\pi T t)} \mathcal{T}\left(\frac{1}{2T} + it\right)}{\int_{-\infty}^{\infty} dt \cosh^{-1}(\pi T t)\, \mathcal{T}\left(\frac{1}{2T} + it\right)}. \quad (11)$$

This ratio has the same universality range and applicability conditions as R_L. For the $\mathcal{T}\left(\frac{1}{2T} + it\right) \sim \cosh^{-\alpha}(\pi T t)$ transmission coefficient, it becomes

$$R_C^{\Delta T} = \frac{2\alpha}{1+\alpha}. \quad (12)$$

We further explore universality of the $R_C^{\Delta T}$ ratio for elastic and inelastic tunneling regimes, as well as breaking of this universality, for the SYK dot in [59].

We introduce further $\frac{\delta S_h^{\Delta T}}{T^2 G} = \frac{\mathcal{N}_3}{T^3 \mathcal{L}_0} = L_2 R_H^{\Delta T}$, where $L_2 = \pi^2$ is the second extended Lorenz number for FL. $R_H^{\Delta T}$ is the deviation of this ratio from the FL value,

$$R_H^{\Delta T} = \frac{2\alpha + 6\alpha^2}{3 + 4\alpha + \alpha^2}. \quad (13)$$

With Eqs. (10), (11) and (13), one can trivially find the three remaining ratios between the symmetric coefficients ($\mathcal{L}_2$, $\mathcal{N}_1$ and $\mathcal{N}_3$).

The same type of universal relations holds between the antisymmetric coefficients. Coefficients $\mathcal{L}_1$, $\mathcal{N}_0$ and $\mathcal{N}_2$ do not vanish only if the system is away from the hole-particle symmetric regime, this deviation can be characterized by the spectral asymmetry parameter $\mathcal{E}$ [59]. In most cases, all the antisymmetric coefficients are proportional to $\sim \mathcal{E}$ for weak particle-hole asymmetry. Nevertheless, this proportionality cancels in their ratios (up to $O(\mathcal{E}^2)$ corrections), which remain finite even in the $\mathcal{E} \to 0$ limit [88]. These ratios can be found by introducing small spectral asymmetry into the transmission

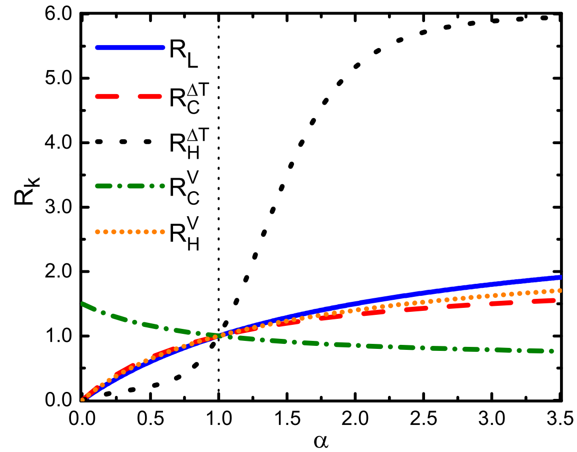

FIG. 2. Extended Lorenz ratios as functions of α. R_k stands for R_L (solid blue line), $R_C^{\Delta T}$ (dashed red line), $R_H^{\Delta T}$ (dotted black line), R_C^V (dash-dotted green line), R_H^V (short-dotted orange line). All ratios are normalized by the extended Lorenz numbers L_i, $i = 0,..,4$ such that $R_k = 1$ at $\alpha = 1$ (Fermi liquid regime, thin vertical dotted line).

coefficient $\mathcal{T}\left(\frac{1}{2T} + it\right)$. This generates odd-in-$t$ terms in $\mathcal{T}$, so the antisymmetric coefficients are nonzero. Subsequently, one can take the $\mathcal{E} \to 0$ limit. In addition to the even part (which does not play a role for the antisymmetric coefficients), there is the odd component of the transmission coefficient that contributes to these ratios $\mathcal{T}_{odd}\left(\frac{1}{2T} + it\right) \sim \frac{i\mathcal{E}t}{\cosh^{\alpha}(\pi T t)}$ [89], and the vanishing $\mathcal{E}$ cancels in the nominator and denominator, providing universal asymptotic relation.

For the ratio $\frac{\delta S_c^{SN}}{G_T} = \frac{T\mathcal{N}_0}{\mathcal{L}_1} = L_3 R_C^V$, one has $L_3 = \frac{1}{3}$,

$$R_C^V = \frac{6}{\pi^2} \frac{\int_{-\infty}^{\infty} dx\, x^2 \cosh^{-1-\alpha}(x)}{\int_{-\infty}^{\infty} dx\, x \sinh(x) \cosh^{-2-\alpha}(x)} = \quad (14)$$
$$\frac{6}{\pi^2} \frac{{}_4F_3\left(\frac{1+\alpha}{2}, \frac{1+\alpha}{2}, \frac{1+\alpha}{2}, 1+\alpha; \frac{3+\alpha}{2}, \frac{3+\alpha}{2}, \frac{3+\alpha}{2}; -1\right)}{\sqrt{\pi}(1+\alpha)^{-1} 2^{-\alpha} \Gamma^{-2}\left(\frac{1+\alpha}{2}\right) \Gamma^3\left(\frac{3+\alpha}{2}\right) \Gamma^{-1}\left(\frac{2+\alpha}{2}\right)},$$

where $\Gamma(\cdot)$ is the Gamma function, ${}_4F_3(\cdot;\cdot;\cdot)$ is the generalized hypergeometric function.

The ratio $\frac{\mathcal{N}_2}{T\mathcal{L}_1} = L_4 R_H^V$, contains $L_4 = \frac{12+\pi^2}{9} \simeq 2.43$,

$$L_4 R_H^V = \frac{4\int_{-\infty}^{\infty} dx\, x^2 \cosh^{-3-\alpha}(x)}{\int_{-\infty}^{\infty} dx\, x \sinh(x) \cosh^{-2-\alpha}(x)} - \pi^2 L_3 R_C^V + 4. \quad (15)$$

Using this expression, we can define $\frac{\delta S_h^{SN}}{T^2 G_T} = L_4 R_H^V - 4$. The explicit form of R_H^V is a cumbersome combination of the generalized hypergeometric functions, so we rather plot the exact value of R_H^V in Fig. 2 along with all other extended Lorenz ratios. All these ratios are reproduced numerically from the exact expressions for $\alpha = \frac{1}{2}$ in [59].

Having established these relations between the symmetric and antisymmetric coefficients, we can identify other characteristics of transport that acquire certain universality. Let us consider the Fano factor of charge transport, which constitutes a ratio between the shot noise and charge current due to the voltage bias, $F_c^{SN} = S_c/I_c$. Note that in the zero temperature limit this ratio becomes exactly unity, which is a universal property of a weak tunneling contact - this can be seen from Eqs. (1) and (2) for an arbitrary $\mathcal{T}(\varepsilon)$, as the Fermi-Dirac distribution becomes an idempotent function $n_L(\varepsilon+\Delta V, T) \underset{T\to 0}{=} \theta(\varepsilon+\Delta V)$. For the linear response regime, which is valid at $\Delta T, \Delta V \ll T$, let us subtract the equilibrium component of the noise, defining $F_c^{SN} = \delta S_c^{SN}/G$. We introduce the delta-T Fano factor as the ratio between the delta-T charge noise and the charge current induced due to temperature imbalance, $F_c^{\Delta T} = \delta S_c^{\Delta T}/G_T$. We have

$$F_c^{SN} F_c^{\Delta T} = \frac{\mathcal{N}_1}{\mathcal{L}_0}\frac{\mathcal{N}_0}{\mathcal{L}_1} = L_1 L_3 R_C^{\Delta T} R_C^V = const. \quad (16)$$

We can further use the Fano factors for the heat current in the same scheme ($F_h = S_h/TI_h$), obtaining

$$F_h^{SN} F_h^{\Delta T} = \frac{\mathcal{N}_2 - 4\mathcal{L}_1}{\mathcal{L}_1}\frac{\mathcal{N}_3}{\mathcal{L}_2} = \frac{L_2 R_H^{\Delta T}}{L_0 R_L}\left(L_4 R_H^V - 4\right). \quad (17)$$

Furthermore, we can write the thermopower $\mathcal{S}$ as

$$\mathcal{S} = \frac{G_T}{G} = \frac{F_c^{SN}}{L_3 R_C^V} = \frac{1}{L_3 R_C^V}\frac{\mathcal{N}_0}{\mathcal{L}_0}. \quad (18)$$

Within the universality regime, the shot noise Fano factor provides us with the same information as the Seebeck coefficient. As is evident from Eqs. (16) and (17), a suppression of the Fano factor for the shot noise leads to an enhancement of the Fano factor for the delta-T noise (see also [90]).

Conclusions. – The introduced *noise coefficients* provide a complete characterization of the zero-frequency noise power in mesoscopic and nanoscopic transport (Eq. (6)) in a system weakly coupled to a metallic contact. There are seven independent differential observables that characterize current and noise. Indeed, the Jonhson-Nyquist charge, mixed and heat noise exactly replicate the differential transport coefficients - conductance, thermoelectric coefficient and heat conductance, correspondingly. Moreover, the mixed shot noise and the charge delta-T noise, and the heat shot noise and the mixed delta-T noise are connected though the reciprocity relations Eq. (7), similar to the Onsager reciprocity relation that connects the Seebeck and Peltier coefficients. Therefore, the mixed noise does not have independent components. It leaves only four potentially independent noise coefficients, which can be treated on the same footing with the three independent transport coefficients characterizing currents. These coefficients are grouped into two categories: symmetric and antisymmetric ones with respect to their dependence on the spectral asymmetry. As we have shown, the generalized Wiedemann-Franz law, the relation that connects two symmetric transport coefficients, is just one of the relations that connect all the symmetric transport and noise coefficients with each other as long as the transmission coefficient of the system can be approximated as a single-parameter function. The same consideration holds for the antisymmetric coefficients, so in this universality regime all transport and noise features of the system are determined by just two independent observables. This redundancy of the noise features can be used to extract the same information about the system from different measurements. In particular, the thermoelectric measurements can be substituted by the shot noise measurements, which require only the voltage bias instead of tuning between the voltage and temperature biases. The established relations between the Fano factors signify that the delta-T noise can be a valuable experimental signature in cases when the shot noise is suppressed. These findings provide new tools for experimental studies of strongly correlated systems and understanding properties of non-Fermi liquid materials, quantum information probes in nanotransport [91–94], studies of holographic systems [95], design of quantum heat engines [55, 96, 97], insights into quantum Hall and fractional quantum Hall devices [66, 98–106], and thermoelectric experiments in cold atoms [107, 108]. An interesting open question is how the relations between the noise and thermoelectrics behave beyond the weak tunneling limit, especially in the opposite limit of an open quantum point contact (QPC). A rigorous quantum field theoretical QPC investigation of the noise including large scale numerical simulations [109] can be carried in future studies.

Acknowledgements – We thank Liliana Arrachea, Diana Dulić, Dmitry Efremov, Deepak Karki, Thierry Martin, Thanh Nguyen, Jerome Rech, Subir Sachdev, Herre van der Zant for stimulating discussions. A.I.P. appreciates the hospitality of ICTP Trieste, where part of this work was carried out. The work of M.N.K. is conducted within the framework of the Trieste Institute for Theoretical Quantum Technologies (TQT).

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